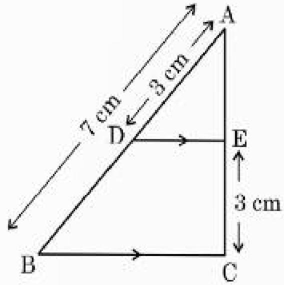
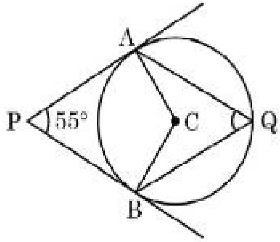
MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 30/6/1)

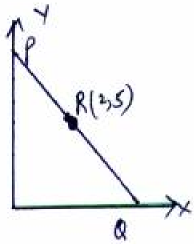
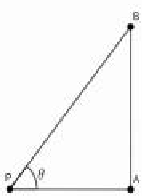
Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	If $p^2 = \frac{32}{50}$, then p is a/an (A) whole number (B) integer (C) rational number (D) irrational number	
Sol.	(C) rational	1
2.	The distance of the point (– 6, 8) from *x*-axis is (A) 6 units (B) – 6 units (C) 8 units (D) 10 units	
Sol.	(C)8 units	1
3.	The number of quadratic polynomials having zeroes –5 and –3 is (A) 1 (B) 2 (C) 3 (D) more than 3	
Sol.	(D)more than 3	1
4.	The point of intersection of the line represented by $3x - y = 3$ and y-axis is given by (A) (0, –3) (B) (0, 3) (C) (2, 0) (D) (–2, 0)	
Sol.	(A) (0, – 3)	1

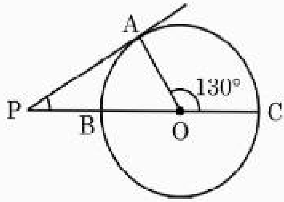
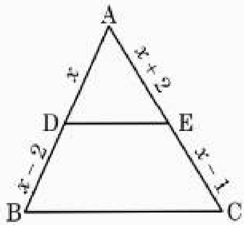
5.	The circumferences of two circles are in the ratio 4 : 5. What is the ratio of their radii ? (A) 16 : 25 (B) 25 : 16 (C) $2:\sqrt{5}$ (D) 4 : 5	
Sol.	(D) 4 : 5	1
6.	If α and β are the zeroes of the polynomial $x^2 - 1$, then the value of $(\alpha + \beta)$ is (A) 2 (B) 1 (C) –1 (D) 0	
Sol.	(D) 0	1
7.	$\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$, in simplified form, is : (A) $\tan^2\theta$ (B) $\sec^2\theta$ (C) 1 (D) –1	
Sol.	(D) – 1	1
8.	If $\Delta PQR \sim \Delta ABC$; PQ = 6 cm, AB = 8 cm and the perimeter of ΔABC is 36 cm, then the perimeter of ΔPQR is (A) 20.25 cm (B) 27 cm (C) 48 cm (D) 64 cm	
Sol.	(B) 27 cm	1
9.	If the quadratic equation $ax^2 + bx + c = 0$ has two real and equal roots, then 'c' is equal to (A) $\frac{-b}{2a}$ (B) $\frac{b}{2a}$ (C) $\frac{-b^2}{4a}$ (D) $\frac{b^2}{4a}$	
Sol.	(D) $\frac{b^2}{4a}$	1

