First Year Model Exam.

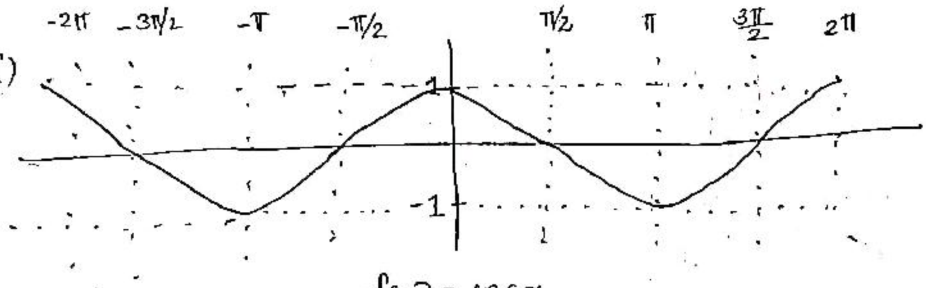
1) $A = \{-1, 1\}, \quad B = \{0, 2, 3\}$

(i) $A \times B = \{(-1,0), (-1,2), (-1,3), (1,0), (1,2), (1,3)\}$.

(ii) No. of relations $= 2^{pq} = 2^{2\times 3} = 2^6 = \underline{\underline{64}}$

(iii) Subsets of B are: $\{0\}, \{2\}, \{3\}, \{0,2\}, \{0,3\}, \{2,3\}, \{0,2,3\}, \phi$

2)
(i)

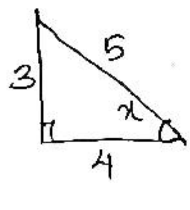


$f(x) = \cos x$.

From the graph it is clear that $\cos x$ increases from -1 to 0 in $\left[\pi, \frac{3\pi}{2}\right]$

$\therefore$ option c

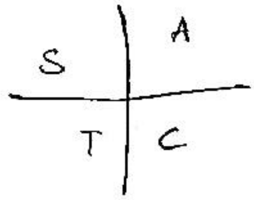
(ii) $\sin x = \frac{3}{5}$



$\sin x = \dfrac{\text{opp. side}}{\text{hypotenuse}} = \dfrac{3}{5}$

$\therefore$ other side $= \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$.

$\therefore \cos x = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \underline{\underline{\dfrac{-4}{5}}}$



[$\because$ $\cos x$ is $-$ve in 2nd quadrant].

(iii) $\frac{5\pi}{3} = \frac{5 \times 180^\circ}{3} = 5 \times 60^\circ = \underline{\underline{300^\circ}}$ [put $\pi = 180^\circ$]

3) $\frac{4-3x}{2} \geq \frac{1-x}{4} - 2.$

$\frac{4-3x}{2} \geq \frac{1-x-8}{4}$

$\frac{2(4-3x)}{2 \times 2} \geq \frac{-7-x}{4}$

$\frac{8-6x}{4} \geq \frac{-7-x}{4}$

$8 - 6x \geq -7 - x$

$-6x + x \geq -7 - 8$

$-5x \geq -15$

$x \leq \frac{-15}{-5} = 3.$

$\therefore$ Solution is $\underline{\underline{(-\infty, 3]}}$

4) (i) Total 5 card combinations $= 52C_5$

$4C_1$	$48C_4$.

Total aces = 4.

1 ace from 4 $= 4C_1$

4 from rest 48 $= 48C_4$.

$\therefore$ Total required combinations $= 4C_1 \times 48C_4$

$= \frac{\cancel{4}}{1} \times \frac{\overset{8}{\cancel{48}} \times 47 \times 46 \times 45}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4}}$

$= \underline{\underline{7,78,320}}$

(ii) $^nC_4 = {^nC_6}$,

$\therefore n = 4+6 = 10$

$^nC_8 = {^{10}C_8} = {^{10}C_2} = \frac{\overset{5}{\cancel{10}} \times 9}{1 \times \cancel{2}} = \underline{\underline{45}}$

$\boxed{\because {^{10}C_8} = {^{10}C_2}}$

$^nC_r = {^nC_{n-r}}$.

