DIRECTORATE OF GOVERNMENT EXAMINATIONS, CHENNAI – 600 006

HSE SECOND YEAR PUBLIC EXAMINATION – MARCH/APRIL-2023

MATHEMATICS MARKING SCHEME – ENGLISH MEDIUM

GENERAL INSTRUCTIONS

MAXIMUM MARKS :90

1. The answers given in the marking scheme are Text book and Solution book bound.

2. **If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous) such answers should be given full credit with suitable distribution.**

3. Follow the footnotes which are given under certain answer schemes.

4. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula (for the stage mark 2*). This mark (*) is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalized.

5. In the case of Part II, Part III and Part IV, if the solution is correct then award full mark directly. The stage mark is essential only if the part of the solution is incorrect.

6. Answers written only in Black or Blue ink should be evaluated

PART – I

1. One mark to write the correct option and the corresponding answer.
2. If one of them (answer or option) is wrong, then award zero mark only.

CODE A			CODE B		
Q.NO	**OPTION**	**ANSWER**	**Q.NO**	**OPTION**	**ANSWER**
1.	(b)	$\rho(A) = n$	1.	(a)	$\frac{1}{e^2}$
2.	(d)	1	2.	(b)	$x \in \left[\frac{1}{2}, 1\right]$
3.	(b)	$x \in \left[\frac{1}{2}, 1\right]$	3.	(a)	3
4.	(a)	$y = kx$	4.	(a)	$\frac{8}{3}$
5.	(a)	3	5.	(b)	z
6.	(c)	0	6.	(b)	log 2
7.		Mere Attempt	7.	(d)	2
8.	(d)	2	8.	(d)	1
9.	(a)	$\frac{1}{e^2}$	9.	(a)	$\frac{-\pi}{6}$
10.	(d)	Z	10.	(d)	- 4
11.	(a)	$\frac{8}{3}$	11.	(d)	n
12.	(a)	$\frac{\pi}{2}$	12.	(c)	5
13.	(c)	5	13.	(c)	2
14.	(c)	2	14.	(a)	$y = kx$
15.	(b)	z	15.	(b)	$2ab$
16.	(d)	- 4	16.	(a)	$\frac{\pi}{2}$
17.	(b)	log 2	17.	(b)	$\rho(A) = n$
18.	(d)	n	18.		Mere Attempt
19.	(a)	$\frac{-\pi}{6}$	19.	(d)	Z
20.	(b)	$2ab$	20.	(c)	0

PART – II

Q.NO	CONTENT	MARKS
21.	$\left.\begin{matrix} \lvert z+3+4i\rvert \le 7 \\ \lvert z+3+4i\rvert \ge 3 \end{matrix}\right\}$	1
	$3 \le \lvert z+3+4i\rvert \le 7$	1
22.	$\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}} = \frac{p+q}{\sqrt{pq}}+\frac{\sqrt{n}}{\sqrt{l}}$	1
	$= -\sqrt{\frac{n}{l}}+\sqrt{\frac{n}{l}}$	1
	= 0	
23.	$c = \pm\sqrt{a^2(1+m^2)} = \pm\sqrt{9(1+16)}$	1
	$c = \pm 3\sqrt{17}$	1
24.	$dv = 4\pi(10)^2(-0.1)$	1
	$= -40\pi cm^3$	1
25.	$\int_b^\infty \frac{dx}{a^2+x^2} = \left[\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]_b^\infty$	1
	$= \frac{1}{a}\left[\frac{\pi}{2}-\tan^{-1}\frac{b}{a}\right]$	1
26.	$\hat{n} = \frac{3\hat{\imath}-4\hat{\jmath}+5\hat{k}}{5\sqrt{2}}$	1
	$\vec{r}.\left(\frac{3\hat{\imath}-4\hat{\jmath}+5\hat{k}}{5\sqrt{2}}\right) = 7$	1
27.	$A \vee B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	1
	$A \wedge B = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$	1
28.	$AA^T = A^T A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	1
	A is orthogonal	1
29.	m = 5	1
	Equation of the tangent is 5x – y – 3 = 0	1
30.	$e^{\cos\theta+i\sin\theta} = e^{\cos\theta}e^{i\sin\theta}$	1
	$= e^{\cos\theta}\cos(sin\theta) + ie^{\cos\theta}\sin(sin\theta)$	1

Important Note for Part – III and Part – IV

In an answer to a question, between any two particular stages of marks (greater than one) if a student starts from a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for the stage.