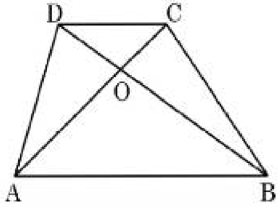
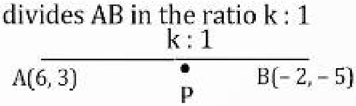
10.	In the given figure, DE‖BC. If AD = 3 cm, AB = 7 cm and EC = 3 cm, then the length of AE is  (A) 2 cm (B) 2.25 cm (C) 3.5 cm (D) 4 cm	
Sol.	(B) 2·25 cm	1
11.	A bag contains 5 pink, 8 blue and 7 yellow balls. One ball is drawn at random from the bag. What is the probability of getting neither a blue nor a pink ball ? (A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{7}{20}$ (D) $\frac{13}{20}$	
Sol.	(C) $\frac{7}{20}$	1
12.	The volume of a right circular cone whose area of the base is 156 cm^2 and the vertical height is 8 cm, is (A) 2496 cm^3 (B) 1248 cm^3 (C) 1664 cm^3 (D) 416 cm^3	
Sol.	(D) 416 cm^3	1
13.	3 chairs and 1 table cost ₹ 900; whereas 5 chairs and 3 tables cost ₹ 2,100. If the cost of 1 chair is ₹ x and the cost of 1 table is ₹ y, then the situation can be represented algebraically as (A) $3x + y = 900$, $3x + 5y = 2100$ (B) $x + 3y = 900$, $3x + 5y = 2100$ (C) $3x + y = 900$, $5x + 3y = 2100$ (D) $x + 3y = 900$, $5x + 3y = 2100$	
Sol.	(C) $3x + y = 900, 5x + 3y = 2100$	1

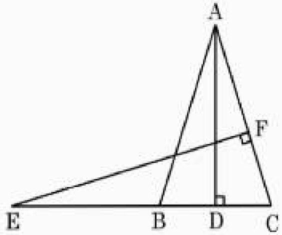
14.	In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of $\angle AQB$ is  (A) $62\frac{1}{2}°$ (B) 125° (C) 55° (D) 90°	
Sol.	(A) $62\frac{1}{2}°$	1
15.	A card is drawn at random from a well shuffled deck of 52 playing cards. The probability of getting a face card is (A) $\frac{1}{2}$ (B) $\frac{3}{13}$ (C) $\frac{4}{13}$ (D) $\frac{1}{13}$	
Sol.	(B) $\frac{3}{13}$	1
16.	If θ is an acute angle of a right angled triangle, then which of the following equation is **not** true ? (A) $\sin\theta \cot\theta = \cos\theta$ (B) $\cos\theta \tan\theta = \sin\theta$ (C) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ (D) $\tan^2\theta - \sec^2\theta = 1$	
Sol.	(D) $\tan^2\theta - \sec^2\theta = 1$	1
17.	If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then (A) a = –7, b = –1 (B) a = 5, b = –1 (C) a = 2, b = – 6 (D) a = 0, b = – 6	
Sol.	(D) a = 0, b = - 6	1
18.	If the sum of the first n terms of an A.P be $3n^2 + n$ and its common difference is 6, then its first term is (A) 2 (B) 3 (C) 1 (D) 4	
Sol.	(D) 4	1

	Assertion – Reason Based Questions : In question numbers **19** and **20**, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option out of the following : (A) Both Assertion (A) and Reason (R) are true; and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true; but Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.	
19.	**Statement A (Assertion) :** If $5 + \sqrt{7}$ is a root of a quadratic equation with rational co-efficients, then its other root is $5 - \sqrt{7}$. **Statement R (Reason) :** Surd roots of a quadratic equation with rational co-efficients occur in conjugate pairs.	
Sol.	(A)	1
20.	**Statement A (Assertion) :** For $0 < \theta \leq 90°$, $\operatorname{cosec} \theta - \cot \theta$ and $\operatorname{cosec} \theta + \cot \theta$ are reciprocal of each other. **Statement R (Reason) :** $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$	
Sol.	(A)	1
	SECTION – B Section – **B** consists of Very Short Answer (VSA) type of questions of **2** marks each.	
21(A).	(A) Show that 6^n can not end with digit 0 for any natural number 'n'.	
Sol.	If 6^n ends with digit 0, it would be divisible by 5. So, prime factorization of 6^n would contain 5. But $6^n = (2 \times 3)^n$, the only prime factorization of 6^n are 2 and 3 as per fundamental theorem of Arithmetic . There is no other prime in the factorization of 6^n. So, there is no natural number n for which 6^n ends with digit zero.	2
	OR	
21(B)	Find the HCF and LCM of 72 and 120.	