5) (i) No. of terms $= \underline{\underline{2n+1}}$

(ii) $\left(x + \frac{3}{x}\right)^4 = {4C_0}\, x^4 + {4C_1}\, x^3\left(\frac{3}{x}\right) + {4C_2}\, x^2\left(\frac{3}{x}\right)^2 + {4C_3}\, x\left(\frac{3}{x}\right)^3 + {4C_4}\left(\frac{3}{x}\right)^4$

$= 1\cdot x^4 + \frac{4\cdot x^3 \times 3}{x} + \frac{6\cdot x^2 \cdot 9}{x^2} + \frac{4\cdot x \cdot 27}{x^3} + \frac{1\cdot 81}{x^4}$

$= x^4 + 12x^2 + 54 + \frac{108}{x^2} + \frac{81}{x^4}$

6) (i) intercept form is $\frac{x}{a} + \frac{y}{b} = 1$.

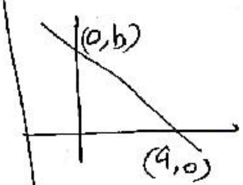
$\therefore \frac{x}{-4} + \frac{y}{3} = 1$

$\frac{3x - 4y}{-12} = 1$

$3x - 4y = -12$

$\underline{\underline{3x - 4y + 12 = 0}}$

Intercept form



$\frac{x}{a} + \frac{y}{b} = 1$

1 → 0
1 1 → 1
1 2 1 → 2
1 3 3 1 → 3
1 4 6 4 1 →

(ii) $d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right| = \left|\frac{3(0) - 4(0) + 12}{\sqrt{3^2+4^2}}\right|$

$(x_1, y_1) = (0,0) =$ origin

$= \left|\frac{12}{\sqrt{9+16}}\right| = \frac{12}{\sqrt{25}} = \underline{\underline{\frac{12}{5} \text{ units}}}$

7) $r = 5,$

centre $= (x, 0)$, point $= (2, 3)$

radius $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - x)^2 + (3 - 0)^2}$

$= \sqrt{(2 - x)^2 + 3^2}$

$= \sqrt{4 - 4x + x^2 + 9}$

$r. = \sqrt{x^2 - 4x + 13} = 5$

Squaring both sides, $x^2 - 4x + 13 = 5^2 = 25$

$x^2 - 4x + 13 - 25 = 0$

$x^2 - 4x - 12 = 0.$

$(x - 6)(x + 2) = 0$

$x = 6,$ or $x = -2$

$-6 \times 2 = -12$
$-6 + 2 = -4$

$\therefore$ centre $= (6, 0)$ or $(-2, 0)$; $r = 5.$

$\therefore$ Equations are:

$(x - 6)^2 + (y - 0)^2 = 5^2.$

$x^2 - 12x + 36 + y^2 = 25$

$x^2 + y^2 - 12x + 36 - 25 = 0$

$x^2 + y^2 - 12x + 11 = 0$

$(x + 2)^2 + (y - 0)^2 = 5^2$

$x^2 + 4x + 4 + y^2 = 25$

$x^2 + y^2 + 4x + 4 - 25 = 0$

$x^2 + y^2 + 4x - 21 = 0.$

$\therefore$ Equations of circles are

$x^2 + y^2 - 12x + 11 = 0$ and

$x^2 + y^2 - 4x - 21 = 0$

8) (i) $\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1.$

option A

(ii) $\lim_{x \to 0} \left(\frac{1-\cos x}{\sin x}\right) = \lim_{x \to 0} \frac{\cancel{2}\sin^2\left(\frac{x}{2}\right)}{\cancel{2}\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)} = \lim_{x \to 0} \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$

$= \lim_{x \to 0} \tan\left(\frac{x}{2}\right)$

$= \lim_{\frac{x}{2} \to 0} \tan\left(\frac{x}{2}\right)$

$= \underline{\underline{0}}.$

$1-\cos x = 2\sin^2\frac{x}{2}$

$\sin x = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

$\lim_{x \to 0} \tan x = 0$

9) (i) $(-1, 3] = \{x : x \in R, -1 < x \leq 3\}.$

(ii) $A \cap A' = \phi$

option B.

