DEPARTMENT MODEL QUESTION - I

HIGHER SECONDARY – SECOND YEAR

MATHEMATICS

Time : 3 Hours | *Max. Marks : 200*

SECTION – A   40 x 1 = 40

(i) All questions are compulsory.
(ii) Each question carries 1 mark.
(iii) Choose the most suitable answer from the given four alternatives.

(1) If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then the rank of AA^T is

(1) 3 (2) 0 (3) 1 (4) 2

(2) If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse then the values of k

(1) k is any real number (2) $k = -4$ (3) $k \neq -4$ (4) $k \neq 4$

(3) If the equation $-2x + y + z = l$
$x - 2y + z = m$
$x + y - 2z = n$
such that $l + m + n = 0$, then the system has

(1) a non-zero unique solution (2) trivial solution
(3) Infinitely many solutions (4) No Solution

(4) Which of the following is not true regarding the non-singular matrices A, B

(1) A (adj A) = (adj A)A (2) $(AB)^{-1} = B^{-1}A^{-1}$ (3) $(AB)^T = B^TA^T$ (4) $(A^T)^{-1} \neq (A^{-1})^T$

(5) If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $\left(\vec{a} + \vec{b}\right)$ is a unit vector if

(1) $\theta = \frac{\pi}{3}$ (2) $\theta = \frac{\pi}{4}$ (3) $\theta = \frac{\pi}{2}$ (4) $\theta = \frac{2\pi}{3}$

(6) The shortest distance of the point (2, 10, 1) from the plane $\vec{r} \cdot \left(3\vec{i} - \vec{j} + 4\vec{k}\right) = 2\sqrt{26}$ is

(1) $2\sqrt{26}$ (2) $\sqrt{26}$ (3) 2 (4) $\frac{1}{\sqrt{26}}$

(7) If the magnitude of moment about the point $\vec{j} + \vec{k}$ of a force $\vec{i} + a\vec{j} - \vec{k}$ acting through the point $\vec{i} + \vec{j}$ is $\sqrt{8}$ then the value of a is

(1) 1 (2) 2 (3) 3 (4) 4

(8) If the points (3, 2, – 4), (9, 8, – 10) and (λ, 4, – 6) are collinear then λ is

(1) 5 (2) – 5 (3) $\frac{5}{2}$ (4) 10

(9) The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is

(1) (0, 0, – 4) (2) (1, 0, 0) (3) (0, 2, 0) (4) (1, 2, 0)

(10) The centre and radius of the sphere given by $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$ is

(1) (– 3, 4, – 5), 49 (2) (– 6, 8, – 10), 1 (3) (3, – 4, 5), 7 (4) (6, – 8, 10), 7

(11) The value of $\left[\frac{-1 + i\sqrt{3}}{2}\right]^{100} + \left[\frac{-1 - i\sqrt{3}}{2}\right]^{100}$ is

(1) 2 (2) 0 (3) – 1 (4) 1

(12) If the point represented by the complex number iz is rotated about the origin through the angle $\frac{\pi}{2}$ in the counter clockwise direction then the complex number representing the new position is

(1) iz (2) $-iz$ (3) $-z$ (4) z

(13) If $\frac{1-i}{1+i}$ is a root of the equation $ax^2 + bx + 1 = 0$, where a, b are real then (a, b) is

(1) (1, 1) (2) (1, – 1) (3) (0, 1) (4) (1, 0)

(14) Which of the following is incorrect?

(1) $|Z_1 Z_2| = |Z_1| \, |Z_2|$ (2) $|Z_1 + Z_2| \le |Z_1| + |Z_2|$

(3) $|Z_1 - Z_2| \le |Z_1| - |Z_2|$ (4) $|Z_1 + Z_2|^2 = (Z_1 + Z_2)\left(\overline{Z_1} + \overline{Z_2}\right)$

(15) The axis of the parabola $y^2 - 2y + 8x - 23 = 0$ is

(1) $y = -1$ (2) $x = -3$ (3) $x = 3$ (4) $y = 1$

(16) The line $4x + 2y = c$ is a tangent to the parabola $y^2 = 16x$ then c is

(1) – 1 (2) – 2 (3) 4 (4) – 4

(17) The area of the triangle formed by the tangent at any point on the rectangular hyperbola $xy = 72$ and its asymptotes is

(1) 36 (2) 18 (3) 72 (4) 144

(18) The centre of the ellipse is (4, – 2) and one of the focus is (4, 2). The other focus is

(1) (4, 0) (2) (8, 0) (3) (4, – 6) (4) (8, – 6)

(19) The parametric equations of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ are

(1) $x = a \sin^3 \theta$; $y = a \cos^3 \theta$ (2) $x = a \cos^3 \theta$; $y = a \sin^3 \theta$

(3) $x = a^3 \sin \theta$; $y = a^3 \cos \theta$ (4) $x = a^3 \cos \theta$; $y = a^3 \sin \theta$

