MATHEMATICS

Time allowed : 3 hours ***Maximum Marks : 100***

GENERAL INSTRUCTIONS :

1. *All questions are compulsory*

2. *The question paper consists of 30 questions divided into four sections - A, B, C and D. Section A comprises of ten questions of 1 mark each, Section B comprises of five questions of 2 marks each, Section C comprises of ten questions of 3 marks each and Section D comprises of five questions of 6 marks each.*

3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

3. *There is no overall choice. However, an internal choice has been provided in one question of 2 marks, three questions of 3 marks each and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.*

4. *In question on construction, the drawings should be neat and exactly as per the given measurements.*

5. *Use of calculators is not permitted.*

QUESTION PAPER CODE 30/1/1

SECTION - A

Question numbers 1 to 10 carry 1 mark each.

1. Has the rational number $\frac{441}{2^2.5^7.7^2}$ a terminating or a non-terminating decimal representation?

2. If α, β are the zeroes of a polynomial, such that $\alpha + \beta = 6$ and $\alpha\beta = 4$, then write the polynomial.

3. If the sum of first p terms of an A.P. is $ap^2 + bp$, find its common difference.

4. In Fig. 1, S and T are points on the sides PQ and PR, respectively of Δ PQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of Δ PST and Δ PQR.

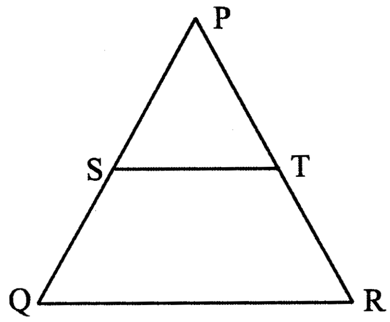


Fig. 1

5. In Fig. 2, Δ AHK is similar to Δ ABC. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC.

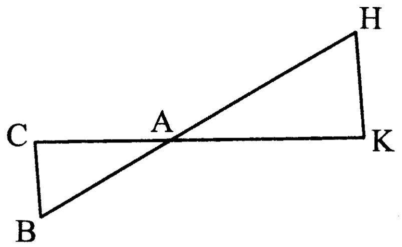


Fig. 2

6. If 3x = cosec θ and $\frac{3}{x}$ = cot θ, find the value of $3\left(x^2 - \frac{1}{x^2}\right)$.

7. If P(2, p) is the mid-point of the line segment joining the points A(6, -5) and B(-2, 11), find the value of p.

8. If A(1, 2), B(4, 3) and C(6, 6) are the three vertices of a parallelogram ABCD, find the coordinates of the fourth vertex D.

9. The slant height of a frustum of a cone is 4 cm and the perimeters (circumferences) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

10. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting a red face card.

Section B

Question Numbers 11 to 15 carry 2 marks each.

11. If two zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

12. Find the value af k for which the following pair of linear equations have infinitely many solutions:

    $2x + 3y = 7; \qquad (k - 1)x + (k + 2)y = 3k$

13. In an A.P., the first term is 2, the last term is 29 and sum of the terms is 155. Find the common difference of the A.P.

14. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

15. Without using trigonometric tables, find the value of the following expression:

    $$\frac{\sec(90° - \theta).\text{cosec}\,\theta - \tan(90°\ \theta)\cot\theta + \cos^2 25° + \cos^2 65°}{3\tan 27°.\ \tan 63°}$$

Or

Find the value of cosec 30° geometrically.

Section C

Question Numbers 16 to 25 carry 3 marks each.

16. Prove that $2 - 3\sqrt{5}$ is an irrational number.

17. The sum of numerator and denominator of a fraction is 3 less than twice the denominator. If each of the numerator and denominator is decreased by 1, the fraction becomes $\frac{1}{2}$. Find the fraction.

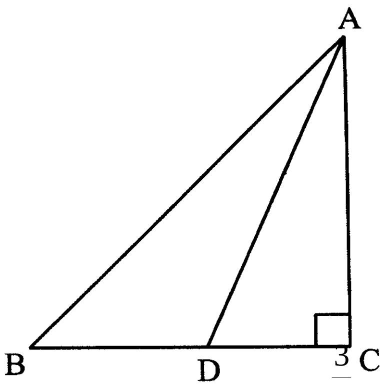
Or

Solve the following pair of equations:

$$\frac{4}{x} + 3y = 8; \qquad \frac{6}{x} - 4y = -5$$

18. In an A.P., fhe sum of first ten terms is –150 and the sum of its next ten terms is –550. Find the A.P.

19. In Fig. 3, ABC is a right triangle, right angled at C and D is the mid-point of BC. Prove that $AB^2 = 4AD^2 - 3AC^2$.



$\frac{3}{4}$

Fig. 3

20. Prove the following:

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \tan A + \cot A$$

Or

Prove the following:

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

21. Construct a triangle ABC in which BC = 8 cm, $\angle B = 45°$ and $\angle C = 30°$. Construct another triangle similar to ΔABC such that its sides are   of the corresponding sides of ΔABC.

22. Point P divides the line segment joining the points A(2, 1) and B(5, –8) such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k.

23. If R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a), then prove that x + y = a + b.

24. In Fig. 4, the boundary of shaded region consists of four semicircular arcs, two smallest being equal. If diameter of the largest is 14 cm and that of the smallest is 3.5 cm, calculate the area of the shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

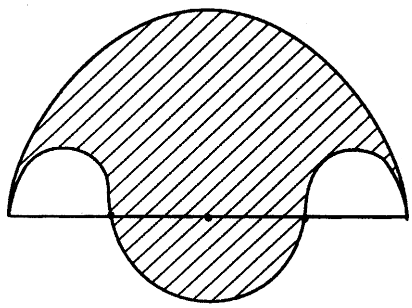
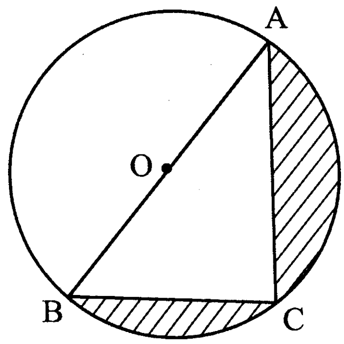


Fig. 4

Or

Find the area of the shaded region in Fig. 5, if AC = 24 cm, BC = 10 cm and O is the centre of the circle.