Sol.	$72 = 2^3 \times 3^2$ $120 = 2^3 \times 3 \times 5$ HCF = 24 LCM=360	 1 1
22.	A line intersects y-axis and x-axis at point P and Q, respectively. If R(2, 5) is the mid-point of line segment PQ, then find the coordinates of P and Q.	
Sol.	 Let the coordinates of P and Q be (0, y) and (x, 0) respectively. ∵ R(2, 5) is the midpoint of PQ $\frac{0+x}{2} = 2$ and $\frac{y+0}{2} = 5$ $\therefore x = 4, y = 10$ P(0, 10) and Q(4, 0)	$\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$
23.	Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the sun is such that $\tan\theta = \frac{6}{7}$.	
Sol.	 Pole of height AB = 18 m AP = length of shadow In Δ APB, $\tan\theta = \frac{18}{AP}$ $\frac{6}{7} = \frac{18}{AP}$ $\Rightarrow AP = 21$ m	 1 $\frac{1}{2}$ $\frac{1}{2}$

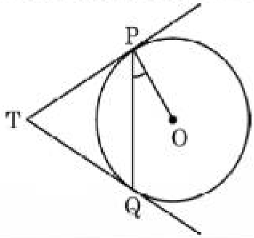
24.	In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If ∠AOC = 130°, then find the measure of ∠APB, where O is the centre of the circle.  	
Sol.	∠ AOB = 180° – 130° = 50° ∠ OAP = 90° ∴ ∠ APB = 180 – (50° + 90°) = 40°	$\frac{1}{2}$ $\frac{1}{2}$ 1
25(A).	In the given figure, ABC is a triangle in which DE∥BC. If AD = x, DB = $x-2$, AE = $x+2$ and EC = $x-1$, then find the value of x.  	
Sol.	In Δ ABC, DE ∥ BC $\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$ $x(x-1) = (x+2)(x-2)$ $x^2 - x = x^2 - 4 \Rightarrow x = 4$	1 1
	OR	

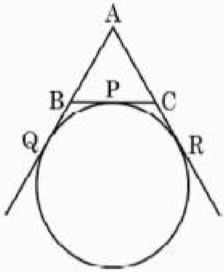
25(B).	Diagonals AC and BD of trapezium ABCD with AB‖DC intersect each other at point O. Show that $\frac{OA}{OC} = \frac{OB}{OD}$.  	
Sol.	In Δ AOB and Δ COD, $\angle OAB = \angle OCD$ $\angle OBA = \angle ODC$ Therefore, Δ AOB ~ Δ COD $\therefore \frac{OA}{OC} = \frac{OB}{OD}$	$1\frac{1}{2}$ $\frac{1}{2}$
	SECTION – C Section – **C** consists of Short Answer (SA) type of questions of **3** marks each.	
26.	Find the ratio in which the line segment joining the points A(6, 3) and B(–2, –5) is divided by *x*-axis.	
Sol.	Let P(x, 0) be the point on x axis which divides AB in the ratio k : 1  $\frac{-5k+3}{k+1} = 0 \Rightarrow k = \frac{3}{5}$ Ratio is 3 : 5	$\frac{1}{2}$ 2 $\frac{1}{2}$
27(A).	Find the HCF and LCM of 26, 65 and 117, using prime factorisation.	
Sol.	26= 13 x 2 65= 13 x 5 117= 13 x 3 x 3	1

