Mathematics 2005 (Outside Delhi)

SECTION A

Question numbers 1 to 10 carry 3 marks each.

Q. 1. Add the difference of

$$\left(\frac{x-4}{x-3} \text{ and } \frac{x+7}{x+8}\right) \text{ to } \frac{x+8}{x^2+5x-24}$$

Ans:(1/x+8)

Q. 2. Find the sum of all two digit odd positive numbers. **Ans:(2475)**

Q. 3. Solve for x and y:

$$\frac{x}{a}+\frac{y}{b}=2$$
$$ax-by = a^2 - b^2$$

Ans:(y =b)

Or

A two digit number is four times the sum of its digits and twice the product of the digits. Find the number. **Ans:(36)**

Q. 4. Find a and b so that the polynomials:

$$p(x) = (x^2 +3x+2)(x^2 +2x+a) \text{ and }$$
$$Q(x) = (x^2 +7x+12)(x^2 +7x+b)$$

Ans: (a= -3, b=6)

Q. 5. Solve for x:

$$a^2 +b^2x^2 +b^2x -a^2x -1= 0$$

Ans: $$x = \frac{-1}{a^2} \text{ or } x = \frac{1}{b^2}$$

Or

Solve for x:

$$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$$

$(x \neq 2,)4$

Ans:(x=5/2 or x=5)

Q. 6. The 8 th term of an Arithmetic progression is zero. Prove that its 38th term is triple its 18 th term.

Q. 7. The cash price of a machine is Rs 9,000. It is also available at Rs. 2,200 cash down payment followed by five equal monthly installments of Rs 1,400 each. Find the rate of interest under the installments plan.

Ans:(12% p.a.)

Q. 8. Deepak borrowed a sum of money and returned it in three equal quarterly installments of Rs 1,40,608. If the rate of interest charged is 16% per annum compounded quarterly, find the sum borrowed. Also find the total interest charged.

Ans:(Rs. 3,90,200, Rs. 31,624)

Q. 9. The perpendicular from vertex A on the side BC of triangle ABC intersects BC at point D such that DB = 3 CD. Prove that 2 AB 2 = 2 AC 2 + BC 2 .

Q.10. In the given figure, find the length of DE if

AE=15cm, DB = 4 cm and CD = 9 cm. **Ans:(DE = 12 cm or 3 cm)**
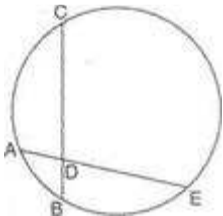
SECTION B

Question numbers 11 to 20 carry 4 marks each.

Q. 11. Solve the following system of equations graphically:

$$2x - y = 4$$
$$3y - x = 3$$

Find the points where the lines meet the y-axis.

Ans: B(0, -4); C(0, 1)

Q. 12. A two digit number is such that the product of its digits is 15. If 18 is added to the number, the digits interchange their places, find the number. **Ans:(35)**

Q. 13. The base radius and height of a right circular solid cone are 2 cm and 8 cm respectively. It is melted and recast into spheres of diameter 2 cm each.

Find the number of spheres so formed. **Ans:(8)**

Q. 14. Prove that:

$$\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$$

Evaluate:

$$\frac{\sec^2 54^0 - \cot^2 36^0}{\cos ec^2 57^0 - \tan^2 33^0} + 2 \sin^2 38^0 . \sec^2 52^0 - \sin^2 45^0$$

Ans:(5/2)

Q. 15. Construct a quadrilateral ABCD with AB = 3 cm, AD = 2.7 cm, BD = 3.6 cm, ∠B= 120° and BC = 4.2 cm. Construct another quadrilateral A'BC'D' similar to quadrilateral ABCD so that diagonal BD' & 4.8 cm

Q. 16. Prove that the points (0, 0); (5, 5) and (-5, 5) are vertices of a right isosceles triangle.

Or

If the point P(x, y) is equidistant from the points A (5, 1) and B (-1, 5), prove that 3x - 2y.