(20) $\lim\limits_{x \to 0} \frac{a^x - b^x}{c^x - d^x}$

(1) ∞ (2) 0 (3) $\log \frac{ab}{cd}$ (4) $\frac{\log (a/b)}{\log (c/d)}$

(21) The function $f(x) = 1 - x^2$ in $0 \le x \le 1$ has

(1) no absolute maximum (2) no absolute minimum

(3) no local maximum (4) absolute and local maximum

(22) The function $f(x) = x^2 - x + 1$ in [0, 1] is

(1) an increasing function (2) a monotonic function

(3) a decreasing function (4) neither decreasing nor increasing

(23) If $u = \frac{1}{\sqrt{x^2 + y^2}}$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to

(1) $\frac{1}{2} u$ (2) u (3) $\frac{3}{2} u$ (4) $-u$

(24) If $u = y \sin x$, then $\frac{\partial^2 u}{\partial x \, \partial y}$ is equal to

(1) $\cos x$ (2) $\cos y$ (3) $\sin x$ 4) 0

(25) The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is

(1) 48 (2) 24 (3) 12 (4) 96

(26) The curved surface area of a sphere of radius 5, intercepted between two parallel planes of distance 2 and 4 from the centre is

(1) 20π (2) 40π (3) 10π (4) 30π

(27) Volume of solid obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major and minor axes are in the ratio

(1) $b^2 : a^2$ (2) $a^2 : b^2$ (3) $a : b$ (4) $b : a$

(28) The value of $\int_{-\pi/4}^{\pi/4} x^3 \sin^2 x \, dx$ is

(1) 0 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

(29) $y = cx - c^2$ is the general solution of the differential equation

(1) $(y')^2 - xy' + y = 0$ (2) $y'' = 0$ (3) $y' = c$ (4) $(y')^2 + xy' + y = 0$

(30) The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{1/3} = x$ is

(1) of order 2 and degree 1
(2) of order 1 and degree 2
(3) of order 1 and degree 6
(4) of order 1 and degree 3

(31) Solution of $\frac{dx}{dy} + mx = 0$, is

(1) $x = ce^{my}$ (2) $x = ce^{-my}$ (3) $x = my + c$ (4) $x = c$

(32) The particular integral of $(D^2 + 7D + 12)y = e^{2x}$ is

(1) $30e^{2x}$ (2) $\frac{1}{22} e^{2x}$ (3) $22\, e^{2x}$ (4) $\frac{1}{30} e^{2x}$

(33) Which of the following is a tautology?

(1) $p \vee q$ (2) $p \wedge q$ (3) $p \vee \sim p$ (4) $p \wedge \sim p$

(34) $p \leftrightarrow q$ is equivalent to

(1) $p \rightarrow q$ (2) $q \rightarrow p$ (3) $(p \rightarrow q) \vee (q \rightarrow p)$ (4) $(p \rightarrow q) \wedge (q \rightarrow p)$

(35) The value of $[3] +_{11} ([5] +_{11} [6])$ is

(1) [0] (2) [1] (3) [2] (4) [3]

(36) Which of the following is a statement?

(1) Give me a cup of tea
(2) Wish you all success
(3) How beautiful you are!
(4) The set of rational numbers is finite

(37) A random variable X has the following probability distribution

X	0	1	2	3	4	5
P(X = x)	1/4	$2a$	$3a$	$4a$	$5a$	1/4

Then $P(1 \le x \le 4)$ is

(1) $\frac{10}{21}$ (2) $\frac{2}{7}$ (3) $\frac{1}{14}$ (4) $\frac{1}{2}$

(38) The distribution function $F(X)$ of a random variable X is

(1) a decreasing function
(2) non decreasing function
(3) a constant function
(4) increasing first and then decreasing

(39) In a Poisson distribution if $P(X = 2) = P(X = 3)$ then the value of its parameter λ is

(1) 6 (2) 2 (3) 3 (4) 0

(40) A property of cumulative distribution function $F(x)$ is

(1) it is strictly increasing
(2) $0 < F(x) < 1$, $-\infty < x < \infty$
(3) $F(-\infty) = -1$
(4) $F(\infty) = 1$

SECTION – B

(i) Answer 10 questions.

(ii) Question 55 is compulsory and choose any 9 questions from the remaining

(iii) Each question carries 6 marks.

(41) Examine the consistency of the system of equations $10 \times 6 = 60$

$2x - 3y + 7z = 5, \quad 3x + y - 3z = 13, \quad 2x + 19y - 47z = 32$

(42) Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and verify the result

$A\,(\text{adj } A) = (\text{adj } A)A = |A| \,.\, I$

(43) (a) For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}\,.\,\vec{i})\,\vec{i} + (\vec{r}\,.\,\vec{j})\,\vec{j} + (\vec{r}\,.\,\vec{k})\,\vec{k}$

(b) Find the projection of the vector $7\vec{i} + \vec{j} - 4\vec{k}$ on $2\vec{i} + 6\vec{j} + 3\vec{k}$

(44) Find the vector and cartesian equation of the sphere on the join of the points A and B having position vectors $2\vec{i} + 6\vec{j} - 7\vec{k}$ and $-2\vec{i} + 4\vec{j} - 3\vec{k}$ respectively as a diameter. Find also the centre and radius of the sphere.

