Second Year Model Exam - 2024.

1) $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix}$, $A^T = \begin{bmatrix} 3 & 0 & -2 \\ 2 & 4 & 1 \\ 1 & -1 & 3 \end{bmatrix}$

Let $P = \frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -\frac{1}{2} \\ 1 & 4 & 0 \\ -\frac{1}{2} & 0 & 3 \end{bmatrix}$

$Q = \frac{A-A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{3}{2} \\ -1 & 0 & -1 \\ -\frac{3}{2} & 1 & 0 \end{bmatrix}$

$P+Q = \begin{bmatrix} 3 & 1 & -\frac{1}{2} \\ 1 & 4 & 0 \\ -\frac{1}{2} & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{3}{2} \\ -1 & 0 & -1 \\ -\frac{3}{2} & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix} = A$

where P is symmetric and Q is skew symmetric

2)

(i) We have $\sin\frac{\pi}{6} = \frac{1}{2}$

Multiply both sides with -1

$-\sin\frac{\pi}{6} = -\frac{1}{2}$. $\qquad \because -\sin x = \sin(-x)$

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = \frac{-\pi}{6}$.

PVB of $\cos^{-1}$ is $[0,\pi]$

(ii) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left[\cos\left(\pi+\frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left[-\cos\frac{\pi}{6}\right]$

$= \cos^{-1}\left[\cos\left(\pi-\frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right]$

$= \frac{5\pi}{6}$

$[0,180^\circ]$

$\frac{7\pi}{6} = \frac{7\times \cancel{180}^{30}}{\cancel{6}} = 210^\circ \notin [0,180]$

$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$

$\cos(\pi+x) = -\cos x.$

$= \cos(\pi - x)$

$\frac{5\pi}{6} = \frac{5\times\cancel{180}^{30}}{\cancel{6}} = 150^\circ \in [0,180^\circ]$

$\cos^{-1}(\cos(x)) = x$

3).
$$f(x)=\begin{cases} x^2-1, & x<2 \\ 4, & x=2 \\ 2x-1, & x>2. \end{cases}$$

when $x<2$, $f(x)=x^2-1$, a polynomial function which is continuous
when, $x>2$, $f(x)=2x-1$, a polynomial function which is continuous

at $x=2$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2-1 = 2^2-1 = 4-1 = 3.$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x-1 = 2(2)-1 = 4-1 = 3.$$

$f(2)=4$.

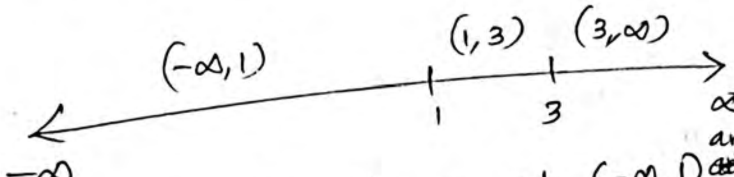
$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) \neq f(2)$

The given function is not continuous at $x=2$, and continuous at all other points.

4) The given graph is of $f'(x)$.
The given graph is of $f'(x)$ ...

It is clear that $f'(x)=0$ at $x=1$, and $x=3$

(i)



From the graph, $f'(x)>0$ at $(-\infty,1)$ and $(3,\infty)$, and $f'(x)<0$ at $(1,3)$

$\therefore$ Given function is $\uparrow$ on $(-\infty,1)$ and $(3,\infty)$
$\downarrow$ on $(1,3)$