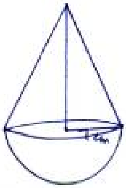
	$\therefore$ HCF = 13 LCM = 13 x 2 x 3 x 5 x 3 = 1170	1 1
	OR	
27(B)	Prove that $\sqrt{2}$ is an irrational number.	
Sol.	Let $\sqrt{2}$ be a rational number. $\therefore \sqrt{2} = \frac{p}{q}$, where $q \neq 0$ and let p & q be co-primes. $2q^2 = p^2 \Rightarrow p^2$ is divisible by 2 $\Rightarrow$ p is divisible by 2 $\Rightarrow p = 2a$, where 'a' is some integer ----- (i) $4a^2 = 2q^2 \Rightarrow q^2 = 2a^2 \Rightarrow q^2$ is divisible by 2 $\Rightarrow$ q is divisible by 2 $\Rightarrow q = 2b$, where 'b' is some integer ----- (ii) (i) and (ii) leads to contradiction as 'p' and 'q' are co-primes. $\therefore \sqrt{2}$ is an irrational number.	½ 1 ½ 1
28.	In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, then prove that ΔABD ~ ΔECF.  	
Sol.	ABC is an isosceles triangle $\therefore AB = AC \Rightarrow \angle B = \angle C$ In Δ ABD and Δ ECF, ∠ ADB = ∠ EFC ∠ ABD = ∠ ECF ∴ Δ ABD ~ Δ ECF	1 1 1

29(A).	The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers.	
Sol.	Let one number be x $\Rightarrow$ another number = 15 – x Therefore, $\frac{1}{x}+\frac{1}{15-x}=\frac{3}{10}$ $\frac{15-x+x}{x(15-x)}=\frac{3}{10} \Rightarrow 150 = 3x(15-x)$ $3x^2 - 45x + 150 = 0$ $x^2 - 15x + 50 = 0 \Rightarrow (x-10)(x-5) = 0$ $\Rightarrow x = 10, 5$ Numbers are 10, 5 or 5, 10	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR	
29(B).	If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2.	
Sol.	$x^2 - 7x + 10 = 0$ $\alpha + \beta = 7, \alpha\beta = 10$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 49 - 20 = 29$ $\alpha^2\beta^2 = (10)^2 = 100$ Quadratic Equation with roots α^2, β^2 is	 $\frac{1}{2}$ 1 1

	$\therefore x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$ i.e. $x^2 - 29x + 100 = 0$	$\frac{1}{2}$
30.	Prove that $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$.	
Sol.	LHS $= \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$ $= 1 + \cos A$ $= \frac{(1 - \cos A)(1 + \cos A)}{(1 - \cos A)}$ $= \frac{1 - \cos^2 A}{1 - \cos A}$ $= \frac{\sin^2 A}{1 - \cos A}$ = RHS	 1 1 1
31.	In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc.	
Sol.	$A = \frac{60}{360} \times \frac{22}{7} \times 21 \times 21 = 231\ cm^2$ Length of arc $= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$ $= 22$ cm	$1\frac{1}{2}$ $1\frac{1}{2}$
	SECTION – D Section – D consists of Long Answer (LA) type questions of **5** marks each.	

32(A).	Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$. 	
Sol.	$TP = TQ$ $\Rightarrow \angle TPQ = \angle TQP$ Let $\angle PTQ$ be θ $\Rightarrow \angle TPQ = \angle TQP = \frac{180° - \theta}{2} = 90° - \frac{\theta}{2}$ Now $\angle OPT = 90°$ $\Rightarrow \angle OPQ = 90° - (90° - \frac{\theta}{2}) = \frac{\theta}{2}$ $\angle PTQ = 2\angle OPQ$	1 $1\frac{1}{2}$ $1\frac{1}{2}$ 1
	OR	

32(B).	A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced at Q and R respectively. Show that $AQ = \frac{1}{2}$ (Perimeter of ΔABC).  	
Sol.	$AQ = AR$ $2AQ = AQ + AR$ $= AB + BQ + AC + CR$ $= AB + AC + (BP + CP)$ $= AB + AC + BC$ $AQ = \frac{1}{2}(AB + AC + BC) = \frac{1}{2}$ (Perimeter of $\triangle ABC$)	1 1 $\frac{1}{2}$ $\frac{1}{2}$ 1 1
33.	A solid is in the shape of a right-circular cone surmounted on a hemisphere, the radius of each of them being 7 cm and the height of the cone is equal to its diameter. Find the volume of the solid.	
Sol.	Radius of cone = radius of hemisphere = 7 cm $\therefore$Height of cone = 14 cm Volume of solid = Volume of hemisphere + volume of cone	 1

