V12M

Virudhunagar District Common Examinations
Common Half Yearly Examination - December 2022

Standard 12

MATHEMATICS

PART - I

Time allowed: 3 hours | Maximum Marks: 90

Note: i) Answer all the questions.
ii) Choose the most appropriate answer from given four alternatives.
iii) Write the option code and the corresponding answer.

1) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be sutch that $\lambda A^{-1} = A$ then λ is

a) 17 b) 14 c) 19 d) 21

2) If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = x+iy$ then $2.5.10\ldots.(1+n^2)$ is

a) 1 b) i c) $1+n^2$ d) x^2+y^2

3) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj } AB| =$

a) −40 b) 80 c) −60 d) −20

4) If $|Z_1| = 1$, $|Z_2| = 2$, $|Z_3| = 3$ and $|9Z_1Z_2+4Z_1Z_3+Z_2Z_3| = 12$ then the value of $|Z_1+Z_2+Z_3|$ is

a) 1 b) 2 c) 3 d) 4

5) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4x - 2\sin^2x + 1$ is

a) 2 b) 1 c) 4 d) ∞

6) If f and g are polynomials of degrees m and n respectively and if $h(x) = f \circ g(x)$ then the degree of h is .....

a) m+n b) m^n c) m^n d) nm

7) If $\cot^{-1}(x) = \frac{2\pi}{5}$, $x \in R$ the value of $\tan^{-1}x =$

a) $-\frac{\pi}{10}$ b) $\frac{\pi}{10}$ c) $\frac{\pi}{5}$ d) $-\frac{\pi}{5}$

8) If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ then x is equal to

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$

9) The eccentricity of the ellipse $(x-3)^2+(y-4)^2 = \frac{y^2}{9}$ is

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$

10) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

a) 3 b) −1 c) 1 d) 9

11) Distance from the origin $3x-6y+2z+7 = 0$

a) 0 b) 1 c) 2 d) 3

12) If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j}$, $\vec{c} = \vec{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$ then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

13) The tangent to the curve $y^2-xy+9 = 0$ is vertical when

a) $y = 0$ b) $y = \pm\sqrt{3}$ c) $y = \frac{1}{2}$ d) $y = \pm 3$

14) $\lim_{x \to 0} \frac{1-\cos x}{x^2} = ...$

a) 1 b) $\frac{1}{2}$ c) -1 d) $-\frac{1}{2}$

15) If $f(x) = \frac{x}{x+1}$ then its differential is given by

a) $-\frac{1}{(x+1)^2}dx$ b) $\frac{1}{(x+1)^2}dx$ c) $\frac{1}{x+1}dx$ d) $\frac{-1}{x+1}dx$

16) If $f(x, y, z) = xy+yz+zx$ then f_z-f_x is equal to

a) z-x b) x-z c) y-z d) y-x

17) The area of the region bounded by x-axis ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

a) πab b) $\frac{\pi}{ab}$ c) $\pi a^2 b$ d) πab^2

18) $\int_0^{\pi/2} \sin^3\theta \cos^5\theta \, d\theta$

a) $-\frac{1}{24}$ b) $\frac{1}{24}$ c) $\frac{1}{12}$ d) $-\frac{1}{12}$

19) The order and degree of the D.E. is $\frac{dy}{dx} = xy = \cot x$

a) (1, 1) b) (2, 2) c) (3, 3) d) (2, 4)

20) The solution of the D.E. $\frac{dy}{dx} = 2xy$ is

a) $y = Ce^{x^2}$ b) $y = 2x^2+c$ c) $y = Ce^{-x^2}+c$ d) $y = x^2+c$

Part-II

Note: Answer any seven questions only. Q.No. 30 is compulsory.

21) Solve (by Cramer's Rule) $5x+2y = 3$; $3x+2y=5$

22) Find the square root of $4+3i$

23) Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

24) Find the latus rectum and the vertex of the hyoperbola $9x^2-16y^2 = 144$

25) Determine whether the three vector $2\vec{i}+3\vec{j}+\vec{k}$, $\vec{i}-2\vec{j}+2\vec{k}$ and $3i+\vec{j}+3\vec{k}$ are coplanar.

26) Suppose $f(x)$ is a differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of $f(7)$

27) If the radius of a sphere, with radius 10 cm has to decrease by 0.1 cm approximately how much will its volume decrease.

28) Find the area of the region bounded between the parabole $y^2 = 4ax$ and its latus rectum?

29) Show that $y = 2x^2$ is the solution of the corresponding differential equation $xy^1 = 2y$.

30) Solve $2x^3-9x^2+10x-3 = 0$

Part-III

Note: i) Answer any seven questions only. ii) Q.No. 40 is compulsory.

31) Find the fourth roots of the unity.

32) If P and Q are the roots of the equation $x^2+px+q = 0$ and $x^2+p^1x+q^1 = 0$ have a common root. Show that it must be equal to $\frac{pq^1 - p^1q}{q-q^1}$ (or) $\frac{q\ \ q^1}{p^1-p}$

33) Evaluate: $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

34) A room 34 m long is constructed to be a whispering gallery. The room has an elliptical ceiling, If the maximum height of the ceiling is 8 m determine, where the foci an located.

35) Prove that $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\left[\vec{a}\vec{b}\vec{c}\right]^2$

36) Expand $\tan x - \pi/2 < x < \pi/2$ using Mclaurin Series.

37) Find the approximate value of $\sqrt[4]{15}$

38) Evaluate: $\int_0^1 x^3e^{-2x}\,dx$

39) Solve: $\cos x\frac{dy}{dx}+y\sin x = 1$

40) If $A = \begin{bmatrix}5 & 3\\ -1 & -2\end{bmatrix}$ show that $A^2-3A-7I_2 = 0$ then find A^{-1}.

Part-IV

Note: Answer all the questions:

41) Solve by Cramer's rule: $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}=1;\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0;\ \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

(OR)

Solve: $x\frac{dy}{dx} = y - \cos^2\left(y/x\right)$

42) i) Show that $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imagnary

ii) Solve: $z^3+27 = 0$

(OR)

If $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$ then pove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y} = 1$.

43) Solve $(x-4)(x-7)(x-2)(x+1) = 16$

(OR)

A ladder 17 meter long is learning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 meters from the wall (i) how fast is the top of the ladder moving down the wall? (ii) at what rate, the area of the triangle formed by the ladder wall and the floor is changing?

44) Solve: $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

(OR)

Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points (3, 6, −2), (−1, −2, 6) and (6, 4, −2)

45) Evaluate: $\int_{\pi/8}^{3\pi/8}\frac{1}{1+\sqrt{\tan x}}$

(OR)

Prove by Vector method, that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent.

46) Parabolic cable of a 60 m portion of the road bed of a suspension bridge are positioned as hanging bridge. Vertical cables are to be spaced every 6 m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.

(OR)

Find the dimension of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

47) Find the vertex, focus, equation of directrix and the length of the latus rectum of the parabola $x^2-2x+8y+17=0$

(OR)

The rate of increases in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours.

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