227

Reg. No. : ....

Name : ....

SECOND YEAR HIGHER SECONDARY
MODEL EXAMINATION, FEBRUARY – 2024

Part – III

Time : 2 Hours

MATHEMATICS (SCIENCE)

Cool-off time : 15 Minutes

Maximum : 60 scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Questions from 1 to 8 carries 3 scores each. Answer only 6 questions. **(6 × 3 = 18)**

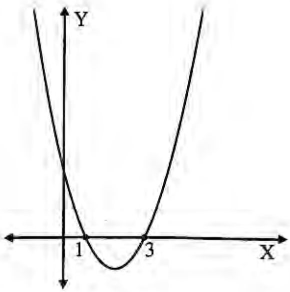
1. Express the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

2. (i) Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$. **(1)**

   (ii) Find the principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$. **(2)**

3. Discuss the continuity of the function $f(x) = \begin{cases} x^2 - 1 & , \ x < 2 \\ 4 & , \ x = 2 \\ 2x - 1 & , \ x > 2 \end{cases}$

4. Consider a third degree polynomial function f(x). The figure given below is the graph of f'(x).



   (i) Find the intervals on which the function is increasing and decreasing. **(2)**

   (ii) Identify the point of local maxima and minima. **(1)**

5. Evaluate $\int \frac{x-1}{x^2-4x-5}\,dx$.

6. (i) If A(1, 5, 3) and B(4, 5, 7) be two points, then find vector $\overrightarrow{AB}$. **(1)**

(ii) Find a unit vector in the direction of $\overrightarrow{AB}$. **(1)**

(iii) Which of the following is a vector perpendicular to $\overrightarrow{AB}$? **(1)**

(a) $15\hat{i}+20\hat{k}$ (b) $15\hat{i}-20\hat{k}$

(c) $-5\hat{j}$ (d) $5\hat{k}$

7. (i) Let $f : \mathbb{N} \to \mathbb{N}$ be function defined by $f(x) = \begin{cases} 1 & , x=1 \\ x-1 & , x>1 \end{cases}$.

Choose the correct answer. **(1)**

(a) f is one-one and onto.

(b) f is many-one and onto.

(c) f is one-one but not onto.

(d) f is neither one-one nor onto.

(ii) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x$ is invertible and hence write the inverse function. **(2)**

8. A die is thrown twice and the sum of numbers appearing is observed to be 9. What is the conditional probability that the number 5 has appeared at least once ?

Questions from 9 to 16 carries 4 scores each. Answer only 6 questions. **(6 × 4 = 24)**

9. Consider the relation R = {(1, 3), (1, 1), (2, 2), (3, 3), (3, 1)} defined on set A = {1, 2, 3}.

   (i) Show that R is an equivalence relation. **(3)**

   (ii) Find the equivalence classes. **(1)**

10. (i) Find the value of x and y, if $\begin{bmatrix} x & 3 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} 1 & 0 \\ y & 3 \end{bmatrix}$. **(2)**

    (ii) If $A = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -3 \end{bmatrix}$ be two matrices, then find AB. **(2)**

11. (i) The volume of a cube is increasing at a rate of 24 cm^3/s. How fast is the surface area increasing when the length of an edge is 6 cm ? **(3)**

    (ii) Find the local minimum value of the function $f(x) = |x| - 2, x \in \mathbb{R}$. **(1)**

12. Find the area enclosed by the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ using integration.

13. (i) Which of the following is a solution of the differential equation y'' – y' = 0 ? **(1)**

    (a) $y = e^x + 1$ (b) $y = e^{-x+1}$

    (c) $y = \sin x + 1$ (d) $y = \sin(x + 1)$

    (ii) Find the general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{9 - y^2}}{x}$. **(3)**

14. (i) Find the projection of vector $\vec{a} = 2\hat{i} + \hat{j}$ on the vector $\vec{b} = 2\vec{a}$. **(1)**

(ii) Find the area of the parallelogram whose adjacent sides are given by the vectors $3\hat{i} + \hat{j} + 4\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$. **(3)**

15. Find the shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{2}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$.

16. (i) If A and B are independent events with P(A) = 0.4 and P(B) = 0.5, then find P(neither A nor B). **(1)**

(ii) A man is known to speak truth 4 out of 5 times. He throws a die and reports that it is 5. Find the probability that it is actually a five. **(3)**

Questions from 17 to 20 carries 6 scores each. Answer only 3 questions. **(3 × 6 = 18)**

17. Solve the system of linear equations using matrix method :

$x + 2y + z = 18$

$2x + y + z = 5$

$x - 3y + 4z = 3$

18. Find $\frac{dy}{dx}$ of the following :

(i) $y = \sqrt{\sin x}$ **(1)**

(ii) $x = a(t - \sin t),\ y = a(1 + \cos t)$ **(3)**

(iii) $y = x^x$ **(2)**

19. Evaluate the following :

(i) $\int e^{x} \sin x \, dx$ **(3)**

(ii) $\int_{0}^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} \, dx$ **(3)**

20. Solve the linear programming problem graphically :

Maximise : $Z = 3x + 7y$

Subject to the constraints :

$$3x + 4y \leq 60$$

$$x + y \geq 10$$

$$x - 3y \leq -6$$

$$x \geq 0$$