[Use π = 3.14]



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25. Cards bearing numbers 1, 3, 5, _ _ _, 35 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card bearing

(i) a prime number less than 15.

(ii) a number divisible by 3 and 5.

Section D

Question Numbers 26 to 30 carry 6 marks each.

26. Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46, find the integers.

Or

The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers.

27. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Using the above, prove the following:

If the areas of two similar triangles are equal, then prove that the triangles are congruent.

28. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of the foot of the tower is 30,°. Find the height of the tower.

29. A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459 \frac{3}{7}$ cm^3. The radii of its lower and upper circular ends are 8 cm and 20 cm respectively. Find the cost of metal sheet used in making the container at the rate of Rs. 1.40 per square centimeter. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Or

A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of base of the cone is 21 cm and its volume is $\frac{2}{3}$ of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

30. Find the mean, mode and median of the following frequency distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	7	10	12	8	5

QUESTION PAPER CODE 30/1

SECTION - A

Question numbers 1 to 10 carry 1 mark each.

1. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.

2. If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.

3. If the sum of the first q terms of an A.P. is $2q + 3q^2$, what is its common difference?

4. In Figure 1, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.

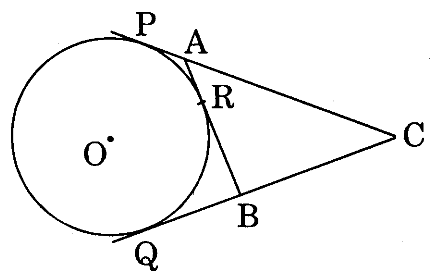

Figure 1

5. In Figure 2, DE || BC in ΔABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.

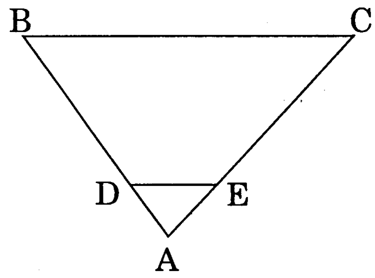

Figure 2

6. If 5x = sec θ and $\frac{5}{x}$ = tan θ, find the value of $\left(x^2 - \frac{1}{x^2}\right)$.

7. What is the distance between the points A(c, 0) and B(0, - c) ?

8. In a Δ ABC, right-angled at C, AC = 6 cm and AB = 12 cm. Find $\angle$A.

9. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, write the height of the frustum.

10. A die is thrown once. What is the probability of getting a number greater than 4 ?

SECTION B

Question numbers 11 to 15 carry 2 marks each.

11. For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$?

12. Find the value of m for which the pair of linear equations

    $2x + 3y - 7 = 0$ and $(m - 1)x + (m + 1)y = (3m - 1)$ has infinitely many solutions.

13. Find the common difference of an A.P. whose first term is 4, the last term is 49 and the sum of all its terms is 265.

14. In Figure 3, there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.

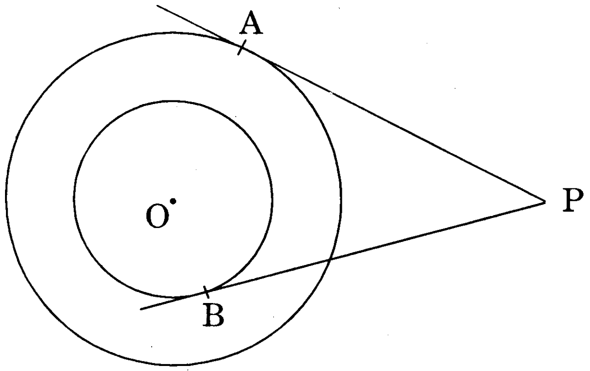

Figure 3

15. Without using trigonometric tables, evaluate the following:

$$\frac{\cos 70^\circ}{3 \sin 20^\circ} + \frac{4\left(\sec^2 59^\circ - \cot^2 31^\circ\right)}{3} - \frac{2}{3}\sin 90^\circ$$

OR

Find the value of sec 60° geometrically.

SECTION C

Question numbers 16 to 25 carry 3 marks each.

16. Prove that $\sqrt{3}$ is an irrational number.

17. Solve the following pair of linear equations for x and y :

$$\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2$$

$$x + y = 2ab$$

OR

The sum of the numerator and the denominator of a fraction is 4 more than twice the numerator. If 3 is added to each of the numerator and denominator, their ratio becomes 2: 3. Find the fraction.

18. In an A.P., the sum of its first ten terms is – 80. and the sum of its next ten terms is – 280. Find the A.P.

19. In Figure 4, ABC is an isosceles triangle in which AB = AC. E is a point on the side CB produced, such that FE $\perp$ AC. If AD $\perp$ CB, prove that:

$$AB \times EF = AD \times EC.$$

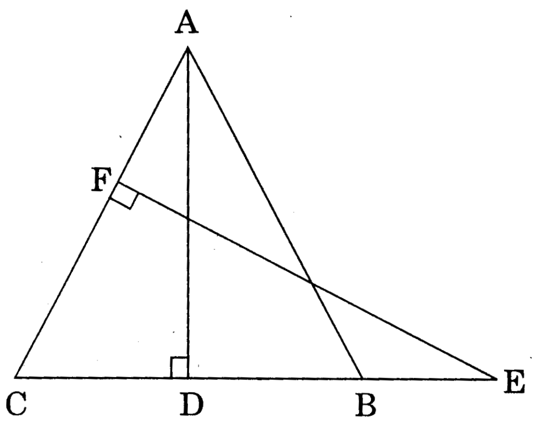

Figure 4

20. Prove the following:

$$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$$

OR

Prove the following:

$$\sin A (1 + \tan A) + \cos A (1 + \cot A) = \sec A + \operatorname{cosec} A$$

21. Construct a triangle ABC in which AB = 8 cm, BC = 10 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of ΔABC.

22. Point P divides the line segment joining the points A(–l, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x – y + 2 = 0, find the value of k.