	 $=\frac{2}{3}\pi(7)^3+\frac{1}{3}\pi(7)^2 14$ $=\frac{1}{3}\times\frac{22}{7}\times 7\times 7(14+14)$ $=\frac{154}{3}\times 28=\frac{4312}{3}\,cm^2$ or $1437.33\ cm^2$	$1\frac{1}{2}+1\frac{1}{2}$ 1
34(A).	The ratio of the 11th term to the 18th term of an A.P. is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms.	
Sol.	$\frac{a+10d}{a+17d}=\frac{2}{3}$ $3a+30d=2a+34d \Rightarrow a=4d$ Therefore, $\frac{a+4d}{a+20d}=\frac{4d+4d}{4d+20d}=\frac{8d}{24d}=\frac{1}{3}$ $\frac{S_5}{S_{21}}=\frac{\frac{5}{2}[2a+4d]}{\frac{21}{2}[2a+20d]}=\frac{5[8d+4d]}{21[8d+20d]}$ $=\frac{5\times 12d}{21\times 28d}=\frac{5}{49}$ or $S_5:S_{21}=5:49$	1 1 1 1 1
	OR	
34(B).	If the sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.	
Sol.	$S_6=36 \Rightarrow \frac{6}{2}[2a+5d]=36$	**1**

	$\Rightarrow 2a + 5d = 12$ ------------ (1)	1
	$S_{16} = 256 \Rightarrow \frac{16}{2}[2a + 15d] = 256$	
	$\Rightarrow 2a + 15d = 32$ ------------ (2)	1
	Solving (1) and (2)	
	$d = 2$	1
	$a = 1$	
	$S_{10} = \frac{10}{2}[2(1) + 9(2)]$	
	$= 100$	1
35.	250 apples of a box were weighed and the distribution of masses of the apples is given in the following table :	

Mass (in grams)	80 – 100	100 – 120	120 – 140	140 – 160	160 – 180
Number of apples	20	60	70	x	60

(i) Find the value of x and the mean mass of the apples. **3**

(ii) Find the modal mass of the apples. **2**

Sol.

(i) $20 + 60 + 70 + x + 60 = 250$

$x = 250 - 210 = 40$ — 1

Mass	80 – 100	100 – 120	120 – 140	140 – 160	160 – 180	Total
No. of apples f_i	20	60	70	x = 40	60	250
x_i	90	110	130	150	170	
$x_i f_i$	1800	6600	9100	6000	10200	33700

