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HALF YEARLY EXAM - 2023-24
MATHEMATICS CLASS: X

ANSWER KEY

1)

a) $(3,0)$ on x-axis, y coordinate should be 0

b) distance = 3 units.

2)

a) $\angle ADC = \angle CBE$

$= \underline{105^\circ}$ (cyclic quad. concept)

b) $\angle ADC + \angle ABC = 180^\circ$ (opposite angles of a cyclic quadrilateral supplementary)

3) a) $PQ = \sqrt{2^2 - 1^2}$

$= \underline{\sqrt{3}}$

b) Since $QR : PQ : PR = 1 : \sqrt{3} : 2$

So its opposite angles are $30^\circ, 60^\circ, 90^\circ$

$\therefore \angle QRP = 60^\circ$

4) Given $4a + 4e = 48$, also $a = e$

$4a + 4a = 48$

$8a = 48$

$\therefore a = \frac{48}{8}$

$= \underline{6 \text{ cm}}$

$\therefore$ Base area $= a^2$

$= 6^2$

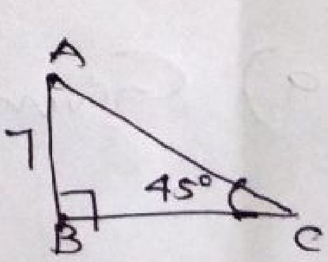
$= \underline{36 \text{ cm}^2}$

5) Fig.

6) Here $\angle A = 45^\circ$

(angle sum property of a $\triangle^{le}$)

$\therefore \triangle ABC$ is isosceles



Here angles are 45°, 45°, 90° so its opp. sides are in the ratio $1:1:\sqrt{2}$

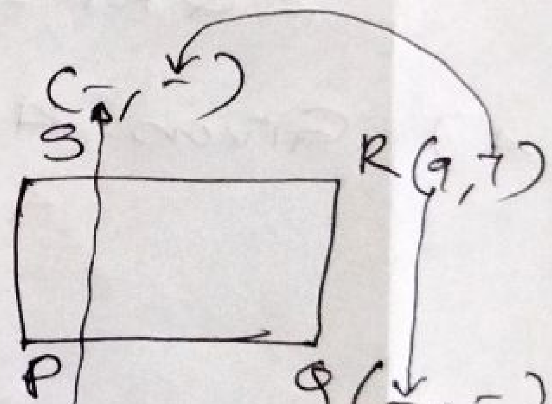
a) $AC = 7\sqrt{2}$ cm

b) Area of square $= (\text{Side})^2$
$= (AC)^2$
$= (7\sqrt{2})^2$
$= \underline{98 \text{ cm}^2}$

7) Given $a = 10$ cm, $e = 13$ cm

a) $l^2 = e^2 - (\frac{a}{2})^2$
$= 13^2 - 5^2$
$= 169 - 25$
$= 144$
$\therefore l = \sqrt{144}$
Slant ht. $= \underline{12 \text{ cm}}$

b) L.S.A $= 2al$
$= 2 \times 10 \times 12$
$= \underline{240 \text{ cm}^2}$



8) a) $Q = (9, 3)$
$S = (4, 7)$

b) Here $PQ = |9-4|$
$= |5|$
$= \underline{5 \text{ units}}$

9) Given $x_8 - x_5 = 12$
$3d = 12$
$\therefore d = \frac{12}{3} = \underline{4}$