23. If the points (p, q); (m, n) and (p – m, q – n) are collinear, show that pn = qm.

24. The rain-water collected on the roof of a building, of dimensions 22 m x 20 m, is drained into a cylindrical vessel having base diameter 2 m and height 3.5 m. If the

vessel is full up to the brim, find the height of rain-water on the roof.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

In Figure 5, AB and CD are two perpendicular diameters of a circle with centre O. If OA = 7 cm, find the area of the, shaded region.

$\left[\text{Use } \pi = \frac{22}{7}\right]$

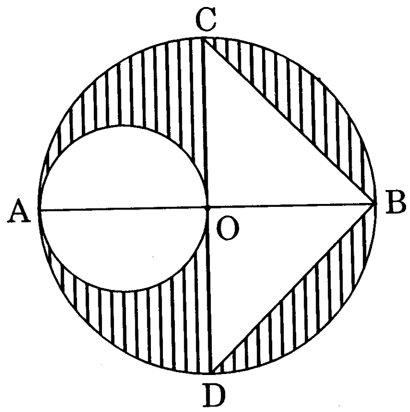

Figure 5

25. A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears

    (i) a two digit number,

    (ii) a number which is a perfect square.

SECTION D

Question numbers 26 to 30 carry 6 marks each.

26. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

27. In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

Using the above, do the following:

In an isosceles triangle PQR, PQ = QR and $PR^2 = 2\ PQ^2$. Prove that $\angle Q$ is a right angle.

28. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732]

OR

The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.

29. The surface area of a solid metallic sphere is 616 cm^2. It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed. $\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

The difference between the outer and inner curved sttrface areas of a hollow right circular cylinder, 14 cm long, is 88 cm^2. If the volume of metal used in making the cylinder is 176 cm^3, find the outer and inner diameters of the cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

30. Draw 'less than ogive' and 'more than ogive' for the following distribution and hence find its median.

Class	20–30	30– 40	40–50	50–60	60–70	70–80	80–90
Frequency	8	12	24	6	10	15	25

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Secondary School Examination

March — 2010

Marking Scheme — Mathematics 30/1/1

General Instructions

1. The Marking Scheme provides general guidelines to reduce subjectivity and maintain uniformity among large number of examiners involved in the marking. The answers given in the marking scheme are the best suggested answers.

2. Marking is to be done as per the instructions provided in the marking scheme. (It should not be done according to one's own interpretation or any other consideration.)Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. Some of the questions may relate to higher order thinking ability. These questions will be indicated to you separately by a star mark. These questions are to be evaluated carefully and the students' understanding / analytical ability may be judged.

5. The Head-Examiners have to go through the first five answer-scripts evaluated by each evaluator to ensure that the evaluation has been done as per instructions given in the marking scheme. The remaining answer scripts meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

6. If a question is attempted twice and the candidate has not crossed any answer, only first attempt is to be evaluated. Write 'EXTRA' with second attempt.

7. A full scale of marks 0 to 80 has to be used. Please do not hesitate to award full marks if the answer deserves it.

QUESTION PAPER CODE 30/1/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. Terminating   2. $x^2 - 6x + 4$ 3. $d = 2a$

4. 1 : 9   5. 5cm   6. $\frac{1}{3}$ 1×10

= 10m

7. $p = 3$ 8. (3.5)   9. $48cm^2$

10. $\frac{3}{26}$

SECTION - B

11. $p(x) = x^3 - 4x^2 - 3x + 12$

$\sqrt{3}$ and $-\sqrt{3}$ are zeroes of $p(x) \Rightarrow (x^2-3)$ is a factor of $p(x)$ ½ m

$$\left(x^3 - 3x - 4x^2 + 12\right) \div \left(x^2 - 3\right) = x - 4$$ 1m

$\Rightarrow x = 4$ is the third zero of $p(x)$ ½ m

12. For a pair of linear equations to have infinitely

many solutions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ½ m

$$\therefore \overset{(i)}{\frac{2}{k-1}} = \overset{(ii)}{\frac{3}{k+2}} = \overset{(iii)}{\frac{7}{3k}}$$

From (i) and (ii) getting k = 7 1 m

k = 7 satisfies (ii) and (iii) and (i) and (iii) also ½ m

$\therefore$ k = 7

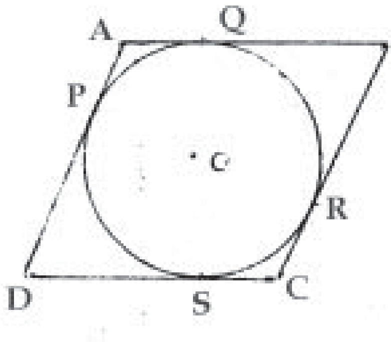
13. Here a = 2, ℓ = 29 and s_n = 155 — 1 m

$\therefore\ 155 = \frac{n}{2}[2+29] \Rightarrow n = 10$

Also, 29 = 2 + (10–1) d $\Rightarrow$ d = 3

$\therefore$ Common difference = 3 — 1 m

14.