(ii) From the graph, $f'(x) = (x-1)(x-3)$

$$f'(x) = x^2 - 4x + 3$$

$$\therefore f''(x) = 2x - 4.$$

at $x = 1$, $f''(1) = 2(1) - 4 = 2 - 4 = -2 < 0$

$\therefore$ local maxima at $x = 1$

at $x = 3$, $f''(3) = 2(3) - 4 = 6 - 4 = 2 > 0$

$\therefore$ local minima at $x = 3$

$\therefore$ Point of local minima is $x = 3$

" local maxima is $x = 1$

5) $$\int \frac{x-1}{x^2-4x-5} \, dx = \int \frac{x-1}{(x-5)(x+1)} \, dx.$$

using partial fraction,

let $$\frac{x-1}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$$

$$x - 1 = A(x+1) + B(x-5)$$

put $x = -1$

$$-1 - 1 = A(0) + B(-1-5)$$

$$-2 = B(-6)$$

$$B = \frac{-2}{-6} = \frac{1}{3}.$$

put $x = 5$,

$$5 - 1 = A(5+1) + B(0)$$

$$4 = A(6)$$

$$A = \frac{4}{6} = \frac{2}{3}.$$

$$\therefore \int \frac{x-1}{x^2-4x-5} \, dx = \frac{2}{3}\int \frac{dx}{x-5} + \frac{1}{3}\int \frac{dx}{x+1}$$

$$= \frac{2}{3} \log|x-5| + \frac{1}{3} \log|x+1| + C$$

6) (i) $A(1,5,3)$, $B(4,5,7)$.

$$\vec{AB} = B - A = (4-1)\hat{i} + (5-5)\hat{j} + (7-3)\hat{k}$$
$$= 3\hat{i} + 0\hat{j} + 4\hat{k}$$
$$\vec{AB} = 3\hat{i} + 4\hat{k}$$

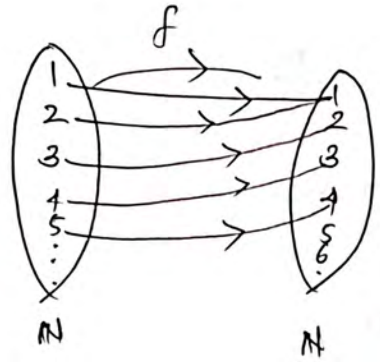
(ii) unit vector $= \dfrac{\vec{AB}}{|\vec{AB}|} = \dfrac{3\hat{i}+4\hat{k}}{\sqrt{3^2+4^2}} = \dfrac{3\hat{i}+4\hat{k}}{\sqrt{25}} = \dfrac{3\hat{i}+4\hat{k}}{5}$

(iii) If $\vec{a}$ and $\vec{b}$ are $\perp$r, then $\vec{a}\cdot\vec{b} = 0$

consider, $\vec{AB}\cdot c = (3\hat{i}+4\hat{k})\cdot(-5\hat{j}) = (3\hat{i}+0\hat{j}+4\hat{k})\cdot(0\hat{i}+-5\hat{j}+0\hat{k})$
$= 0+0+0 = 0$

$\therefore$ option c

7). (i)



1 has 2 images $\therefore$ it is many one.

Range of $f = \mathbb{N}$., $\therefore$ it is onto.

$\therefore$ option (b)

(ii) $f: R \to R$, $f(x) = 4x$.

1-1	onto
Let $f(x_1) = f(x_2)$	let $y = f(x)$
$4x_1 = 4x_2$	$4x = y$
$x_1 = x_2$	$x = \frac{y}{4} \in R$
$\therefore f$ is 1-1	$\therefore f$ is onto.

$\therefore f(x)$ is invertible $\because f$ is 1-1 & onto

$\therefore f^{-1}(x) = \frac{x}{4}$

8) Let A : atleast one number is 5 on the die

$A = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),$
$(5,1),(5,2),(5,3),(5,4),(5,6)\}$

B: Sum of the digits is 9.

$B = \{(3,6),(4,5),(5,4),(6,3)\}$

$P(A \cap B) = \frac{2}{36}$

$P(B) = \frac{4}{36}$

$A \cap B = \{(4,5),(5,4)\}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\left(\frac{2}{36}\right)}{\left(\frac{4}{36}\right)} = \frac{2}{4} = \frac{1}{2}$$

9) (i) $R = \{(1,3),(1,1),(2,2),(3,3),(3,1)\}$.

Reflexive

$(1,1),(2,2),(3,3) \in R$ $\therefore R$ is reflexive

Symmetric

$(1,3) \in R$ and $(3,1) \in R$.

