

MATHEMATICS
Outside Compartment — 2006

SECTION - A

Question numbers 1 to 10 carry 3 marks each.

Q. 1. Using factorisation, express the following as a rational expression in lowest terms:

$$\frac{3x-5}{3x^2-2x-5} - \frac{3x+5}{3x^2+2x-5} + \frac{2x}{x^2-1}$$

Q. 2. Find the HCF of the following polynomials:

$$30x^2y^2(x^4 - x^3 - 2x^2); 42(x^6 - 8x^3) \text{ and } 24x^6y^6(x^2 - 5x + 6)$$

Q. 3. Which term of the arithmetic progression 5, 15, 25, will be 130 more than its 31st term?

Q. 4. In an AP, the sum of first n terms is $\frac{5n^2}{2} + \frac{3n}{2}$, Find its 20th term.

Q. 5. Solve the following equations for x and y:

$$\frac{a^2}{x} - \frac{b^2}{y} = 0 \quad \frac{a^2b}{x} + \frac{b^2a}{y} = a = b \quad x, y \neq 0 \quad \text{Or}$$

The sum of the numerator and the denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction

Q. 6. Rewrite the following as a quadratic equation in x and then solve for x:

$$\frac{4}{x} - 3 = \frac{5}{2x+3} \quad x \neq 0, -\frac{3}{2}$$

Q. 7. An article is available for Rs. 7,000 cash or for Rs. 1,900 cash down payment and six monthly installments of Rs. 920 each. Compute the rate of interest under the installments scheme.

Q. 8. A man pays back a bank loan of Rs. 10,815 in three equal half yearly installments. If rate of interest is $13\frac{1}{3}\%$ per annum compounded half yearly, find the amount of each installment.

Q. 9. In figure 1, S and T trisect the side QR of a right triangle PQR. Prove that $8 PT^2 = 3 PR^2 + 5 PS^2$.

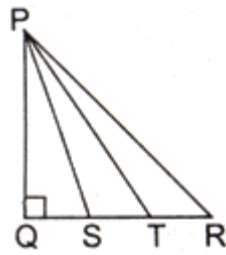


Fig. 1

Or

If BL and CM are medians of a triangle ABC right-angled at A, then prove that $4(BL^2 + CM^2) = 5 BC^2$.

Q. 10. In figure 2, O is the centre of the circle. Prove that

$$\angle AOB = 2[\angle ABC + \angle CAB]$$

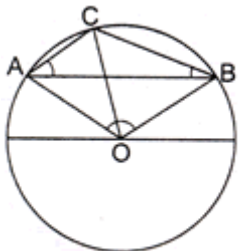


Fig. 2

SECTION - B

Q. 11. Solve the following equations graphically for x and y:

$$3x + 2y = 12 \text{ and } 5x - 2y = 4$$

Find the coordinates of the points where the lines meet the y-axis.

Q. 12. Construct a triangle ABC in which $BC = 6 \text{ cm}$, $\angle A = 60^\circ$ and altitude through A is 4.2 cm. How many such triangles are possible?

Q. 13. Without using trigonometric tables evaluate the following:

$$\frac{\sec 39^\circ}{\operatorname{cosec} 51^\circ} + \frac{2}{\sqrt{3}} \tan 17^\circ \tan 38^\circ \tan 60^\circ \tan 52^\circ$$

Or

Prove that : $\left(1 + \frac{1}{\tan^2 A}\right) \left(1 + \frac{1}{\cot^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$

Q. 14. A 2-digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number. *Or*

A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Q. 15. Find the mean of the following frequency distribution:

Classes	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	15	18	21	29	17

Q. 16. The expenditure of a firm on various heads is represented by the pie chart shown in Figure 3.



Fig. 3

If the total expenditure of the firm is Rs. 72 lakh. find out the following:

- Expenditure on equipment
- Difference between expenditure on wages and miscellaneous expenditure
- Total expenditure on rent and stationery
- Difference between wages and equipment expenditure.