As parallelogram ABCD circumscribes a circle with centre O

$\therefore$ AB + CD = BC + AD

[$\because$ AQ = AP, BQ = BR, CR = CS, PD = DS] — ½ m

As ABCD is a parallelogram $\Rightarrow$ AB = DC

and BC = AD

2AB = 2AD or AB = AD — 1 m

$\therefore$ ABCD is a rhombus (As AB = BC = CD = AD) — ½ m

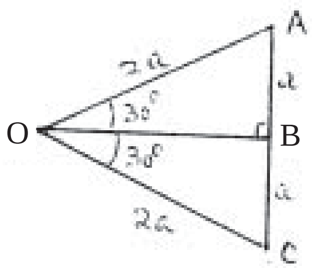
15. $\sec(90^\circ - \theta) = \operatorname{cosec}\theta, \tan(90^\circ - \theta) = \cot\theta, \cos 65^\circ = \sin 25^\circ$

and $\tan 63^\circ = \cot 27^\circ$ — 1 m

$\therefore$ Given expression becomes

$$\frac{(\operatorname{cosec}^2\theta - \cot^2\theta) + (\cos^2 25^\circ + \sin^2 25^\circ)}{3\tan 27^\circ \cot 27^\circ} = \frac{1+1}{3} = \frac{2}{3}$$ — 1 m

OR



Correct Fig. — ½ m

Draw rt . Δ OBA, in which $\angle$BOA = 30^o

Take OA = 2a . Replicate Δ OCB on

the other side of OB $\Rightarrow$ $\angle$ AOC = 60^o — ½ m

and OC = 2a

$\therefore$ Δ AOC is equilateral Δ and AB = a

$$\text{cosec } 30° = \frac{OA}{AB} = \frac{2a}{a} = 2$$ 1m

$\therefore$ cosec $30^o = 2$

SECTION -C

16. Let $2-3\sqrt{5} = x$, where x is a rational number ½ m

$\therefore$ $2-x = 3\sqrt{5}$ or $\frac{2-x}{3} = \sqrt{5}$ ....(i) ½ m

As x is a rational number, so is $\frac{2-x}{3}$ 1 m

$\therefore$ LHS of (i) is rational but RHS of (i) is irrational

$\therefore$ Our supposition that x is rational is wrong

$\Rightarrow$ $2-3\sqrt{5}$ is irrational 1 m

17. Let the fraction be $\frac{x}{y}$, $y \neq 0$

$\therefore$ According to the question $x + y = 2y - 3$ 1 m

or $x = y - 3$.....................................(i)

Also, $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1$..(ii) 1 m

From (i) and (ii), $x = 4$, $y = 7$

$\therefore$ Required fraction = $\frac{4}{7}$ 1 m

OR

$\frac{4}{x} + 3y = 8$ .... (i), $\frac{6}{x} - 4y = -5$ ....(ii)

(i) $\times$ 3 $\Rightarrow$ $\frac{12}{x} + 9y = 24$ and (ii) $\times$ 2 $\Rightarrow$ $\frac{12}{x} - 8y = -10$ 1½ m

Solving to get y = 2 and x = 2 1½ m

18. $S_{10} = -150$ and $S_{20} = -(150+550) = -700$ ½ m

$\therefore \quad -150 = 5(2a + 9d)$ and $-700 = 10(2a + 19d)$ 1m

$\Rightarrow \quad 2a + 9d = -30$ and $2a + 19d = -70$

$\Rightarrow \quad d = -4$ and $a = 3$ 1m

$\therefore$ A . P is $\quad 3, -1, -5, \ldots\ldots\ldots\ldots$ ½ m

19. $AB^2 = AC^2 + BC^2$ ....(i) and $AD^2 = AC^2 + DC^2$

$$= AC^2 + \frac{BC^2}{4} \left(\because DC = \frac{1}{2}BC\right)$$ 1+½ m

or $4\,AD^2 = 4\,AC^2 + BC^2$ .... (ii) ½ m

From (i) and (ii), $AB^2 = AC^2 + 4\,AD^2 - 4\,AC^2$

or $AB^2 = 4\,AD^2 - 3\,AC^2$ 1 m

20. The given expression can be written as

$$\frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}} + \frac{1}{\frac{\sin A}{\cos A}\left(1-\frac{\sin A}{\cos A}\right)}$$ 1 m

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$ ½ m

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A(\sin A - \cos A)} = \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$$ 1 m

$$= \tan A + \cot A + 1$$ ½ m

OR

$$\text{LHS} = \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$ ½ m

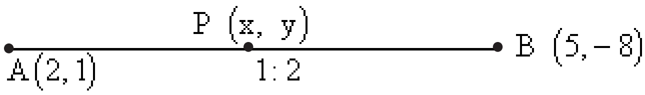
$$= \frac{1-\sin^2 A}{\sin A} \cdot \frac{1-\cos^2 A}{\cos A} = \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} = \sin A \cos A$$ 1½ m

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \frac{1}{\dfrac{\sin^2 A}{\sin A \cos A} + \dfrac{\cos^2 A}{\sin A \cos A}} = \frac{1}{\tan A + \cot A} = \text{RHS}$$ 1 m

21. Correct construction of triangle Δ ABC 1 m

Correct construction of triangle similar to Δ ABC 2m

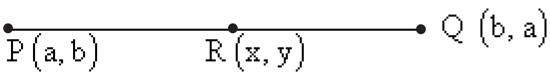
22. $\frac{AP}{AB} = \frac{1}{3} \Rightarrow \frac{AP}{PB} = \frac{1}{2}$ ½ m



$x = \frac{4+5}{3} = 3, \quad y = \frac{-8+2}{3} = -2 \Rightarrow P(3, -2)$ 1+½ =1½ m

P lies on $2x - y + k = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$ 1 m

23.



If P, R, Q are collinear, ar (Δ PRQ) = 0 ½ m

$\therefore \quad a(y - a) + x(a - b) + b(b - y) = 0$

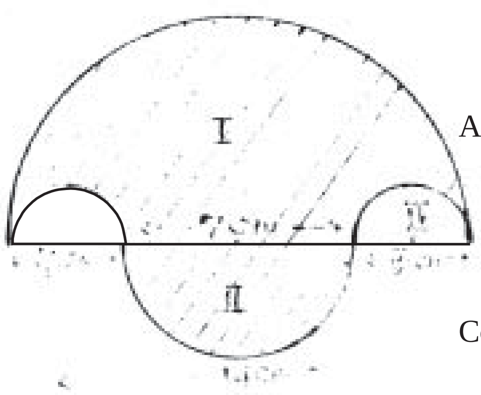
or $\quad ay - a^2 + ax - bx + b^2 - by = 0$

or, $(a - b)(x + y) = a^2 - b^2$ 1 m

$\Rightarrow \quad x + y = a + b$ ½ m

24. Area of semi-circle I $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \text{cm}$

$= 77 \text{ cm}^2$ ½ m



Area of semi-circle II $= \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{77}{4} \text{ cm}^2$ ½ m

