12

Reg. No.

Time : 3.00 hrs.

Half-Yearly Examination - 2022

MATHEMATICS

Max. Marks : 90

PART - A

I. Answer all the following **20 x 1 = 20**

1. If $A = \begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$ then $9I_2 - A$ is equal to a) $\frac{A^{-1}}{2}$ b) A^{-1} b) $2A^{-1}$ d) $3A^{-1}$
2. If $A^T . A^{-1}$ is symmetric, then A^2 is equal to a) A^T b) A^{-1} c) $(A^{-1})^2$ d) $(A^T)^2$
3. If $|z| = 1$ then the value of $\frac{1+z}{1+\bar{z}}$ is a) $\bar{z}$ b) z c) 1 d) $\frac{1}{z}$
4. z_1, z_2, z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is a) 0 b) 1 c) 2 d) 3
5. If α and β are the roots of the equation $x^2 - 6x + 7 = 0$ then the quadratic equation with roots $\frac{-1}{\alpha}, \frac{-1}{\beta}$ is a) $7x^2 + 6x + 1 = 0$ b) $7x^2 - 6x + 1 = 0$ c) $7x^2 - 6x - 1 = 0$ d) $7x^2 + 6x - 1 = 0$
6. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1 = 0$ is a) 1 b) 2 c) 4 d) ∞
7. If $\cot^{-1}(x) = \frac{2\pi}{5}$ for some $x \in R$ then the value of $\tan^{-1}(x)$ is a) $-\frac{\pi}{5}$ b) $\frac{\pi}{5}$ c) $\frac{-\pi}{10}$ d) $\frac{\pi}{10}$
8. If $\sin^{-1}(x) + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ then x is equal to a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$
9. The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is a) (4, 7) b) (4, 9) c) (7, 4) d) (9, 4)
10. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is a) $\frac{3}{4}$ b) $\frac{3}{5}$ c) $\frac{4}{5}$ d) $\frac{4}{3}$
11. If $\vec{a}$ and $\vec{b}$ are parallel vectors then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to a) –1 b) 0 c) 1 d) 2
12. The co-ordiantes of the point where the line $\vec{r} = (6\hat{i} - \hat{j} - 3\hat{k}) + t(-\hat{i} + 4\hat{k})$ meets the plane $\vec{r} . (\hat{i} + \hat{j} - \hat{k}) = 3$ are a) (5, –1, 1) b) (1, 2, –6) c) (7, –1, –7) d) (2, 1, 0)
13. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when a) $y = \frac{1}{2}$ b) $y = 0$ c) $y = \pm\sqrt{3}$ d) $y = \pm 3$
14. The maximum value of the function $f(x) = \sin x + \cos x$ is a) 2 b) $\sqrt{2}$ c) $\sqrt{3}$ d) 1
15. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31? a) 5 b) 31 c) $\frac{1}{5}$ d) $\frac{1}{31}$
16. $f(x, y, z) = xy + yz + zx$ then $f_x - f_x$ is equal toi a) z – x b) y – z c) x – z d) y – x
17. $\int_{-1}^{2} |x| \, dx$ is a) $\frac{7}{2}$ b) $\frac{5}{2}$ c) $\frac{3}{2}$ d) $\frac{1}{2}$
18. $\int_{0}^{\infty} e^{-3x} x^2 \, dx$ is a) $\frac{2}{27}$ b) $\frac{4}{27}$ c) $\frac{5}{27}$ d) $\frac{7}{27}$
19. The solution of $xdy + ydx = 0$ is a) $x^2 + y^2 = k$ b) $xy = k$ c) $x^2y = k$ d) $x^2 + y = k$
20. If sinx is the integrating factor of the linear differential equation $\frac{dy}{dx} + py = Q$ then P is a) cosx b) cotx c) tanx d) long (sinx)

PART - B

Answer any 7 of the following. Question No.30 is compulsory. **7 x 2 = 14**

21. Find the rank of $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$
22. Find the sum of the squares of the roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$
23. Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$

24. Find the equation of the parabola whose vertex is (1, –2) and focus is (4, –2).

25. For any vector $\vec{a}$ prove that $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k}) = 2\vec{a}$

26. Find the values in the interval (1, 2) of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$.

27. Find the linear approximation for the function $h(x) = \frac{x}{x+1}$ at $x_0 = 1$.

28. Evaluate $\int_3^4 \frac{dx}{x^2-4}$

29. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A, B are arbitrary constants.

30. If $\omega \neq 1$ is complex cubic root of unity from a quadratic equation with roots 2ω and $2\omega^2$.

PART - C

Answer any 7 of the following. Q.No.40 is compulsory. **7 x 3 = 21**

31. Solve : $2x + 5y = -2$, $x + 2y = -3$ by using matrix inversion method.

32. Find the square root of the complex number $-5-12i$

33. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$

34. Find the value of $\cos\left[\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{3}{4}\right)\right]$

35. The line $3x + 4y - 12 = 0$ meets the co-ordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

36. Find the point of intersection of the straight lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

37. Evaluate : $\lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

38. If $U = \log(x^3 + y^2 + z^3)$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

39. Solve $\frac{dy}{dx} + 2y = e^{-x}$

40. Evaluate $\int_0^1 \log\left(\frac{1-x}{x}\right) dx$

PART - D

Answer all the following questions. **7 x 5 = 35**

41. a) Test for consistency and solve by using rank method.
$x - y + 2z = 2$; $2x + y + 4z = 7$; $4x - y + z = 4$ **(OR)** b) Solve : $(y^2 - 2xy)\,dx = (x^2 - 2xy)\,dy$

42. a) Find all cube roots of $(\sqrt{3}+i)$ **(OR)** b) If $U = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

43. a) Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ **(OR)**
b) A conical water tank with vertext down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 meters deep?

44. a) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ **(OR)**

b) Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$.

45. a) Find the centre, vertices, foci and directrices of the ellipse $18x^2 + 12y^2 - 144x + 48y + 120 = 0$
(OR)
b) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

46. a) A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre on either sides. **(OR)**

b) Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

47. a) Using second derivative test find the local maximum and local minimum values of
$2x^3 - 3x^2 - 12x + 13$. **(OR)**
b) Find the parametric vector equation and cartesian equation of the plane passing through the point (1, –1, 3) and perpendicular to the planes $x + y - 2z = 8$ and $x - 2y + 7z = 9$