

MATHEMATICS
Delhi Compartment — 2006

SECTION - A

Question numbers 1 to 10 carry 3 marks each.

Q. 1. Using factorisation, express the following as a rational expression:

$$\frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-0}$$

Q 2. If $x - \alpha$ is the HCF of polynomials $P(x) = x^3 - 7x + 6$ and $Q(x) = x^3 - x^2 + x - 1$, find the value of α

Q. 3. If the 10th term of an AP is 52 and the 17th term is 20 more than the 13th term, find the A.P.

Q. 4. Using A.P., find the sum of all 3-digit natural numbers which are multiples of 7.

Q. 5. Solve the following equations for x and y: $mx - ny = m^2 + n^2$ and $x + y = 2m$ *Or*

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi.

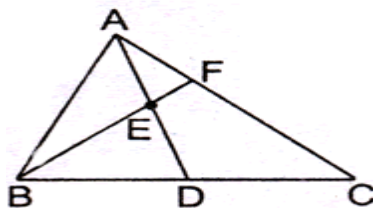
Q. 6. Solve for x and y: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Q. 7. An air-conditioner is available for Rs. 24,000 cash or for Rs. 8,000 as cash down payment followed by 6 monthly instalments of Rs. 2,800 each. Find the rate of interest charged under the instalment scheme.

Q. 8. A computer is available for Rs. 78,600 cash or for Rs. 25,640 cash down payment and three equal half yearly instalments. If the dealer charges interest at the rate of 20% per annum, compounded semi-annually, find the amount of each instalment.

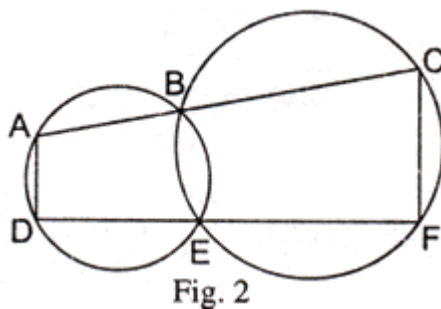
Q. 9. In $\triangle ABC$, AD is a median and E is mid-point of AD (Figure 1). If BE is produced it meets AC at F. Show that $AF = \frac{1}{3} AC$.

Or



P is a point in the interior of rectangle ABCD. If P is joined to each of the vertices of the rectangle, prove that $PB^2 + PD^2 = PA^2 + PC^2$.

Q. 10. In Figure 2, A, B and C are collinear and D, E and F are also collinear. Prove that AD || CF.



SECTION - B

Question numbers 11 to 20 carry 4 marks each.

Q. 11. Solve the following system of linear equations graphically:
 $3x - 2y - 1 = 0$, $2x - 3y + 6 = 0$ Shade the region bounded by the lines and x-axis.

Q. 12. Construct a $\triangle PQR$ in which $QR = 6.5$ cm, $\angle P = 60^\circ$ and median from P to QR is 5 cm long. How many such triangles are possible?

Q. 13. Without using trigonometric tables, evaluate the following:

$$\frac{\cos 220^\circ + \cos^2 70^\circ}{\sec^2 50^\circ - \cot^2 40^\circ} + 2\cos^2 58^\circ - 2\cot 58^\circ \tan 32^\circ - 4\tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ$$

Q. 14. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages.

Q. 15. If the mean of the following distribution is 27, find the value of P:

Classes	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	8	p	12	13	10

Q. 16. The number of persons in a locality speaking different languages is given below. Represent the data by a pie chart.

Languages	Hindi	English	Marathi	Tamil	Bengali
Persons	525	300	200	325	450

Q. 17. Find the probability that a number selected at random from the numbers 1, 2, 3, 35 is a (i) prime number (ii) multiple of 7 (iii) multiple of 3 or 5

Q. 18. Prove that the points (3, 0), (6, 4) and (-1, 3) are vertices of a right-angled triangle. Also, prove that these are the vertices of an isosceles triangle.

Q. 19. In what ratio is the line segment joining the points (-2, -3) and (3, 7) divided by they-axis? Also, find the coordinates of the point of division. *Or*

If A (5, -1), B (-3, -2) and C (-1, 8) are the vertices of triangle ABC, find the length of median through A and the coordinates of the centroid.

Q. 20. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and radius of each hemispherical end is 7cm, find the cost of polishing its surface at the rate of Its. 10 per dm².

SECTION - C

Question numbers 21 to 25 carry 6 marks each.

Q. 21. If PAB is a secant to a circle intersecting the circle at A and B and PT is a tangent, prove that $PA \times PB = PT^2$

Using the above prove the following:

ABC is an isosceles triangle such that $AB = AC$ and the point D is the mid-point of AC. A circle is drawn taking BD as diameter, which intersects AB at E.

Prove that $AE = 1/4 AC$.

Q. 22. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys with the shape of a right circular cone mounted on a hemisphere of radius 3 cm. If the height of the toy is 12cm, find the number of toys so formed. *Or*

A bucket is in the form of a frustum of a cone with a capacity of 12308.8 c.cm of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. (*Use $\pi = 3.14$*)

Q. 23. In a right-angled triangle the square on the hypotenuse is equal to the sum of squares on the other two sides. Prove it.

Using the above prove the following:

In $\triangle ABC$, D is the mid-point of BC and $AE \perp BC$. If $AC > AB$, show that $AB^2 = AD^2 - BC \cdot DE + 1/4 BC^2$.

Q. 24. From a window 15 metres high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the opposite house is 23.66 metres. *Take $\sqrt{3} = 1.732$*

Or

The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is 30° . On advancing 150 metres towards the foot of the tower, the angle of elevation becomes 60° .

Show that the height of the tower is 129 metres. Use $\sqrt{3} = 1.732$

Q. 25. The annual income of Ramesh is Rs. 3,95,000 (excluding HRA). He contributes Rs. 6,000 per month towards provident fund and pays Rs. 4,000 quarterly as LIC premium. How much money should he invest in NSCs so as to get maximum rebate? He donates Rs. 15,000 towards Prime Minister's Relief Fund (100% rebate) and Rs. 10,000 towards Rajiv Gandhi Foundation (50% rebate). If he pays Rs. 3,000 per month as income tax for the first 11 months, find his tax liability in the last month of the year.

Use the following to calculate income tax:

1. Savings: 100% exemption for savings upto Rs. 1,00,000
2. Rate of Income tax :

<i>Slab</i>	<i>Income Tax</i>
i. Taxable income upto Rs.1,00,000	NIL
ii. Taxable income from Rs. 1,00,001 to Rs.1,50,000	10% of the amount by which taxable income exceeds Rs. 1,00,000.
iii. Taxable income from Rs. 1,50,000 to Rs. 2,50,000	Rs. 5,000 + 20% of the amount by which taxable income exceeds Rs. 1,50,000.
iv. Taxable income above Rs. 2,50,000	Rs. 2,50,000 + 30% of the amount by which taxable income exceeds Rs. 2,50,000
v. Education cess	2% of the amount of tax payable.