Combined area of semi-circles III and IV

$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \text{ cm}^2 = \frac{77}{8} \text{ cm}^2$ 1m

$\therefore$ Area of shaded region $= 77\left[\frac{1}{4} - \frac{1}{8} + 1\right] \text{ cm}^2$

$= \frac{693}{8} \text{ cm}^2$ or 86.625 cm^2 1m

OR

$AB^2 = AC^2 + BC^2$ (as $\angle ACB = 90^o$)

$= 24^2 + 10^2 = 26^2 \quad \Rightarrow \quad AB = 26\text{cm}$ ½ m

$\therefore$ Area of semi-circle ACBOA $= \left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) \text{cm}^2$ 1m

$\therefore$ Area of Δ ACB $= \left(\frac{1}{2} \times 24 \times 10\right) \text{cm} = 120 \text{ cm}^2$ ½ m

$\therefore$ Area of shaded region $= \left[\left(\frac{1}{2} \times 3.14 \times 13 \times 13\right) - 120\right] \text{cm}^2$

$= (265.33 - 120)$ or 145.33 cm^2 1m

25. Total number of cards = 18 ½ m

(i) Prime numbers less than 15 are 3, 5, 7, 11, 13 – Five in number ½ m

$\therefore$ P (a prime no. less than 15) $= \frac{5}{18}$ 1m

(ii) numbers divisible by 3 and 5 is only 15 (one in number) ½ m

$\therefore$ P (a no. divisible by 3 and 5) $= \frac{1}{18}$ ½ m

SECTION D

26. Let the three consecutive numbers be x, $x + 1$, $x + 2$ 1 m

According to the question

$$x^2 + (x + 1)(x + 2) = 46$$

or $2x^2 + 3x - 44 = 0 \Rightarrow 2x^2 + 11x - 8x - 44 = 0$ 2 m

$\Rightarrow (x - 4)(2x + 11) = 0$ 1 m

As x is positive $\Rightarrow x = 4 \left(x = \frac{-11}{2} \text{ rejected} \right)$ 1 m

$\therefore$ The numbers are 4, 5, 6 1 m

OR

Let the two numbers be x, y where $x > y$

$\therefore$ $x^2 - y^2 = 88$ ....(i)

Also, $x = 2y - 5$ ....(ii)

2 m

From (i) and (ii), $(2y - 5)^2 - y^2 = 88$

$\Rightarrow 3y^2 - 20y - 63 = 0$ 1 m

or $3y^2 - 27y + 7y - 63 = 0 \Rightarrow (3y + 7)(y - 9) = 0$

$\Rightarrow$ y = 9, – 7/3 1 m

$x = 2y - 5 = 13$ or $x = -\frac{29}{3}$ 1 m

$\therefore$ The numbers are 13, 9 (Rejecting $x = \frac{-29}{3}$

and $y = {}^{-7}\!/_{3}$) 1 m

27. Correctly stated Given, To Prove, Construction and correct Figure $\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$ 2 m

Correct Proof 2 m

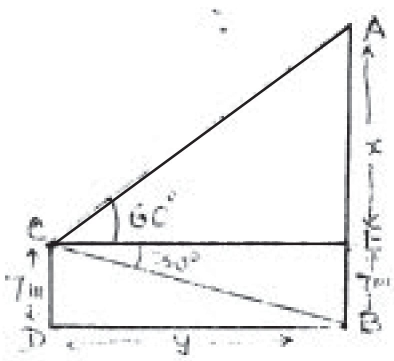
Let ABC and PQR be two similar triangles ½ m

$$\frac{ar\ (ABC)}{ar\ (PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1 \Rightarrow \begin{array}{l} AB = PQ, \\ BC = QR,\ AC = PR \end{array}$$ 1½ m

$\therefore\ \Delta\, ABC \cong \Delta\, PQR\ (\text{by SSS})$ ½ m

28. Let CD be the building and AB, the tower

$\therefore$ Correct Figure 1m



Writing trigonometric equations

(i) $\frac{7}{y} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow y = 7\sqrt{3}$

$= 12.124$ m 1½ m

(ii) $\frac{x}{y} = \tan 60° = \sqrt{3}$ 1½ m

or $\frac{x}{7\sqrt{3}} = \sqrt{3} \Rightarrow x = 21$ 1 m

Height of tower = (21 + 7) m or 28 m 1 m

29. Volume of bucket = $10459\frac{3}{7}\ cm^3$

$= \frac{73216}{7}\ cm^3$

$$\therefore \quad \frac{73216}{7} = \frac{1}{3}\times\frac{22}{7}\times h\left[20^2+8^2+20\times 8\right] \quad \text{1m}$$

$$\therefore\ h = \frac{73216}{7}\times\frac{21}{22}\times\frac{1}{624} = 16\text{ cm} \qquad \text{1m}$$

$$\ell^2 = h^2+(r_2-r_1)^2 = 16^2+(20-8)^2$$

$$= 400$$

$$\Rightarrow \ell = 20\text{ cm} \qquad \text{1m}$$

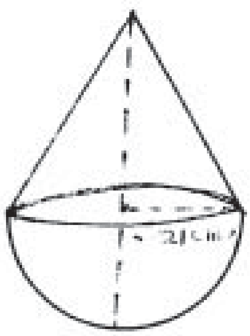
Total surface area of metal sheet used

$$\left.\begin{aligned} &= \frac{22}{7}\times 20\times(20+8)+\frac{22}{7}(8)^2\ \text{cm}^2 \\ &= \left(1760+\frac{1408}{7}\right)\ \text{cm}^2 \end{aligned}\right\} \quad \text{2 m}$$