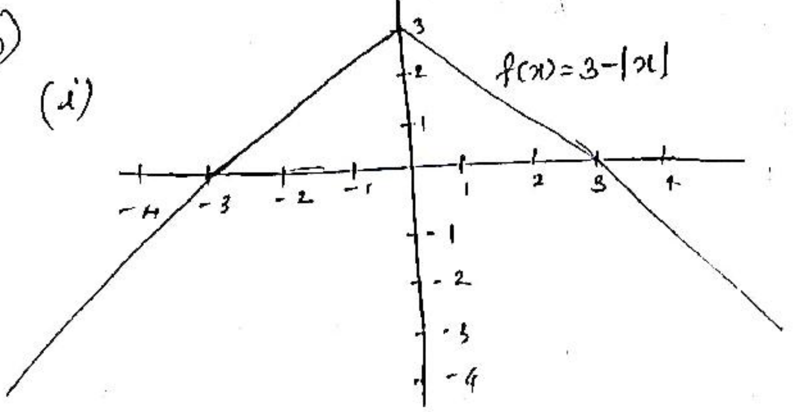
(iii) $A \cup B = \{2, 3, 4, 5\}$

$B - A = \{4, 5\}.$

$A \cap B = \{3\}.$

$(A \cup B)' = \{1, 6\}.$

10) (i)



(ii) Range $= (-\infty, 3]$

(iii) $\sqrt{4-x^2}$

Square root function ≥ 0

$\sqrt{4-x^2} \geq 0$

Squaring both sides.

$4-x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4 \therefore$ ie, $x^2 \leq 2^2$

$\boxed{x^2 \leq a^2 \text{ means } -a \leq x \leq a}$

$\Rightarrow \quad -2 \leq x \leq 2$

$\therefore$ Domain $= [-2, 2]$

11) (i) $z = \dfrac{5+\sqrt{2}i}{1-\sqrt{2}i} = \dfrac{(5+\sqrt{2}i)}{(1-\sqrt{2}i)} \times \dfrac{(1+\sqrt{2}i)}{(1+\sqrt{2}i)}$

$= \dfrac{5+5\sqrt{2}i+\sqrt{2}i+(\sqrt{2}i)^2}{1^2+(\sqrt{2})^2}$

$= \dfrac{5+6\sqrt{2}i+2i^2}{1+2}$

$= \dfrac{5+6\sqrt{2}i-2}{3}$

$= \dfrac{3+6\sqrt{2}i}{3} = \dfrac{3}{3} + \dfrac{6\sqrt{2}i}{3}$

$= 1+i2\sqrt{2}$

(ii) $z^{-1} = \dfrac{\bar{z}}{|z|^2} = \dfrac{4+3i}{4^2+3^2} = \dfrac{4+3i}{16+9} = \dfrac{4+3i}{25} = \dfrac{4}{25}+\dfrac{3i}{25}$

12) (i) INDEPENDENCE

Total letters $= 12$

$N = 3$

$D = 2$

$E = 4$

Total arrangements $= \dfrac{12!}{3!\,2!\,4!} = \dfrac{12\times11\times10\times9\times\cancel{8}^{4}\times7\times\cancel{6}\times5\times\cancel{4!}}{\cancel{4!}\times3\times\cancel{2}\times\cancel{2}}$

$= 16,63,200$

(ii) Total vowels = 5 (4 E + 1 I)

4E+1I (5)	N=3, D=2.
1 unit	7 letters.

taking all vowels as a single unit,

Total arrangements $= \cancel{\frac{7!}{3!}} = \dfrac{8!}{3!\,2!}$

Vowels permute among them in $\dfrac{5!}{4!}$ ways.

$\therefore$ Total no. of arrangements in which vowels occur together

$$= \frac{8!}{3!\,2!} \times \frac{5!}{4!}$$

$$= \frac{8\times7\times6\times5\times\cancel{4}^{2}\times\cancel{3!}}{\cancel{3!}\times\cancel{2!}} \times \frac{5\times\cancel{4!}}{\cancel{4!}}$$

$$= \underline{\underline{16,800}}$$

13) (i) slope of $l_1 = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

(ii) l_1 passes through $(0,0)$, slope $= \dfrac{1}{\sqrt{3}}$

$\therefore$ Eqn is $y = mx$.

$$y = \frac{1}{\sqrt{3}}x$$

$$\sqrt{3}y = x$$

$$x = \sqrt{3}y$$

$$\underline{\underline{x - \sqrt{3}y = 0}}$$

(iii) l_2 passes through $(4,0)$.