Q. 17. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is

- a black queen
- a red card
- a blackjack
- a picture card (jacks, queens and kings are picture cards)

Q. 18. Prove that the points $(-4, -1)$; $(-2, -4)$; $(4, 0)$ and $(2, 3)$ are vertices of a rectangle.

Q. 19. If $(-2, -1)$; $(a, 0)$; $(4, b)$ and $(1, 2)$ are the vertices of a parallelogram, find the values of a and b .

Q. 20. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the capacity of the vessel.

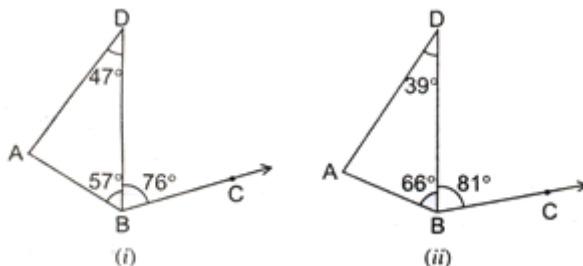
$$\left(\text{Take } \pi = \frac{22}{7} \right)$$

SECTION - C

Question numbers 21 to 25 carry 6 marks each.

Q. 21. If a line is drawn through an end point of a chord of a circle so that the angle formed by it with the chord is equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle. Prove it.

In each of the following, (i) and (ii) (in Figure 4), if a circle is drawn through A, B and D, then using the above theorem, examine in which case is the line BC a tangent.



Q. 22. A man on the top of a vertical tower observes a car moving at uniform speed towards the tower. If it takes 12 minutes for the angle of depression to change from 30° to 45° , how soon, after this, will the car reach the tower?

Or

The angle of elevation of the top of a hill at the foot of the tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30° . If the tower is 50 m high, find the height of the hill.

Q. 23. In a triangle, if the square on one side is equal to the sum of the squares on the remaining two, the angle opposite the first side is a right angle. Prove it.

Using the above prove the following:

If in a $\triangle PQR$ $PS \perp QR$ and $PS^2 = QS \times RS$, then it is right-angled at P.

Q. 24. A bucket made of aluminium sheet is of circular pipe in half hour height 20 cm and its upper and lower ends are of radius 25 cm and 10 cm respectively. Find the cost of making the bucket if the aluminium sheet costs Rs. 70 per 100 cm^2 . (Use $\pi = 3.14$)

Or

Water flows out through a circular pipe, whose internal diameter is 2 cm, at the rate of 0.7 m per second into a cylindrical tank. The radius of whose base is 40 cm. By how much will the level of water in the cylindrical tank rise in half an hour?

Q. 25. The Annual income from salary of Mrs. Usha, who is a senior citizen, is Rs. 3,85,000. She donates Rs. 10,000 to Prime Minister's Relief Fund (100% exemption) and Rs. 10,000 to a Charitable Society (50% exemption). She contributes Rs. 70,000 towards PPF annually and pays a quarterly premium of Rs. 3,500 towards Life Insurance. She also purchases NSCs for Rs. 20,000. She pays Rs. 1,600 per month towards income tax for 11 months. What is her tax liability for the last month of the financial year?

Use the following for calculating income tax:

1. Savings: 100% exemption for savings upto Rs. 1,00,000
2. Rate of Income tax :

<i>Slab</i>	<i>Income Tax</i>
i. Taxable income upto Rs.1,00,000	NIL
ii. Taxable income from Rs. 1,00,001 to Rs.1,50,000	10% of the amount by which taxable income exceeds Rs. 1,00,000.
iii. Taxable income from Rs. 1,50,001 to Rs. 2,50,000	Rs. 5,000 + 20% of the amount by which taxable income exceeds Rs. 1,50,000.
iv. Taxable income above Rs. 2,50,000	Rs. 25,000 + 30% of the amount by which taxable income exceeds Rs. 2,50,000
v. Education cess	2% of the amount of tax payable.