$$\therefore\ \text{Cost of metal sheet} = \text{Rs}\left(1760+\frac{1408}{7}\right)\frac{14}{10}$$

$$= \text{Rs } 2745.60 \qquad \text{1 m}$$

OR



$$\text{Volume of hemisphere} = \frac{2}{3}\times\frac{22}{7}\times(21)^3\ \text{cm}^3 \qquad \text{1 m}$$

$$\therefore\ \text{Volume of cone} = \left(\frac{2}{3}\cdot\frac{2}{3}\ \frac{22}{7}\times 21\times 21\times 21\right)\ \text{cm}^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times(21)^2\times h \qquad \text{1½ m}$$

$$\Rightarrow h = \frac{2\times 2\times\cancel{22}\times 7\times\cancel{21}}{\cancel{22}\times\cancel{21}} = 28\text{ cm} \qquad \text{1 m}$$

$$\therefore\ \ell^2 = h^2+r^2 = 28^2+21^2 = 1225 = (35)^2$$

$$\Rightarrow \ell = 35\text{ cm} \qquad \text{1 m}$$

Surface area of toy = $2\pi r^2 + \pi r \ell$

$$= \left(2\times\frac{22}{7}\times 21\times 21+\frac{22}{7}\times 21\times 35\right)\ \text{cm}^2$$ 1½ m

$= 5082\ \text{cm}^2$

30.

Classes	0–10	10–20	20–30	30–40	40–50	50–60	60–70
class marks (x_i)	5	15	25	35	45	55	65
$d_i = \frac{x_i - 35}{10}$	–3	–2	–1	0	1	2	3
f_i	4	4	7	10	12	8	5
$f_i d_i$	–12	–8	–7	0	12	16	15

$\sum f_i = 50,\ \sum f_i d_i = 16$ 1 m

(i) $\bar{x} = 35 + \frac{16}{50}\times 10 = 38.2$ 1 m

(ii) Modal Class = 40 – 50 ½ m

$\text{Mode} = 40 + \frac{12-10}{24-18}\times 10 = 43.33$ 1½ m

(iii) Median Class = 30 – 40 ½ m

$\text{Median} = 30 + \frac{\frac{50}{2}-15}{10}\times 10 = 40.00$ 1½ m

Note : If a candidate finds any two of the measures of central tendency correctly and finds the third correctly using Empirical formula, give full credit

QUESTION PAPER CODE 30/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1.	Rational	2.	-1	3.	6	
4.	7 cm	5.	2 cm	6.	$\frac{1}{5}$	1×10
						= 10 m
7.	$\sqrt{2}$ c	8.	60^{o}	9.	3 cm	
10.	$\frac{1}{3}$					

SECTION - B

11. When 3 is a zero, we have $2(3)^2 + (3) + k = 0$ — 1 m

$\Rightarrow$ $18 + 3 + k = 0$ or $k = -21$ — 1 m

12. The condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ ½ m

$$\Rightarrow \frac{2}{m-1} = \frac{3}{m+1} = \frac{7}{3m-1}$$ ½ m

solving to get m = 5 — 1 m

13. $265 = \frac{n}{2}[4 + 49] \Rightarrow n = 10$ — 1 m

$49 = 4 + 9d \Rightarrow d = 5$ — 1 m

14. Join OA, OB and OP

$OP^2 = (5)^2 + (12)^2 = 169 \Rightarrow OP = 13$ cm — 1 m

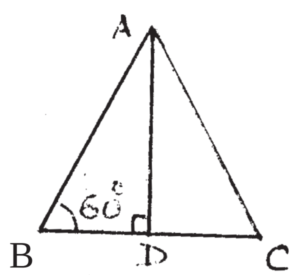
$BP^2 = (13)^2 - (3)^2 = 160 \Rightarrow BP = \sqrt{160}$ cm or $4\sqrt{10}$ cm — 1 m

15. $\cos 70^\circ = \cos(90^\circ - 20^\circ) = \sin 20^\circ$, $\sec^2 59^\circ = \text{cosec}^2 31^\circ$

Given expression = $\dfrac{\sin 20^\circ}{3\sin 20^\circ} + \dfrac{4(\text{cosec}^2 31^\circ - \cot^2 31)}{3} - \dfrac{2}{3}\sin 90^\circ$ 1 m

$= \dfrac{1}{3} + \dfrac{4}{3} - \dfrac{2}{3} = 1$ 1 m

OR



Let ABC be an equilateral triangle and AD $\perp$ BC

If AB = BC = AC = a then BD = $\dfrac{a}{2}$ and $\angle B = 60^\circ$ 1 m

$\sec 60^\circ = \sec B = \dfrac{AB}{BD} = \dfrac{a}{\frac{a}{2}} = 2$ ½ m

Hence $\sec 60^\circ = 2$ ½ m

SECTION -C

16. Let us suppose that $\sqrt{3}$ be a rational number

$\therefore$ Let $\sqrt{3} = \dfrac{p}{q}$, p, q are integers $q \neq 0$ and p and q are coprimes 1 m

$\Rightarrow 3 = \dfrac{p^2}{q^2}$ or $p^2 = 3q^2 \Rightarrow$ 3 divides p^2

Hence 3 divides p .... (i) 1m

Let p = 3a, where a is an integer

$\therefore$ $9a^2 = 3q^2 \Rightarrow q^2 = 3a^2 \Rightarrow$ 3 divides q^2

Hence 3 divides q .... (ii) ½m

But p and q are coprimes, hence a contradiction

$\therefore$ $\sqrt{3}$ is not rational, Hence $\sqrt{3}$ is irrational ½ m

17. Simplifying given equations, we get

$$b^2x + a^2y = a^3b + ab^3; \ x + y = 2ab$$ ½ m

$$\Rightarrow \begin{array}{l} b^2x + a^2y = a^3b + ab^3 \\ \underline{a}^2x \pm a^2y = \underline{2}\, a^3b \\ \hline \end{array}$$

$$\Rightarrow \left(b^2 - a^2\right) x = ab\left(b^2 - a^2\right)$$ ½ m

$$\Rightarrow x = ab$$ 1 m

$$x + y = 2ab \quad \Rightarrow y = ab$$ 1 m

OR

Let the fraction be $\frac{x}{y}$

$\Rightarrow x + y = 2x + 4$ or $x - y + 4 = 0$ .... (i) 1 m

$\frac{x+3}{y+3} = \frac{2}{3} \Rightarrow 3x + 9 = 2y + 6$ or $3x - 2y + 3 = 0$ ....(ii) 1 m