a) $x_{15} - x_9 = (15-9)d$

$= 6d$

$= 6 \times 4$

$= \underline{24}$

b) Given $x_{11} = 45$, $x_{20} = x_{11} + (20-11)d$

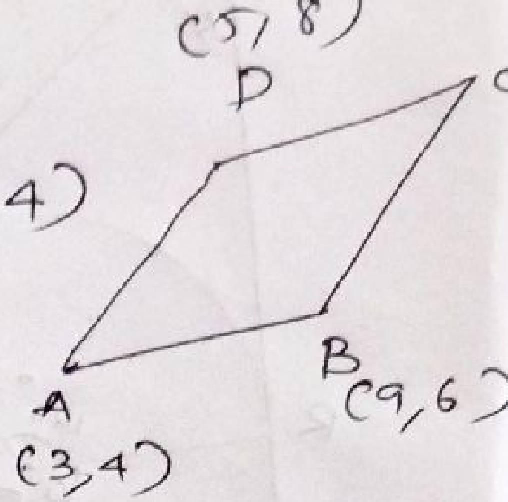
$= 45 + 9d$

$= 45 + 9 \times 4$

$= 45 + 36$

$= \underline{81}$.

10)



a) $C = (9+5-3, 8+6-4)$

$= (11, 10)$

b) Point of intersection of diagonals $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{5+9}{2}, \frac{8+6}{2}\right)$

$= (7, 7)$

Given,

11) No. of white Black & Red balls = 30

$P(W) = \frac{7}{30}$, $P(R) = \frac{3^{\times 3}}{10_{\times 3}}$

$= \frac{9}{30}$

a) No. of white balls = 7

b) $P(\text{Black}) = \frac{30 - W - R}{30}$

$= \frac{30-7-9}{30}$

$= \frac{14}{30}$

$= \frac{7}{15}$

c) If 3 red balls taken out,

No. of red balls in the box = 9 - 3

= 6

Total No. of balls " = 30 - 3

= 27

Then $P(\text{Red}) = \frac{6}{27} = \frac{2}{9}$.

12) Given A. Seq. is 6, 10, 14, ...

a) $d = a_2 - a_1$

$= 10 - 6$

$= \underline{4}$

b) Sum = 510.

$\frac{d}{2}n^2 + (f - \frac{d}{2})n = 510$

$2n^2 + (6-2)n = 510$

$2n^2 + 4n = 510$

$\div 2 \Rightarrow n^2 + 2n = 255$

$n^2 + 2n + 1 = 255 + 1$

$(n+1)^2 = 256$

$n + 1 = \sqrt{256}$

$n + 1 = 16$

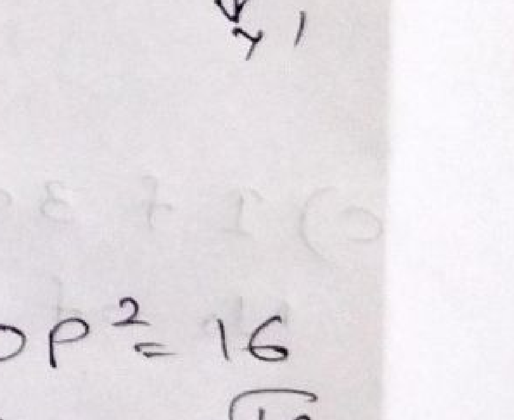
$\therefore n = 16 - 1$

$= \underline{\underline{15}}$.

No. of terms = $\underline{\underline{15}}$.

13)

Given radius = 5 units



a) $B = (3+5, 0)$

$= \underline{(8, 0)}$

$A = (3-5, 0)$

$= \underline{(-2, 0)}$.