$\therefore (a,b) \in R \Rightarrow (b,a) \in R$ $\therefore R$ is symmetric

transitive

$(1,3),(3,1) \in R \Rightarrow (1,1) \in R$.
$(1,3),(3,3) \in R \Rightarrow (1,3) \in R$
$(3,3),(3,1) \in R \Rightarrow (3,1) \in R$
$(3,1),(1,3) \in R \Rightarrow (3,3) \in R$
$(3,1),(1,1) \in R, \Rightarrow (3,1) \in R$

$\therefore$ if $(a,b),(b,c) \in R \Rightarrow (a,c) \in R$

$\therefore R$ is transitive

$\because R$ is Reflexive, symmetric & transitive

R is equivalence relation

(ii) $R = \{(1,3),(1,1),(2,2),(3,3),(3,1)\}$

Equivalence classes are

$[1] = \{1,3\}$

$[2] = \{2\}$

$[3] = \{1,3\}$

10) (i) $\begin{bmatrix} x-1 & 0 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2y & 6 \end{bmatrix}$

$x-1 = 2$ | $2y = -4$

$x = 2+1 = 3$ | $y = -\frac{4}{2} = -2$

$\therefore x = 3, y = -2$

(ii) $AB = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 5\times 2 & 5\times -1 & 5\times -3 \\ -2\times 2 & -2\times -1 & -2\times -3 \\ 3\times 2 & 3\times -1 & 3\times -3 \end{bmatrix} = \begin{bmatrix} 10 & -5 & -15 \\ -4 & 2 & 6 \\ 6 & -3 & -9 \end{bmatrix}$

~~11) (i) Volume of cube = a³ = 24 cm³/s~~

~~dv/ds = 24 = 3a²~~

~~Surface area = 6a² = 2×3a² = 2×24 = 48 cm/s~~

Surface Area, $A = 6a^2$.

11) (i) Volume of cube, $V = a^3$,

$\frac{dv}{dt} = 24$

$\frac{d}{dt}(a^3) = 24$

$3a^2 \times \frac{da}{dt} = 24$

$a^2 \times \frac{da}{dt} = \frac{24}{3} = 8$

$\frac{da}{dt} = \frac{8}{a^2}$

$\frac{dA}{dt} = \frac{d}{dt}(6a^2) = 6 \cdot 2a \cdot \frac{da}{dt}$

$= 12 \cdot a \cdot \frac{da}{dt}$

$= 12 \cdot a \cdot \frac{8}{a^2}$

$\therefore$ at $a = 6$

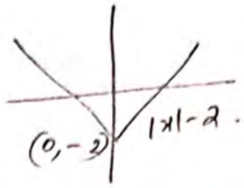
$\frac{dA}{dt} = \cancel{12}^{2} \times \cancel{6} \times \frac{8}{\cancel{6} \times \cancel{6}}$

$= 16\ cm^2/s$

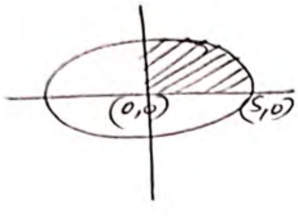
(ii) $f(x) = |x| - 2$

at $x = 0$, $f(x) = |0| - 2 = -2$.

$\therefore$ local minimum $= -2$.



12)



$$\frac{x^2}{25} + \frac{y^2}{9} = 1.$$

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$$

$$\frac{y^2}{9} = 1 - \frac{x^2}{25} = \frac{25 - x^2}{25}$$

$$y^2 = \frac{9}{25}(25 - x^2)$$

$$y = \frac{3}{5}\sqrt{5^2 - x^2}$$

$$\therefore \text{Area} = 4 \times \int_0^5 y\,dx$$

$$\boxed{\because \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)}$$

$$= 4 \times \int_0^5 \frac{3}{5}\sqrt{5^2 - x^2}\,dx$$

$$= \frac{4 \times 3}{5}\int_0^5 \sqrt{5^2 - x^2}\,dx$$

$$= \frac{12}{5}\left[\frac{x}{2}\sqrt{5^2 - x^2} + \frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= \frac{12}{5}\left[\frac{5}{2}\sqrt{5^2 - 5^2} + \frac{25}{2}\sin^{-1}\left(\frac{5}{5}\right) - 0\right]$$