Solving (i) and (ii) to get $x = 5$, $y = 9$ ½ m

$\therefore$ Fraction is $\frac{5}{9}$ ½ m

18. Here $S_{10} = -80$ and $S_{20} = -80 - 280 = -360$ ½ m

$5[2a + 9d] = -80$ or $2a + 9d = -16$ ½ m

$10[2a + 19d] = -360$ or $2a + 19d = -36$ ½ m

Solving to get $d = -2$ and $a = 1$ 1 m

$\Rightarrow$ AP is $1, -1, -3, -5,$ .... ½ m

19. In Δs ABD and ECF, $\angle D = \angle F = 90^0$ and $\angle B = \angle C\ [\because AB = AC]$ 1 m

$\therefore\ \Delta ABD \sim \Delta ECF \quad (AA)$ ½ m

$\Rightarrow \dfrac{AB}{EC} = \dfrac{AD}{EF} \qquad \therefore\ AB \times EF = AD \times EC$ 1+½ m

20. $LHS = \dfrac{(\sin A + \cos A - 1)}{\sin A} \cdot \dfrac{(\cos A + \sin A + 1)}{\cos A}$ 1 m

$= \dfrac{(\sin^2 A + \cos^2 A) + 2 \sin A \cos A - 1}{\sin A\ \cos A}$ 1 m

$= \dfrac{2 \sin A \cos A}{\sin A \cos A} = 2 = RHS$ 1 m

OR

$LHS = \dfrac{\sin A\ (\cos A + \sin A)}{\cos A} + \dfrac{\cos A\ (\sin A + \cos A)}{\sin A}$ 1 m

$= (\sin A + \cos A)\left(\dfrac{\sin^2 A + \cos^2 A}{\cos A \sin A}\right)$ 1 m

$= \dfrac{\sin A + \cos A}{\cos A \sin A} = \sec A + \operatorname{cosec} A \cdot = RHS$ 1 m

21. Constructing ΔABC 1 m

Constructing a triangle (similar to ΔABC with scale facter $\frac{4}{5}$) 2 m

22. $\underset{(-1,3)}{A}$ ——— $\underset{(x,y)}{\overset{K:1}{P}}$ ——— $\underset{(9,8)}{B}$ Let (x,y) be the coordinates of P

$\therefore \quad x = \frac{9k-1}{k+1}, \quad y = \frac{8k+3}{k+1}$ 1 m

P lies on x-y + 2 = 0 $\Rightarrow \frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$ 1m

Solving to get $k = \frac{2}{3}$ 1m

23. Let A (p,q), B (m,n) and C (p–m, q–n) be collinear points

$\therefore$ area Δ ABC = 0 ½ m

$\Rightarrow$ p (n – q + n) + m (q – n – q) + (p – m) (q – n) = 0 1 m

$\Rightarrow$ 2 pn – pq – mn + pq – pn – qm + mn = 0 1 m

$\Rightarrow$ pn = qm ½ m

24. Let h m be the height of water collected on the roof

$\therefore$ Volume of water at the roof = $22 \times 20 \times h \ m^3$............................(i) 1 m

Volume of water in cylinder = $\frac{22}{7} \times (1)^2 \times 3.5 \ m^3$........................(ii) 1m

From (i) and (ii) $h = \frac{1}{40}$ m or 2.5 cm. 1m

OR

Shaded area = Area of circle of radius 7 cm – Area of circle of radius

3.5 cm – area of Δ BCD 1m

$= \left[\frac{22}{7} \times 7 \times 7 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} - \frac{1}{2} \times 14 \times 7\right] cm^2$ 1m

$= [154 - 38.5 - 49] \ cm^2 = 66.5 \ cm^2$ 1m

25. Total number of cards = 89 1m

(i) P (a two digit number) = $\frac{81}{89}$ 1m

(ii) P (a perfect square) = $\frac{8}{89}$ 1m

SECTION D

26. Let the age of sister be x years

$\therefore$ Age of the girl (Elder Sister) = 2 x years ½ m

Four years hence, their ages will be (x + 4) years and (2 x + 4) years 1 m

$\therefore$ $(x + 4)(2x + 4) = 160 \Rightarrow x^2 + 6x - 72 = 0$ 1+1m

$\Rightarrow (x + 12)(x - 6) = 0 \Rightarrow x = 6$ (x = – 12 is rejected) 1½ m

$\therefore$ Ages of two sisters are 6 years and 12 years 1m

27. For correct Given, To Prove, Construction and Figure ½ x 4 = 2 m

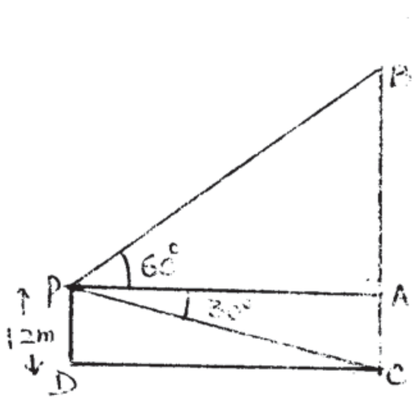
For Correct Proof 2 m

Given $PR^2 = 2PQ^2 = PQ^2 + PQ^2$ ½ m

$= PQ^2 + QR^2$ ($\because$ PQ = QR) ½ m

$\therefore \angle Q = 90^0$ (converse of Pythagoras theorem) 1 m

28.