b) $OA \times OB = OP^2$

$2 \times 8 = OP^2$

$\therefore OP^2 = 16$

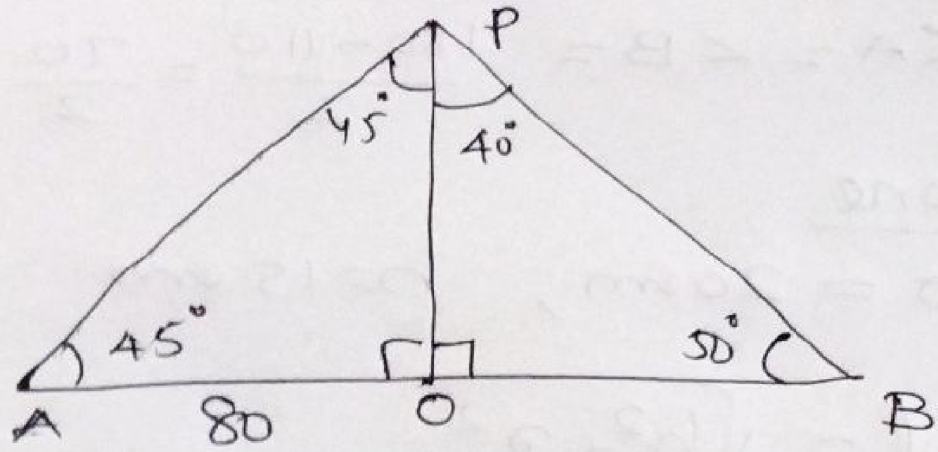
$\therefore OP = \sqrt{16}$

$= \underline{4}$

$\therefore \underline{P = (0, 4)}$

14)

a)



b) In ΔAOP, angles are $45°, 45°, 90°$

So sides are in the ratio $1 : 1 : \sqrt{2}$

Since $OA = 80m$

Here $OP = \underline{80m}$

Height of tower $= \underline{80m}$

c) In ΔOBP, $\tan 40 = \dfrac{\text{opp side}}{\text{adj. side}}$

$0.84 = \dfrac{OB}{OP}$

$0.84 = \dfrac{OB}{80}$

$\therefore OB = 80 \times 0.84$

$= \underline{67.20}$

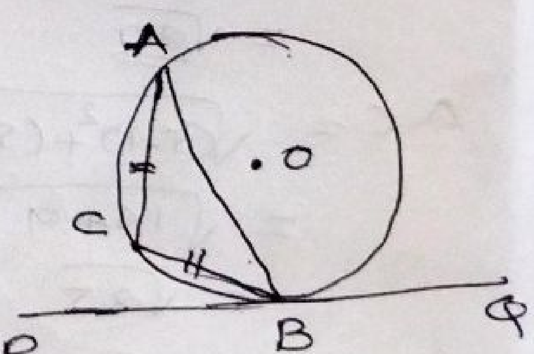
Distance b/w the 2 persons $= AB$

$= OA + OB$

$= 80 + 67.2$

$= \underline{147.2m}$

15) Given $\angle ABP = 70°$



a) $\angle ABQ = 180 - 70$

$= \underline{110°}$

b) $\angle C = \angle ABQ$

$= \underline{110°}$ (angle b/w tangent & chord is equal angle in its opp. arc)

Since $AC = BC$, ΔABC is isosceles

$\angle A = \angle B = \frac{180-110}{2} = \frac{70}{2} = \underline{35^\circ}$

16) Cone

Given $r = 20\,m$, $h = 15\,m$.

a) $l = \sqrt{h^2 + r^2}$

$= \sqrt{15^2 + 20^2}$

$= \sqrt{225 + 400}$

$= \sqrt{625}$

Slant ht. $= \underline{25\,m}$

b) C.S.A $= \pi r l$

$= \pi \times 20 \times 25$

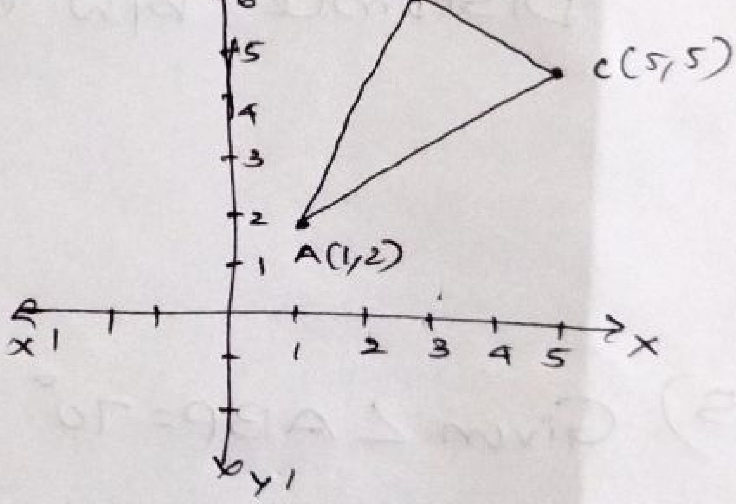
$= \underline{500\pi\,m^2}$

c) Total Cost of Canvas

$= 500\pi \times 60$

$= 500 \times 3.14 \times 60$

$= \underline{94200}$ rupees.