(45) If $\arg(z - 1) = \frac{\pi}{6}$ and $\arg(z + 1) = 2\frac{\pi}{3}$ then prove that $|z| = 1$

(46) Find the two tangents that can be drawn from (1,2) to the hyperbola $2x^2 - 3y^2 = 6$

(47) Determine where the curve $y = x^3 - 3x + 1$ is cancave upward, and where it is concave downward. Also find the inflection points.

(48) (a) Show that e^{-x} is strictly decreasing in R.

(b) Find the Maclaurin's series for $\cos x$.

(49) The time of swing T of a pendulum is given by $T = k\sqrt{l}$ where k is a constant. Determine the percentage error in the time of swing if the length of the pendulum l changes from 32.1 cm to32.0 cm.

(50) Evaluate the following problem using properties of integration $\int_0^3 \frac{\sqrt{x}\,dx}{\sqrt{x} + \sqrt{3-x}}$

(51) Solve the following differential equation : $(D^2 + 3D - 4)\,y = x^2$

(52) Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

(53) Suppose that the probability of suffering a side effect from a certain vaccine is 0.005. If 1000 persons are inoculated. Find approximately the probability that (i) atmost 1 person suffer. (ii) 4, 5 or 6 persons suffer. $[e^{-5} = 0.0067]$

(54) If the height of 300 students or normally distributed with mean 64.5" and standard deviation 3.3" find the height below which 99% of the students lie.

$f(x) = c\, e^{-x^2 + 3x}, \; -\infty < X < \infty.$

(55) (a) If $(a_1 + ib_1)(a_2 + ib_2) \ldots (a_n + ib_n) = A + iB$,

prove that (i) $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \ldots (a_n^2 + b_n^2) = A^2 + B^2$

(ii) $\tan^{-1}\left(\frac{b_1}{a_1}\right) + \tan^{-1}\left(\frac{b_2}{a_2}\right) + \ldots + \tan^{-1}\left(\frac{b_n}{a_n}\right) = n\pi + \tan^{-1}\left(\frac{B}{A}\right), n \in Z$

OR

(b) Show that the set of four matrices $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$ form an abelian group, under multiplication of matrices.

SECTION – C

(i) Answer 10 questions.

(ii) Question 70 is compulsory and choose any 9 questions from the remaining

(iii) Each question carries 10 marks.

(56) Solve the following by using determinant method. $10 \times 10 = 100$

$x + y + 2z = 6 \quad ; \quad 3x + y - z = 2 \quad ; \quad 4x + 2y + z = 8$

(57) Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect and hence find the point of intersection.

(58) Find the vector and cartesian equation of the plane through the point (1, 3, 2) and parallel to the lines

$\frac{x+1}{2} = \frac{y+2}{-1} = \frac{z+3}{3}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+2}{2}$

(59) If α and β are the roots of the equation $x^2 - 2px + (p^2 + q^2) = 0$ and

$\tan\theta = \frac{q}{y+p}$ show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = q^{n-1} \frac{\sin n\theta}{\sin^n\theta}$

(60) A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. find (i) the equation of the comet's orbit (ii) how close does the comet come nearer to the sun? (Take the orbit as open rightward).

(61) Find the eccentricity, centre, foci, vertices of the ellipse $25x^2 + 9y^2 - 50x + 36y - 164 = 0$

(62) A car *A* is travelling from west at 50 km/hr. towards East and car B is travelling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car *A* is 0.3 kilometers and car *B* is 0.4 kilometers from the intersection?

(63) A poster is to have an area of 180 cm^2 with 1 cm margins at the bottom and sides and a 2 cm margin at the top. What dimensions will give the largest printed area?

(64) Trace the following curve : $y = x^3$

(65) Find the area between the curves $y = x^2 - x - 2$, x-axis and the lines $x = -2$ and $x = 4$

(66) Derive the formula for the volume of a right circular cone with radius '*r*' and height '*h*'.

(67) Solve : $(1 + e^{x/y})dx + e^{x/y}(1 - x/y)\, dy = 0$ given that $y = 1$, where $x = 0$

(68) Show that the set *G* of all rational numbers except – 1 forms an abelian group with respect to the operation * given by $a * b = a + b + ab$ for all $a, b \in G$.

(69) Find the Mean and Variance for the following probability density function

$$f(x) = \begin{cases} xe^{-x} & , \text{if } x > 0 \\ 0 & ,\text{otherwise} \end{cases}$$

(70) (a) The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in the city in 1960 and 1,60,000 in 1990 what population may be anticipated in 2020.

(OR)

(b) A satellite is travelling around the earth in an elliptical orbit having the earth at a focus and of eccentricity 1/2 . The shortest distance that the satellite gets to the earth is 400 kms. Find the longest distance that the satellite gets from the earth.