product of slopes of 2 $\perp$r lines, $m_1 \times m_2 = -1$.

$$\frac{1}{\sqrt{3}} \times m_2 = -1$$

$$m_2 = \frac{-1 \times \sqrt{3}}{1} = -\sqrt{3}.$$

$\therefore$ Eqn is, $y - y_1 = m(x - x_1)$

$$y - 0 = -\sqrt{3}(x-4)$$

$$y = -\sqrt{3}x + 4\sqrt{3}$$

$$\sqrt{3}\,x + y - 4\sqrt{3} = 0.$$

14). $\frac{x^2}{2^2} + \frac{y^2}{5^2} = 1$. Compairing with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

$\therefore a^2 = 25,\ a = 5$

$b^2 = 4,\ b = 2.$

$c = \sqrt{a^2 - b^2} = \sqrt{25-4} = \sqrt{21}$

foci $= (0, \pm c) = (0, \pm\sqrt{21})$

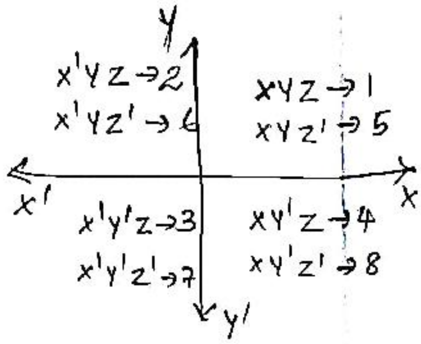
vertices $= (0, \pm a) = (0, \pm 5)$

eccentricity, $e = \frac{c}{a} = \frac{\sqrt{21}}{5}$

length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{5} = \frac{8}{5}$

15) (i) 6^{th} octant $= (x'yz') = (-3, 1, -2)$.

Option B



(ii) Let $A(-2,3,5)$, $B(1,2,3)$, $C(7,0,-1)$.

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$= \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$

$$= \sqrt{3^2 + 1^2 + 2^2}$$

$$= \sqrt{9+1+4} = \sqrt{14}.$$

$$BC = \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$$
$$= \sqrt{6^2+2^2+4^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= \sqrt{14\times4} = 2\sqrt{14}$$

$$AC = \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$$
$$= \sqrt{9^2+3^2+6^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= \sqrt{14\times9} = 3\sqrt{14}.$$

$$AB+BC = \sqrt{14}+2\sqrt{14} = 3\sqrt{14} = \underline{\underline{AC}}.$$

$\therefore$ A, B, C are <u>Collinear</u>.

16) $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

(i) A : No head

$A = \{TTT\}$

B : Exactly 1 head

$B = \{HTT, THT, TTH\}$.

C : atleast 2heads.

$C = \{HHH, HHT, HTH, THH,\}$

(ii) $A \cap B \cap C = \phi$

Yes, they forms mutually exclusive events.

(iii) exactly 2 tails = { THT, TTH, HTT }.

$$\therefore p(2\text{ tails}) = \underline{\underline{\frac{3}{8}}}$$

17) (i) $\sin\frac{\pi}{6} = \frac{1}{2}$.

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
sin	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\left(\frac{1}{2}\right)} = 2.$$

$$\sin\frac{5\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right)$$

$\sin(\pi - x) = \sin x$.

$$= \sin\frac{\pi}{6} = \frac{1}{2}.$$

$$\cot\frac{\pi}{4} = \frac{1}{\tan\frac{\pi}{4}} = \frac{1}{1} = 1.$$

$$\therefore 3\sin\frac{\pi}{6} \times \sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6} \times \cot\frac{\pi}{4} = 3 \times \frac{1}{2} \times 2 - 4 \times \frac{1}{2} \times 1$$

$$= 3 - 2$$

$$= \underline{\underline{1}}.$$

(ii)
$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}.$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$= \frac{2\cos\left(\frac{6x}{2}\right)\cos\left(\frac{2x}{2}\right) + \cos 3x}{2\,\cancel{\cos}\sin\left(\frac{6x}{2}\right)\,\cancel{\sin}\cos\left(\frac{2x}{2}\right) + \sin 3x}$$

$$= \frac{2\cos 3x \cdot \cos x + \cos 3x}{2\sin 3x \cdot \cos x + \sin 3x}.$$

$$= \frac{\cos 3x\,[2\cos x + 1]}{\sin 3x\,[2\cos x + 1]}$$

$$= \frac{\cos 3x}{\sin 3x} = \underline{\underline{\cot 3x}}$$

18) (i) $a = -3$.