17) a) $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(3-1)^2 + (6-2)^2}$

$= \sqrt{2^2 + 4^2}$

$= \underline{\sqrt{20}}$

$AC = \sqrt{(5-1)^2 + (5-2)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= \underline{5}$

$BC = \sqrt{(5-3)^2 + (6-5)^2}$

$= \sqrt{2^2 + 1^2}$

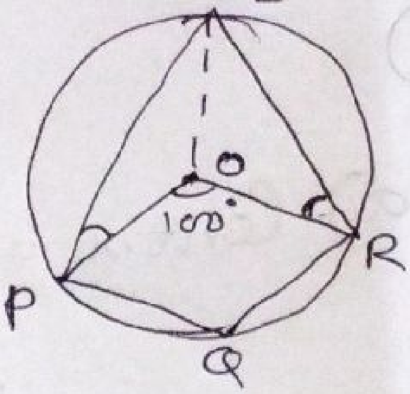
$= \sqrt{5}$.

b) Here $AB^2 + BC^2 = AC^2$ $\therefore \triangle ABC$ is a rt. angled triangle rt angled at B

$(\sqrt{20})^2 + (\sqrt{5})^2 = 5^2$

$20 + 5 = 25$

18)



a) Central angle of arc PSR

$= 360 - 100$

$= \underline{260°}$

b) $\angle PSR = \frac{100}{2}$

$= \underline{50°}$

$\angle PQR = 180 - 50$

$= \underline{130°}$

c) $\angle OPR + \angle ORS = \angle PSR$

$= \underline{50°}$

19) Cone

Given, $R = 12$ cms, $h = 15$ cms.

Sphere $r = 3$ cms.

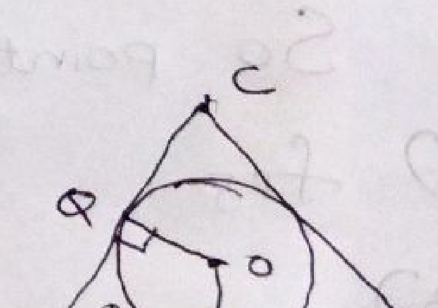
No. of spheres $= \frac{\text{vol. of Cone}}{\text{vol. of 1 sphere}}$

$= \frac{\frac{1}{3}\pi R^2 h}{\frac{4}{3}\pi r^3}$

$= \frac{R^2 h}{4 r^3}$

$= \frac{12 \times 12 \times 15}{4 \times 3 \times 3 \times 3}$

$= \underline{20}$

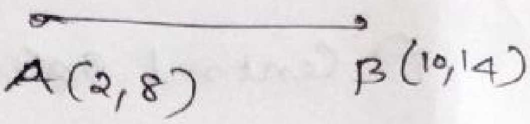


20) a) $\angle POQ = 180 - 50$

$= \underline{130°}$

b) fig.

21)



a) Centre $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{2+10}{2}, \frac{8+14}{2}\right)$

$= \underline{(6, 11)}$

b) radius $= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(6-2)^2+(11-8)^2}$

$= \sqrt{4^2+3^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= \underline{5 \text{ units}}$

c) Distance b/w Centre (6,11) & point (9,15)

distance $= \sqrt{(9-6)^2+(15-11)^2}$

$= \sqrt{3^2+4^2}$

$= \sqrt{9+16}$

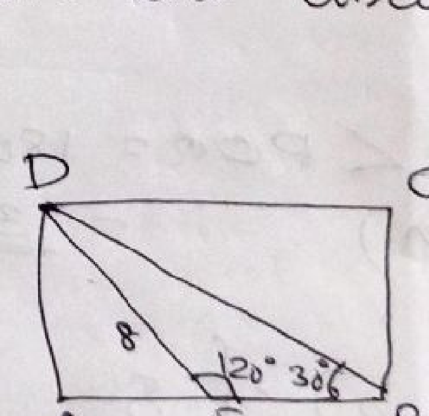
$= \sqrt{25}$

$= 5$

$= \underline{\text{radius}}$

So point (9,15) is on the Circle.

