**MATHEMATICS**
**Outside Delhi — 2006**

## SECTION - A

**Question numbers 1 to 10 carry 3 marks each.**

**Q. 1.** Solve the system of equations:
$bx/a - ay/b + a + b = 0$ and $bx - ay + 2ab = 0$. *Or*

The sum of the digits of a two-digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.

**Q. 2.** Given that : $P = \frac{x+2y}{x+y} + \frac{x}{y}, Q = \frac{x+y}{x-y} - \frac{x-y}{x+y}$ *and* $R = \frac{x+2y}{x+y} - \frac{x}{x+y}$

Express P x Q ÷ R as a rational expression.

**Q. 3.** If $(x + 2)(x - 3)$ is the HCF of the polynomials $p(x) = (x^2 + x - 2)(3x^2 - 8x + c)$ and $q(x) = (x^2 + x - 12)(2x^2 + x + b)$. Find the values of c and b.

**Q. 4.** Using the quadratic formula, solve the equation:
$A^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$ *Or*

The sum of two natural numbers is 8. Determine the numbers if the sum of their reciprocals is 8/15.

**Q. 5.** The $5^{th}$ term of an Arithmetic Progression (A.P.) is 26 and the 10th term is 51. Determine the $15^{th}$ term of the A.P.

**Q. 6.** Find the sum of all the natural numbers less than 100 which are divisible by 6.

**Q. 7.** A household article is available for Rs. 970 cash or Rs. 210 cash down payment followed by three equal monthly installments. If the rate of interest charged under the installment plan is 16% per annum, find the amount of installment.

**Q. 8.** A man borrows Rs. 25,200 from a finance company and has to repay it in two equal annual installments. If the interest is charged at the rate of 10% per annum compounded annually, calculate the amount of installment.

**Q. 9.** In Figure 1, $\angle BAC = 90^0$, $AD \perp BC$.

Prove that $AB^2 + CD^2 = BD^2 + AC^2$.

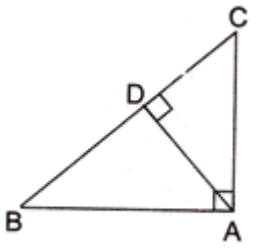


Fig. 1

**Q. 10.** In Figure 2, PT = 6 cm AR = 5 cm.

Find the length of PA.

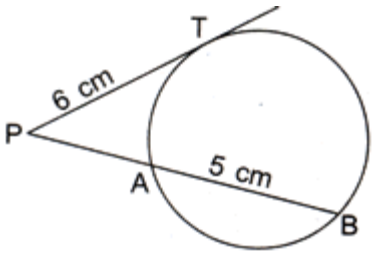


Fig. 2

**SECTION - B**

**Question numbers 11 to 20 carry 4 marks each.**

**Q. 11.** Draw the graphs of the following equations:
3x - 4y + 6 = 0, 3x + y - 9 = 0
Also determine the co-ordinates of the vertices of the triangle formed by these lines and the x-axis.

**Q. 12.** The speed of a boat in still water is 11 kin/hr. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.

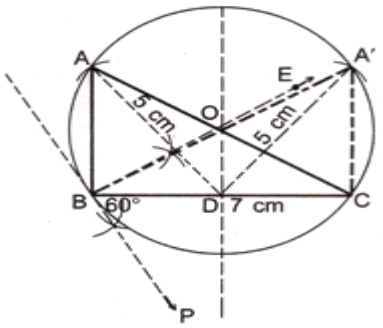
**Q. 13.** A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 58 cm and the diameter of the cylinder is 28cm. Find the total surface area of the sold.
$Use\ \pi = \frac{22}{7}$

**Q. 14.** Prove that $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

*Or*

Evaluate without using Trigonometric tables: $\frac{\operatorname{cosec}^2(90^0-\theta) - \tan^2\theta}{4(\cos^2 48^0 + \cos^2 42^0)} - \frac{2\tan^2 30^0 \sec^2 52^0 \sin^2 38^0}{\operatorname{cosec}^2 700 - \tan^2 20^0}$

**Q. 15.** Construct a triangle ABC in which BC = 7 cm, $\angle A = 60^0$ and median AD = 5 cm. Write the steps of construction also.