$a_4 = (a_2)^2$

$a \cdot r^3 = (ar)^2$

$a \cdot r^3 = a^2 \cdot r^2$.

$\frac{r^3}{r^2} = \frac{a^2}{a}$

$r = a = \underline{\underline{-3}}$.

$\therefore a_7 = a \cdot r^6 = (-3)(-3)^6 = (-3)^7 = \underline{\underline{-2187}}$

(ii) $3, \frac{3}{2}, \frac{3}{4}, \ldots$

$S_n = \frac{3069}{512}$.

$a = 3, \quad r = \frac{1}{2} < 1$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = \frac{3\left[1-\frac{1}{2^n}\right]}{\frac{1}{2}}$

$= \underline{6\left(1-\frac{1}{2^n}\right)}$

$\cancel{= 6 - \frac{6}{2^n}}$

$6\left(1-\frac{1}{2^n}\right) = \frac{3069}{512}$.

$1-\frac{1}{2^n} = \frac{3069}{512 \times 6} = \frac{3069}{3072}$

$-\frac{1}{2^n} = \frac{3069}{3072} - 1 = \frac{3069-3072}{3072} = \frac{-3}{3072}$

$\therefore \cancel{\left(\frac{1}{2}\right)^n} \quad \frac{1}{2^n} = \frac{3}{3072} = \frac{1}{1024} = \frac{1}{2^{10}}$

ie, $\frac{1}{2^n} = \frac{1}{2^{10}}$

$\therefore \underline{\underline{n = 10}}$.

19)

(ii) Let $f(x) = \tan x = \frac{\sin x}{\cos x}$

$$f(x+h) = \tan(x+h) = \frac{\sin(x+h)}{\cos(x+h)}.$$

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cos x \cdot \sin(x+h) - \sin x \cos(x+h)}{\cos(x+h)\cos x}\right]$$

$$\boxed{\sin x \cos y - \cos x \sin y = \sin(x-y)}$$

$$\therefore \sin(x+h)\cos x - \cos(x+h)\sin x = \sin(x+h-x)$$
$$= \sin h$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\sin h}{\cos(x+h)\cos x}\right]$$

$$= \lim_{h\to 0}\left(\frac{\sin h}{h}\right) \times \lim_{h\to 0} \frac{1}{\cos(x+h)\cos x}$$

$$= 1 \times \frac{1}{\cos(x+0)\cos x}$$

$$= \frac{1}{\cos x \times \cos x} = \frac{1}{\cos^2 x} = \underline{\underline{\sec^2 x}}$$

$\therefore$ derivative of $\tan x = \underline{\underline{\sec^2 x}}$

8 (ii) $f(x) = \frac{x+\cos x}{\tan x}$

$\left(\frac{u}{v}\right)' = \frac{vu' - uv'}{v^2}$

$f'(x) = \frac{\tan x \times \frac{d}{dx}(x+\cos x) - (x+\cos x)\frac{d}{dx}(\tan x)}{\tan^2 x}$

$= \cancel{\tan}$

$= \frac{\tan x \cdot (1-\sin x) - (x+\cos x)\sec^2 x}{\tan^2 x}$

$= \frac{\tan x - \tan x \sin x - x\sec^2 x - \cos x \cdot \sec^2 x}{\tan^2 x}$

20)

Class	f_i	x_i	$f_i x_i$	x_i^2	$f_i x_i^2$
10 - 20	2	15	30	225	450
20 - 30	3	25	75	625	1875
30 - 40	8	35	280	1225	9800
40 - 50	14	45	630	2025	28350
50 - 60	8	55	440	3025	24200
60 - 70	3	65	195	4225	12675
70 - 80	2	75	150	5625	11250
	N=40		1800		88600

Mean, $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{1800}{40} = 45$

$$\text{Variance} = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2$$

$$= \frac{88600}{40} - (45)^2$$

$$= 2215 - 2025$$

$$= \underline{\underline{190}}$$

$$S \cdot D = \sqrt{\text{Variance}} = \sqrt{190}$$

$$= \underline{\underline{13.784}}$$