22) fig.

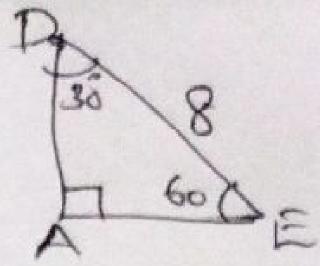


23) a) $\angle AED = 180 - 120$

$= \underline{60°}$.

b) In $\triangle AED$, angles are $30°, 60°, 90$ so sides are $1 : \sqrt{3} : 2$

AE : AD : ED = 1 : √3 : 2



$\frac{AD}{DE} = \frac{\sqrt{3}}{2}$

$\frac{AD}{8} = \frac{\sqrt{3}}{2}$ $\therefore AD = \frac{\sqrt{3}}{2} \times 8$

$= 4\sqrt{3}$ cm

Also $AE = \frac{8}{2} = 4$ cm

c) In △ BED, angles are 30°, 30°, 120°

So it is an isosceles triangle.

$\therefore$ BE = DE

$\therefore$ BE = 8 cm

d) Area of rectangle = l × b

= AB × AD

= (AE + EB) × AD

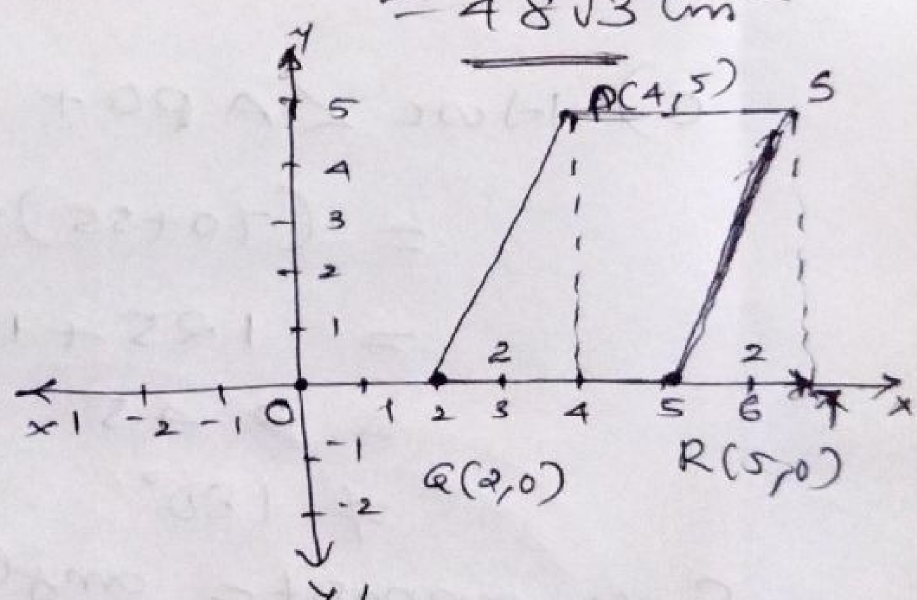
= (4 + 8) × 4√3

= 12 × 4√3

= 48√3 cm²

24)

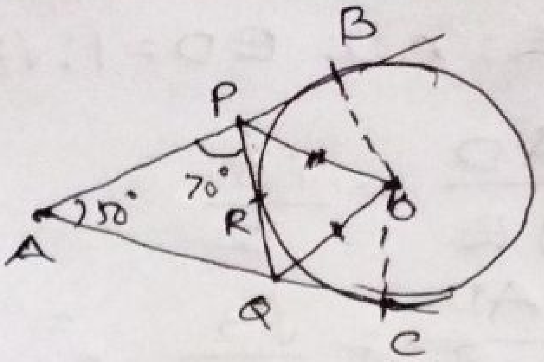
a)



b) S = (5 + 2, 5)