[Correct Fig.] 1 m

Let distance of cliff from the ship be CD

$\therefore \frac{AB}{AP} = \tan 60^0 = \sqrt{3}$ ½ m

$\therefore AB = \sqrt{3}\,(AP)$ ....(i) 1 m

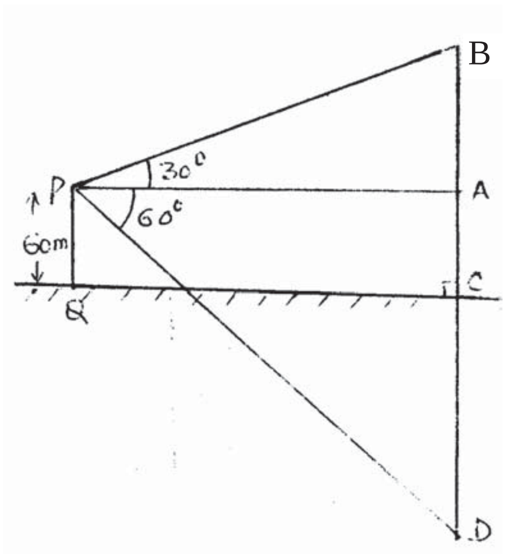
$\frac{AC}{AP} = \tan 30^0 = \frac{1}{\sqrt{3}}$, But AC = 12m ½ m

$\Rightarrow AP = 12\sqrt{3}\text{ m} = 20.784\text{ m}$ 1 m

Putting in (i), $AB = \sqrt{3}\left(12\sqrt{3}\right)\text{m} = 36\text{ m}$ or 35.99 m 1 m

$\therefore$ Height of cliff = 36 + 12 = 48 m 1 m

or 47.99 m



OR

Correct Fig. 1 m

Let P be the given point such that

PQ = 60 m. Let B be the position of cloud

and D be its reflection in the lake

Let BC = h m and AP = x m.

AB = (h – 60) m and AD = (h + 60) m. ½ m

$\therefore$ In Δ PAB, $\frac{AB}{AP} = \tan 30^0$ or $\frac{h\text{-}60}{x} = \frac{1}{\sqrt{3}}$ 1 m

or $x = \sqrt{3}\,(h - 60)\text{ m}$ ½ m

In Δ APD, $\frac{h + 60}{x} = \tan 60^0 = \sqrt{3}$ 1 m

$\therefore\ h + 60 = \sqrt{3}\,x = \sqrt{3}\left[\sqrt{3}\,(h - 60)\right]$ 1m

$\Rightarrow$ h + 60 = 3h – 180 $\Rightarrow$ 2h = 240 $\Rightarrow$ h = 120 m. 1m

$\therefore$ Height of the cloud = 120 m.

29. Here $4 \times \frac{22}{7} \times r^2 = 616 \Rightarrow$ r (radius of sphere) = 7 cm. 1 m

Volume of sphere = $\frac{4}{3}\pi \times (7)^3\ \text{cm}^3$ ....(i) 1 m

Volume of cone = $\frac{1}{3}\pi\ R^2 \times 28\ \text{cm}^3$ ....(ii) 1m

$\Rightarrow \ \frac{1}{3}\pi R^2 \times 28 = \frac{4}{3}\pi (7)^3$ 1m

$\Rightarrow$ R (radius of cone) = 7 cm. 1m

$\therefore$ Diameter = 14 cm. 1m

OR

Here, $2\pi \ \ R\text{-}r \cdot 14 = 88\ cm^2$ 1 m

$\Rightarrow$ $R - r = 1$ cm ....(i)

$V = 176 = \frac{22}{7} \times 14 \left[R^2 - r^2\right]$ 1 m

$\Rightarrow$ $R^2 - r^2 = 4$ i.e $(R + r)\ (R - r) = 4$ 1m

$\Rightarrow$ $R + r = 4$ cm ....(ii) 1m

$\therefore$ Outer radius $= \frac{5}{2}$ cm
Inner radius $= \frac{3}{2}$ cm 1 m

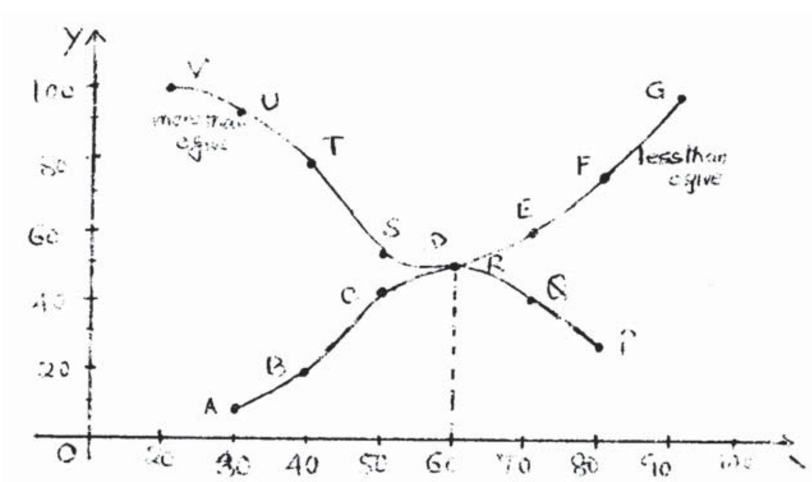
$\therefore$ Outer diameter = 5 cm
Inner diameter = 3 cm 1 m

30. The points for 'less than ogive' are

$\overset{A}{(30,8)},\ \overset{B}{(40,20)},\ \overset{C}{(50,44)},\ \overset{D}{(60,50)},\ \overset{E}{(70,60)},\ \overset{F}{(80,75)},\ \overset{G}{(90,100)}$ 1 m

The points for 'more than ogive' are

$\overset{P}{(80,25)},\ \overset{Q}{(70,40)},\ \overset{R}{(60,50)},\ \overset{S}{(50,56)},\ \overset{T}{(40,80)},\ \overset{U}{(30,92)},\ \overset{V}{(20,100)}$ 1 m



For Correct less than ogive 1½ m

For Correct more than ogive 1½ m

Getting Median = 60 1m