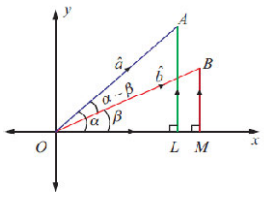
PART – III

Q.NO	CONTENT	MARKS
31.	The equation of parabola is $(x+1)^2 = 4a(y+2)$	1
	$a = \frac{1}{2}$	1
	$x^2 + 2x - 2y - 3 = 0$	1
32.	$a + c = 152 \times 10^6$	1
	$a - c = 94.5 \times 10^6$	1
	Distance of the sun from the other focus = 575×10^5 km.	1
33.	$-1 < 3x - 1 < 0$	1
	$0 < 3x < 1$	1
	$0 < x < \frac{1}{3}$	1
34	$\cos\alpha = \frac{2}{3}$, $\cos\beta = \frac{2}{3}$, $\cos\gamma = -\frac{1}{3}$	2
	$\alpha = \beta = \cos^{-1}\left(\frac{2}{3}\right)$, $\gamma = \cos^{-1}\left(-\frac{1}{3}\right)$	1
35.	$(125-2)^{2/3} \approx (5^3)^{2/3} + \frac{2}{3}\frac{1}{(5^3)^{1/3}}(-2)$	2
	≈ 24.73	1
36.	$\cos y \, dy = e^x \left(\log x + \frac{1}{x}\right) dx$	1
	$\sin y = e^x \log x + c$	2
37.	$\lvert F(\alpha)\rvert = 1$	1
	$[F(\alpha)]^{-1} = \begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix}$	1
	$= F(-\alpha)$	1

38.	<table><tr><th>p</th><th>q</th><th>$p \to q$</th><th>$q \to p$</th></tr><tr><td>T</td><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td><td>T</td></tr><tr><td>F</td><td>T</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td><td>T</td></tr></table> (1) (1) $p \to q \not\equiv q \to p$	1+1 1
39.	$z = 5 + i$ $z^{-1} = \frac{5}{26} - i\frac{1}{26}$	1 2
40.	$\Delta = (c + a - b)^2 - 4(b + c - a)(a + b - c)$ $\Delta = 4(c - a)^2$ (perfect square) The roots are rational	1 1 1

PART – IV

Q. NO	CONTENT	MARKS
41.a	$z^3 = -8i$	
	$z = 2\left[\cos\left(\frac{4k\pi-\pi}{6}\right) + i\sin\left(\frac{4k\pi-\pi}{6}\right)\right]$ k = 0, 1 2	2
	When k = 0 , $z = 2\left[\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right] = \sqrt{3} - i$	1
	When k = 1 , $z = 2\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right] = 2i$	1
	When k = 2 , $z = 2\left[-\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right] = -\sqrt{3} - i$	1
	OR	
41.b	$\frac{dx}{dy} + \frac{1}{y}x = -\frac{1}{y(1+y^2)}$	1
	$P = \frac{1}{y}$, $Q = -\frac{1}{y(1+y^2)}$	1
	$e^{\int p dy}$ = y	1
	$xy = -\tan^{-1} y + c$	2