= (7, 5)

25)



a) $\angle BPQ = 180 - 70$

$= 110^\circ$

b) $\angle OPQ = \dfrac{110^\circ}{2}$

$= 55^\circ$

~~Since~~ $\angle AQP = 180 - (50 + 70)$

$= 60^\circ$

$\therefore \angle \cancel{R}QP = 180 - 60$

$= 120^\circ$

$\therefore \angle OQP = \dfrac{120}{2}$

$= 60^\circ$

c) Here $\angle APO + \angle AQO$

$= (70 + 55) + (60 + 60)$

$= 125 + 120$

$= 245$

$\neq 180^\circ$

Since opposite angles are not Supplementary

$\therefore$ quad. APOQ is not cyclic.

26) Given $x_1 = 12$, $x_1 + x_2 + x_3 = 51$

a) $x_2 = \dfrac{51}{3}$ middle term $= \dfrac{\text{Sum}}{\text{No.}}$

$= 17$

b) Here $d = x_2 - x_1$

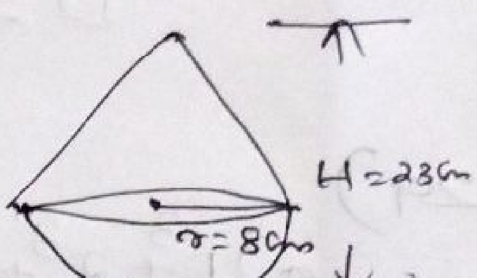
$= 17 - 12 = 5$

$\therefore x_8 = f+7d$
$= 12+7\times5$
$= 12+35$
$= \underline{47}$

c) Sum = middle term × No. of terms)

$\therefore S_{15} = x_8 \times 15$
$= 47\times15$
$= \underline{705}$

27)



a) ht. of Cone = 23 − 8
= $\underline{15cm}$

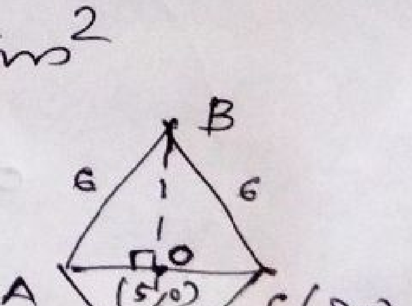
b) $l = \sqrt{h^2+r^2}$
$= \sqrt{15^2+8^2}$
$= \sqrt{225+64}$
$= \sqrt{289}$

Slant ht. = 17 cm

C.S.A of Cone + C.S.A of Hemisph

c) Surface area $= \pi r l + 2\pi r^2$
$= \pi\times8\times17+2\pi\times8^2$
$= 136\pi + 128\pi$
$= \underline{264\pi\ cm^2}$

28)



Here AC = |8 − 2|

a) = |6|
= 6 units

b) mid point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
$= \left(\frac{8+2}{2}, \frac{0+0}{2}\right) = \underline{(5,0)}$

In rt. $\triangle^{le}$ AOB, $OB = \sqrt{AB^2 - OA^2}$

$= \sqrt{6^2 - 3^2}$

$= \sqrt{36 - 9}$

$= \sqrt{27}$

$= \underline{3\sqrt{3}}$.

