

N.S.
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(Pages : 3)

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MATHEMATICS—Paper I

Total Score : 50

Time : 2 hours

Note.—(1) Answer all questions.

(2) Give the calculations and drawings on the right margin against the answers.

(3) In the answers, irrationals such as π or $\sqrt{2}$ may be used as they are, without conversion to decimals, unless otherwise specified.

SCORES

[For questions 1—4, choose the correct answer from those given in brackets.]

1. What is distance between -2 and 2 in the number line ? 1/2

[0 ; -4 ; 4 ; 2 .]

2. By which of the following, should the polynomial $p(x)$ be divided to get $p\left(\frac{1}{2}\right)$ as remainder ? 1/2

[$x - 2$; $2x - 1$; $x + 2$; $2x + 1$.]

3. The area of the lid of a hemispherical bowl is 10 square centimetres. What is the surface area of the entire bowl with the lid ? 1/2



[20 sq.cm. ; 30 sq.cm. ; 40 sq.cm. ; 45 sq.cm.]

4. The co-ordinates of two points P and Q are $(4, 0)$, $(8, 0)$. What are the co-ordinates of the midpoint of PQ ? 1/2

[(6, 0) ; (0, 6) ; (12, 0) ; (0, 12).]

[For questions 5—8 give answers in one word.]

5. What is the value of x if $x^2 - 2x + 1 = 0$. 1/2

6. If the lateral faces of a square pyramid are equilateral triangles of side 2 centimetres, what is its slant height ? 1/2

7. Find one solution of the equation $ax^2 + bx + c = 0$, given $2x - 3$ is a factor of the polynomial $ax^2 + bx + c$. 1/2

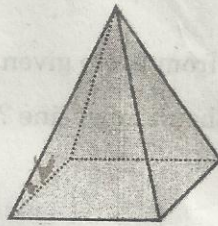
8. By melting a cone and recasting, how many cones of half the radius and half the height can be made ? 1/2

[In the answers to the following questions, write detailed steps whenever necessary.]

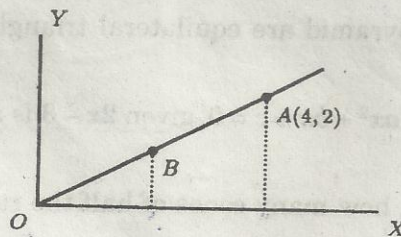
9. Find those real numbers for which the square of the number is equal to double the number. 1

[P.T.O.]

10. What is the remainder on dividing $(x^2 + 3x + 1) + (x^2 - 2x + 1)$ by $x - 1$? 1
11. Prove that if $x - 1$ is a factor of $ax^3 + bx^2 + cx + d$, then $a + b + c + d = 0$. 1
12. Solve the equation $x^2 + 6x + 8 = 0$. 1
13. Prove that the equation $x^2 + x + 1 = 0$ has no solutions. $1\frac{1}{2}$
14. Four isosceles triangles each of base 8 centimetres and altitude 5 centimetres are joined to make a square pyramid. What is the height of the pyramid? $1\frac{1}{2}$

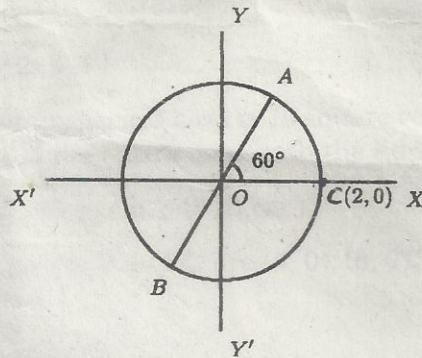


15. Find the real number x satisfying the equation, $|2x + 1| = |2x - 3|$. $1\frac{1}{2}$
16. O and A are points with co-ordinates $(0, 0)$ and $(4, 0)$. An isosceles triangle is to be constructed above the x -axis with OA as base and altitude 3 units. What should be the co-ordinates of the third vertex. $1\frac{1}{2}$
17. What should be the number k so that $kx^3 - 2x^2 - x + 2$ has $x - 1$ as a factor? Find also the other first degree factors of the polynomial, when k is given this value. 2
18. Prove that the triangle with vertexes $(0, 0)$, $(2, 0)$ and $(1, \sqrt{3})$ is equilateral. 2
19. Find all the integers x for which $|x - 2| < 3$. 2
20. Prove that if n is an even number, then both $x - 1$ and $x + 1$ are factors of $x^n - 1$. 2
21. In the picture, O is the origin and A is $(4, 2)$. B is the midpoint of OA. Prove that the co-ordinates of B are $(2, 1)$. 2



22. Prove that the sum of any real number and its reciprocal cannot be equal to $\frac{3}{2}$. 2
23. A petrol tank is in the shape of a cylinder with hemispheres of the same radius as the base of the cylinder attached to both ends. If the total length of the tank is 5 metres and the base radius of the cylinder is 1 metre, how many litres of petrol can it hold? 3

24. Write the polynomial $x^3 - 6x^2 + 11x - 6$ as a product of first degree polynomials. 3
25. A solid hemisphere of radius 6 centimetres is melted and recast into a cone of the same radius. What is the height of the cone? 3
26. A tangent is drawn to a circle from a point outside the circle. The length of the tangent (from the external point to the point of contact) is 1 centimetre more than the radius of the circle and 1 centimetre less than distance between the centre and the external point. What is the radius of the circle? 3
27. From the top of a cone of base radius 10 centimetres and height 24 centimetres, a cone of slant height 13 centimetres is cut-off. What is the volume of the remaining frustum of the cone? 3
28. The length and breadth of a rectangular garden are 80 metres and 40 metres. A path runs outside the garden along its boundary. The area of this path is 1,300 square metres. What is the width of the path? 3
29. In the picture, the origin O is the center of the circle and $C(2, 0)$ is a point on the circle. A , B are the endpoints of a diameter. Find the co-ordinates of the points A and B . 3



30. The base radius of a wooden cylinder is 30 centimetres and its height is 16 centimetres. The base radius of another wooden cylinder of equal volume is 20 centimetres. Find the height of this cylinder. From each of these cylinders, a sphere of the largest possible size is cut out. Find the volume of the spheres.