Q. 17. The line joining the points (2, 1) and (5, -8) is trisected at the points P and Q. If point P lies on the line 2x - y + k = 0, find the value of k.

Ans:(k = -8)

Q. 18. If the mean of the following data is 18-75 find the value of p:

X 1	10	15	P	25	30
F 1	5	10	7	8	2

Ans:(p = 20)

Q. 19. The data on mode of transport used by students to come to school are given below:

Mode of transport	Bus	Cycle	Train	Car	Scooter
Number of students	120	180	240	80	100

Represent the above data by a pie-chart.

Q. 20. A bag contains 8 red, 6 white and 4 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is:

(i) red or white (ii) not black (ii) neither white nor black. **Ans:(7/9, 7/9, 4/9)**

SECTION C

Question numbers 21 to 25 carry 6 marks each.

Q. 21. Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Use the above in the following:

In a trapezium ABCD, O is the point of intersection of AC and BD, ABII CD and AB = 2 CD. If the area of AAOB = 84 cm2, find the area of ACOD.

Ans:(21 cm2)

Q. 22. Two pillars of equal height stand on either side of a roadway which is 150 m wide. From a point on the roadway between the pillars, the elevations of the top of the pillars are 60° and 30°. Find the height of the pillars and the position of the point. **Ans:(64-88m)**

Or

A man on the deck of a ship is 10 m above water level. He observes that the angle of elevation of the top of a hill is 60° and the angle of depression of the base of the hill is 30°. Calculate the distance of the hill from the ship and the height of the hill.

Ans:(17-3 m, 40m)

Q. 23. A tent is in the form of a cylinder of diameter 4-2 m and height 4 m, surmounted by a cone of equal base and height 2-8 m. Find the capacity of the tent

and the cost of canvas for making the tent at Rs 100

per sq. m. **Ans:(Rs. 7,590)**

Or

If the radii of the ends of a bucket, 45 cm high, are 28 cm and 7 cm, determine the capacity and total surface area of the bucket. **Ans:(5616-6 cm2)**

Q. 24. PAB is a secant to a circle intersecting it at A and B and PT is a tangent to the circle. Prove that PA x PB = PT **2** .

Use the above in the following:

Two circles intersect each other at A and B. The common chord AB is produced to meet common tangent PQ to the circle at D. Prove that DP = DQ.

Q. 25. Dr. Salim is a senior citizen aged 67 years. He earns Rs. 21,000 per month. He donates Rs. 6,000 to the Prime Minister Relief Fund (100% relief) and Rs. 4,000 to an educational institution (50% relief). He contributes Rs. 60,000 towards PPF and purchases NSC worth Rs. 15,000. He pays income tax of Rs. 600 per month for the first 11 months of the year. Find the income tax to be paid by him in the last month of the year.

Use the following for calculating income tax:

For calculating income tax, use the following:

(a) Standard Deduction :   (i) Rs. 30,000 if incomes is up to Rs.5,00,000
                                            (ii)Rs. 20,000 if income is more than Rs.5,00,000

(b) Rates of income tax :

Slab	*Income tax*
(i) Upto Rs.50,000	No tax
(ii) From Rs.50,001 to Rs. 60,000	10% of the amount exceeding Rs. 50,000
(iii)From Rs.60,001 to Rs. 1,50,000	Rs.1,000+20% of the amount exceeding Rs. 60,000
(iv) Above Rs.1,50,000	Rs.19,000+30% of the amount exceeding Rs.1,50,000
(c) Rebate in tax	(i) 20% of the savings subject to a maximum of Rs.14,000 if the gross incomes is upto Rs.1,50,000. (ii) 15% of the savings subject to a maximum is upto Rs.10,500 if the gross income is between Rs.1,50,001 to Rs.5,00,000.
(d) Educational cess	2% of the income tax payable
(e) Special Rebate	For female, maximum Rs. 5,000 over and above the rebate on savings.

Ans:(Rs.1,254)