$\therefore B = (5, \underline{3\sqrt{3}})$

$D = (5, \underline{-3\sqrt{3}})$

29) a) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1+2+3+4+5)^2 = \left(\frac{5 \times 6}{2}\right)^2$

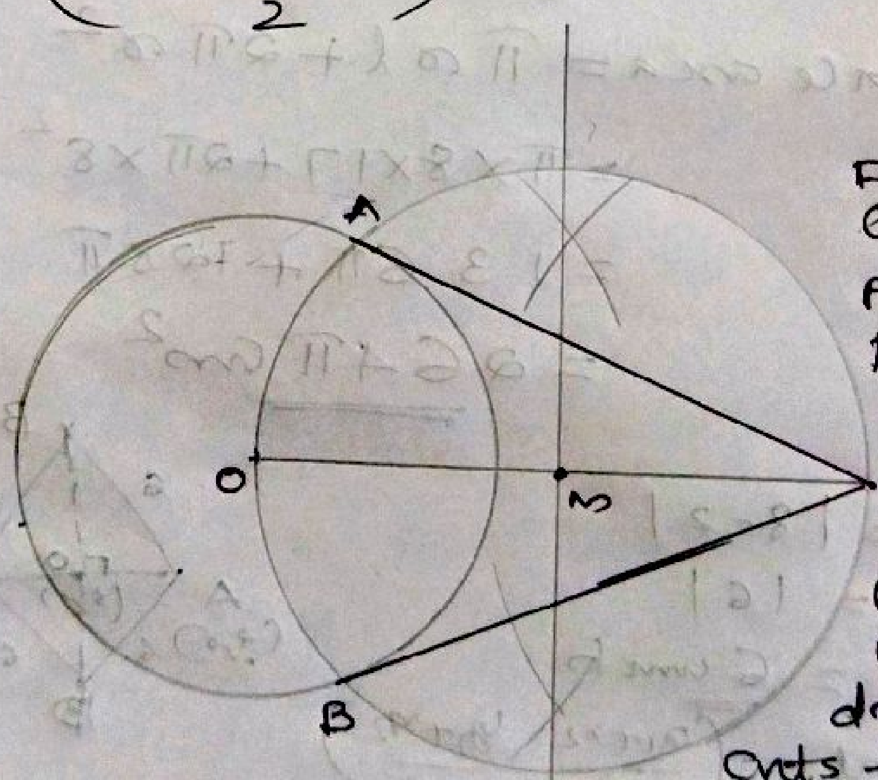
b) $x = 7$

c) $y = 9$

d) $\left(\frac{100 \times 101}{2}\right)^2$

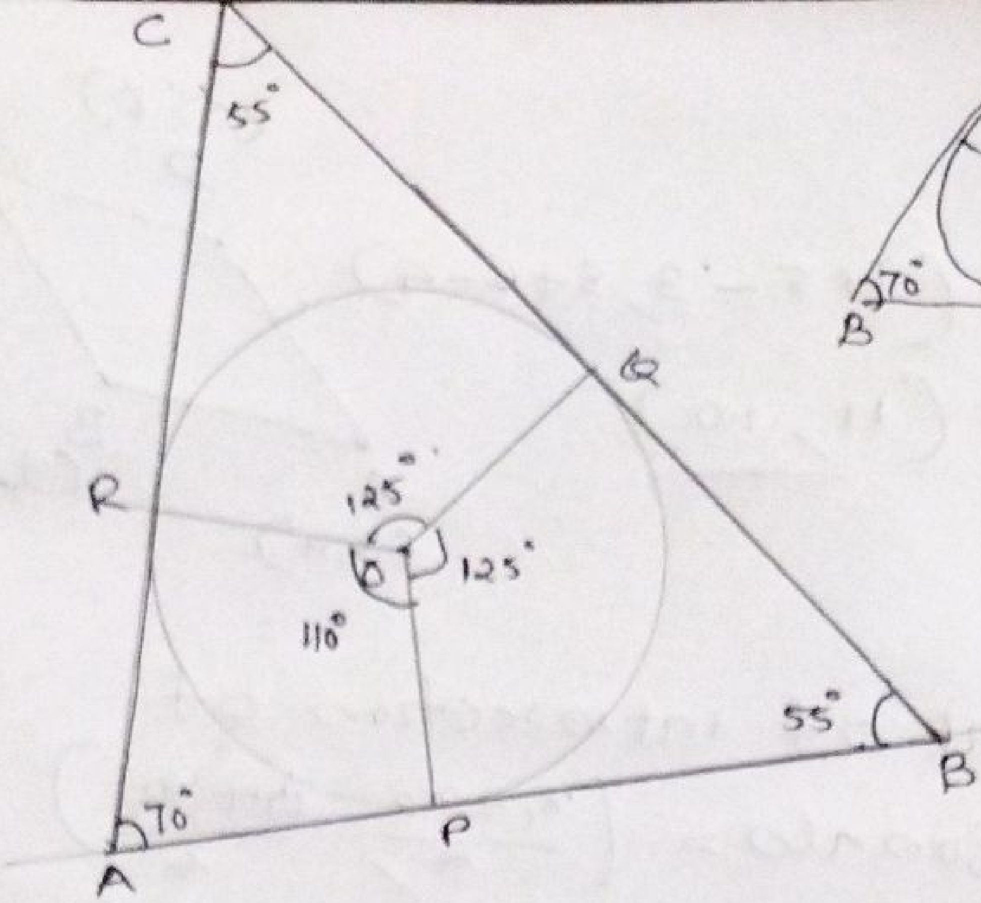
e) $\left(\frac{n(n+1)}{2}\right)^2$

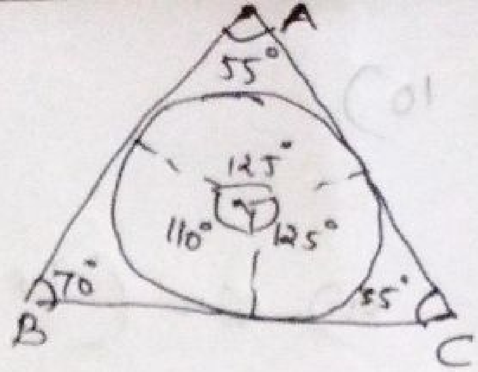
5)



First draw the given circle of rad. 3cm, mark P such that OP = 7.5cm. Draw ⊥ bisector to OP, then we get the midpt of OP as M. Draw a circle centre at M MO = MP as radius draw