**Q. 16.** Show that the points A(6, 2), B(2, 1), C(1, 5) and D(5, 6) are the vertices of a square. *Or*

Find the co-ordinates of the point equidistant from three given points A(5, 3), B(5, -5) and C(1, -5).

**Q. 17.** Find the value of p for which the points (- 5, 1), (1, p) and (4, - 2) are collinear.

**Q. 18.** The Arithmetic Mean of the following frequency distribution is 47. Determine the value of p.

| Classes | 0 -20 | 20 - 40 | 40 - 60 | 60 -80 | 80 - 100 |
|---|---|---|---|---|---|
| Frequency | 8 | 15 | 20 | p | 5 |

**Q. 19.** The following table shows the monthly expenditure of a company. Draw a pie-chart for the data.

| **Item** | **Amount (in Rs.)** |
|---|---|
| Wages | 4800 |
| Materials | 3200 |
| Taxation | 2400 |
| Adm. Expenditure | 3000 |
| Miscellaneous | 1000 |

**Q. 20.** A card is drawn at random from a well- shuffled deck of playing cards. Find the probability that the card drawn is

i. a card of spades or an ace
ii. a red king
iii. neither a king nor a queen
iv. either a king or a queen.

**SECTION - C**

***Question numbers 21 to 25 carry 6 marks each.***

**Q. 21.** Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

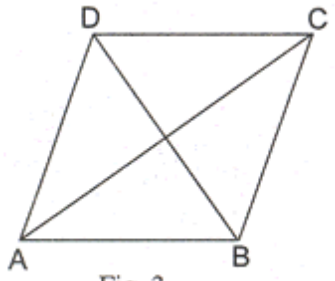


Fig. 3

Making use of the above, prove the following:
In Fig. 3, ABCD is a Fig. 3 rhombus. Prove that $4AB^2 = AC^2 + BD^2$.

**Q. 22.** Prove that I a line touches a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments. Using the above, do the following:

AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30^0$. The tangent at C intersects AR produced in a point I Prove that BC = RD.

**Q. 23.** A bucket made up of a metal sheet is in the form of a frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of Rs. 20 per litre and the cost of the metal sheet used, if it costs Rs. 10 per 100 $cm^2$. $(Use \pi = 3.14)$ *Or*

Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 cm.

**Q. 24.** A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as $30^0$. Calculate the distance of the hill from the ship and the height of the hill. *Or*

From a window x metres high above the ground in a street, the angles of elevation and depression of the top and foot of the other house on the opposite side of the street are $\alpha \text{ and } \beta$ respectively. Show that the height of the opposite house is x $(1+\tan \alpha \cot \beta)$ *metres.*

**Q. 25.** Mrs. Ruchi's salary is Rs. 32,250 per month exclusive of HRA. She donates Rs. 12,000 to Prime Minister's Relief Fund (100% exemption). She also donates Rs. 6,000 to a school and gets a relief of 50% on this donation. She contributes Rs. 5,000 per month towards her Provident Fund. She pays a quarterly premium of Rs. 2,500 towards her LIC policy and invests Rs. 25,000 in NSCs. If Rs. 2,700 is the tax deducted each month from her salary for 11 months, find the tax deducted from her salary in the last month of the year.

Use the following for calculating income tax:

1. Savings: 100% exemption for savings upto Rs. 1,00,000
2. Rate of Income tax for Ladies:

| | *Slab* | *Income Tax* |
|---|---|---|
| i. | Upto Rs. 1,35,000 | No tax |
| ii. | From Rs. 1,35,001 to Rs. 1,50,000 | 10% of the taxable income above Rs. 1,35,000 |
| iii. | From Rs. 1,50,001 to Rs. 2, 50,000 | Rs. 1,500 + 20% of the income exceeding Rs. 1,50,000 |
| iv. | Rs. 2, 50,001 and above | Rs. 21,5000 + 30% of the amount exceeding Rs. 2,50,000 |

3. Education cess: 2% of the income tax