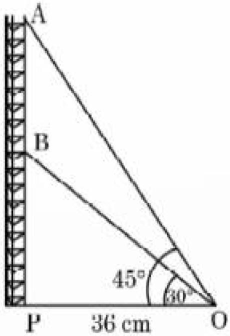
1 for correct table

Mean mass $= \frac{33700}{250} = 134{\cdot}8$ — 1

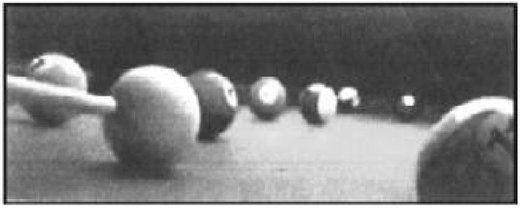
Mean mass = 134.8 g — $\frac{1}{2}$

	(ii) Modal class = 120-140 $$\text{Mode} = 120 + \frac{(70-60)}{(140-60-40)} \times 20$$ $= 125$ Hence modal mass = 125 gm	1 $\frac{1}{2}$
	SECTION – E **3** Case Study Based Questions. Each question is of **4** marks.	
36.	A coaching institute of Mathematics conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, there are 20 poor and 5 rich children, whereas in batch II, there are 5 poor and 25 rich children. The total monthly collection of fees from batch I is ₹ 9000 and from batch II is ₹ 26,000. Assume that each poor child pays ₹ x per month and each rich child pays ₹ y per month.  Based on the above information, answer the following questions : (i) Represent the information given above in terms of x and y. (ii) Find the monthly fee paid by a poor child. **OR** Find the difference in the monthly fee paid by a poor child and a rich child. (iii) If there are 10 poor and 20 rich children in batch II, what is the total monthly collection of fees from batch II ?	

Sol.	(i) $20x + 5y = 9000$ $5x + 25y = 26000$	1
	(ii) Solving the equations $x = 200$ Monthly fee paid by poor child = ₹200	2
	OR	
	(ii) getting $x=200$ and $y= 1000$	$1+\frac{1}{2}$
	Difference in the fee = $1000 - 200 =$ ₹ 800	$\frac{1}{2}$
	(iii) $10x + 20y = 10(200) + 20(1000)$ = ₹ 22000	1

37.	Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O. Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.  Based on the above information, answer the following questions : (i) Find the length of the wire from the point O to the top of Section B. (ii) Find the distance AB. **OR** Find the area of ΔOPB. (iii) Find the height of the Section A from the base of the tower.	
Sol.	(i) In Δ OBP, $\cos 30° = \frac{OP}{OB}$ $\frac{\sqrt{3}}{2} = \frac{36}{OB} \Rightarrow OB = \frac{72}{\sqrt{3}}$ $= 24\sqrt{3}$ cm	$\frac{1}{2}$ $\frac{1}{2}$

	(ii)In Δ OBP, $\tan 30° = \frac{PB}{36} \Rightarrow PB = \frac{36}{\sqrt{3}}$	
	$PB = 12\sqrt{3}$	1
	In Δ OAP, $\tan 45° = \frac{AP}{36} \Rightarrow AP = 36$ cm	$\frac{1}{2}$
	$AB = AP - PB = 36 - 12\sqrt{3} = 12(3 - \sqrt{3})$ cm	$\frac{1}{2}$
	OR	
	(ii)Area of Δ OPB $= \frac{1}{2} \times OP \times PB$	
	$= \frac{1}{2} \times 36 \times 12\sqrt{3} = 216\sqrt{3}\ \text{cm}^2$	1+1
	(iii) AP = 36 cm	1

38.	"Eight Ball" is a game played on a pool table with 15 balls numbered 1 to 15 and a "cue ball" that is solid and white. Of the 15 numbered balls, eight are solid (non-white) coloured and numbered 1 to 8 and seven are striped balls numbered 9 to 15.  The 15 numbered pool balls (no cue ball) are placed in a large bowl and mixed, then one ball is drawn out at random. Based on the above information, answer the following questions : (i) What is the probability that the drawn ball bears number 8 ? (ii) What is the probability that the drawn ball bears an even number ? **OR** What is the probability that the drawn ball bears a number, which is a multiple of 3 ? (iii) What is the probability that the drawn ball is a solid coloured and bears an even number ?	
Sol.	(i)P (drawing ball bearing number 8) = $\frac{1}{15}$	1
	(ii)Even numbers = 2, 4, 6, 8, 10, 12, 14	$\frac{1}{2}$
	No. of favourable outcomes = 7	
	P (even number ball) = $\frac{7}{15}$	$1\frac{1}{2}$
	OR	
	(ii)Multiples of 3 are 3, 6, 9, 12, 15	$\frac{1}{2}$

	No. of favourable outcomes = 5 $\therefore$ P(multiple of 3) $= \frac{5}{15} = \frac{1}{3}$ (iii) Solid colour and even number 2, 4, 6, 8 P(solid colour and bear an even no.) $= \frac{4}{15}$	$1\frac{1}{2}$ 1