a circle, which cuts first circle at A & B. Join PA, PB which are the required tangents.

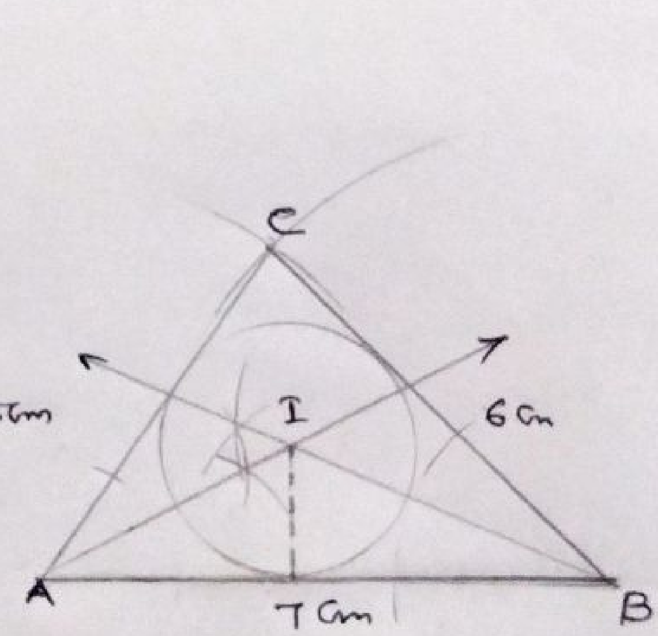
20)





First draw the given circle, then draw a
radius segment OP, then take angle 125°
thus we get ∠POQ as 125°, then from OQ
take angle 125°, so that ∠QOR = 125°, draw
tangents through P, Q, R, we get the required Δ

22)



First draw the given Δ ABC, then draw angle
bisectors to ∠A & ∠B, I is the point of intersection
of angle bisectors, which is the incentre, from
that point to any one side as radius we can
draw the required incircle.

Incircle radius = 1.5 cm.