42.a	Rough diagram 	1
	$\hat{a} = \cos\alpha\,\hat{\imath} + \sin\alpha\,\hat{\jmath}$	1
	$\hat{b} = \cos\beta\,\hat{\imath} + \sin\beta\,\hat{\jmath}$	1
	Remaining part	2
	OR	
42.b	$k = \frac{1}{400}$	1
	$F(x) = \begin{cases} 0, & for\ x < 200 \\ \frac{x}{400} - \frac{1}{2}, & for\ 200 \le x \le 600 \\ 1, & for\ x > 600 \end{cases}$	1+1+1
	$P(300 \le x \le 500) = \frac{1}{2}$	1
43.a	It represents an ellipse	1
	$\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$	1
	Center = (4 , -2)	1
	Foci = $(4 , -2 \pm \sqrt{6})$	1
	Vertices = $(4 , -2 \pm 3\sqrt{2})$	1
	OR	
43.b	Let $\cos^{-1} x = \alpha$ and $\cos^{-1} y = \beta$	1
	$\alpha + \beta = \pi - \cos^{-1} z$	1
	$\cos(\pi - \cos^{-1} z) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$	1
	$\sqrt{1-x^2}\sqrt{1-y^2} = xy + z$	1
	$x^2 + y^2 + z^2 + 2xyz = 1$	1
44. a	$[A \mid B] = \begin{pmatrix} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 9 & 3 & 1 & 8 \end{pmatrix}$	1
	a = 1 , b = 3 and c = -10	1+1+1
	The equation is $y = x^2 + 3x - 10$	
	The point P(7,60) satisfies this equation , the boy will meet his friend.	1
	OR	

44 b	$x^2 = \frac{16}{3}$ and $y^2 = \frac{4}{6}$	1
	$m_1 = -\frac{x}{4y}$ and $m_2 = \frac{x}{2y}$	1 + 1
	m_1 x $m_2 = \frac{-x^2}{8\,y^2}$ = -1	1
	The given curves cut orthogonally	1
45.a	$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$, $\vec{u} = 2\imath - \hat{\jmath} + 4\hat{k}$, $\vec{v} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$	1
	Vector e$quation$ $\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + s(2\hat{\imath} - \hat{\jmath} + 4\hat{k}) + t(\hat{\imath} + 2\hat{\jmath} + \hat{k})$	2*
	Cartesian equation $9x - 2y - 5z + 4 = 0$	2*
	OR	
45 b	$3x^2 - 10x + 3$ is a factor of $6x^4 - 5x^3 - 38x^2 - 5x + 6$	1
	$6x^4 - 5x^3 - 38x^2 - 5x + 6 = (3x^2 - 10x + 3)(2x^2 + px + 2)$	1
	$p = 5$	1
	The other factor is $2x^2 + 5x + 2$	1
	The roots are $\frac{1}{3}$, 3, $-\frac{1}{2}$, -2	1
46. a		1+1+1+1

p	q	r	$\neg p$	$\neg q$	$(\neg q \vee r)$	$p \rightarrow (\neg q \vee r)$	$\neg p \vee (\neg q \vee r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(1) (1) (1) (1)

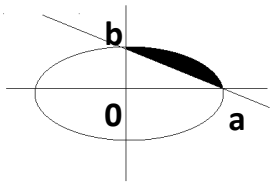
$p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ — 1

Note: i) The order of rows and columns need not be same as in the scheme

ii)If any mistake in the table then award two marks for the table

	OR	

A

46. b	Let P be the amount at t years. $P = Ce^{0.05t}$ $C = 10000$ $P = 10000e^{0.075}$	 2 1 2
47. a	Let $f(x) = \frac{logx}{x}$, $x > 0$ $f'(x) = \frac{1 - logx}{x^2}$ $x = e$ $f''(x) = \frac{-x - (1 - logx)(2x)}{x^4}$ $f''(e) = \frac{-1}{e^3} < 0$ At x = *e* , f(x) has maximum and the maximum value = $\frac{1}{e}$	 1 1 1 1 1
	OR	
47. b	 Rough diagram Required area = $\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}dx - \int_0^a \frac{b}{a}(a - x)dx$ $= \frac{b}{a}\left[\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$ $= \frac{ab}{4}[\pi - 2]$ Note: one can find the area of the remaining part , award suitable marks.	1 1 1 2