$$= \frac{12}{5}\left[\frac{25}{2}\sin^{-1}(1)\right]$$

$$\boxed{\sin^{-1}(1) = \frac{\pi}{2}}$$

$$= \frac{\cancel{12}^{\,3}}{\cancel{5}} \times \frac{\cancel{25}^{\,5}}{\cancel{2}} \times \frac{\pi}{\cancel{2}}$$

$$= 15\pi \text{ sq. units}$$

3/4

13) (i) $y' - y' = 0$

option (a) $y = e^x + 1$

$y' = e^x$

$y'' = e^x$

$\therefore y'' - y' = e^x - e^x = 0$

(ii) $\frac{dy}{dx} = \frac{\sqrt{9-y^2}}{x}$

Variable separable form

$$\frac{dy}{\sqrt{9-y^2}} = \frac{dx}{x}.$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{dy}{\sqrt{9-y^2}} = \int \frac{dx}{x}.$$

$$\sin^{-1}\left(\frac{x}{3}\right) = \log|x| + C$$

14) (i) Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\vec{b} = 2\vec{a}$
$= 2(2\hat{i}+\hat{j})$
$= 4\hat{i}+2\hat{j}$

$$\vec{a}\cdot\vec{b} = (2\hat{i}+\hat{j})\cdot(4\hat{i}+2\hat{j})$$
$$= 8+2 = 10$$

$$|\vec{b}| = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}.$$

$$\therefore \text{Projection} = \frac{10}{\sqrt{20}} = \frac{10}{\sqrt{5\times 4}} = \frac{10}{2\sqrt{5}} = \frac{5}{\sqrt{5}}$$
$$= \sqrt{5}$$

(ii)

Area $= |\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(1+4) - \hat{j}(3-4) + \hat{k}(-3-1)$$
$$= \hat{i}(5) - \hat{j}(-1) + \hat{k}(-4)$$
$$= 5\hat{i}+\hat{j}-4\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{5^2+1^2+4^2} = \sqrt{25+1+16} = \sqrt{42}$$

$\therefore$ Area $= \sqrt{42}$ sq. units.

15) $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{2}$, $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+1}{2}$.

$\therefore \vec{r} = (\hat{i}+\hat{j}) + \lambda(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{r} = (2\hat{i}+\hat{j}-\hat{k}) + \mu(3\hat{i}-5\hat{j}+2\hat{k})$

Shortest distance $= \left| \frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|} \right|$

$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+10) - \hat{j}(4-6) + \hat{k}(-10+3)$

$= \hat{i}(8) - \hat{j}(-2) + \hat{k}(-7)$

$= 8\hat{i} + 2\hat{j} - 7\hat{k}$

$|\vec{b_1}\times\vec{b_2}| = \sqrt{8^2+2^2+7^2} = \sqrt{64+4+49} = \sqrt{117}$

$\vec{a_2} - \vec{a_1} = \hat{i} + 0\hat{j} - \hat{k}$.

$(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2}) = (\hat{i}+0\hat{j}-\hat{k})\cdot(8\hat{i}+2\hat{j}-7\hat{k})$

$= 8 + 0 + 7 = 15$

$\therefore$ S.D $= \frac{15}{\sqrt{117}}$ units

16) (i) $P(A) = 0.4$, $P(B) = 0.5$

$P(\text{neither A nor B}) = P(A\cup B)' = 1 - P(A\cup B)$

$= 1 - 0.7$

$= 0.3$

~~$P(A\cup B) = P(A) + P(B)$~~

~~$= 0.4 + 0.5$~~

~~$= 0.9$~~

$P(A\cap B) = P(A) \times P(B)$

$= 0.4 \times 0.5$

$= 0.20$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.9 - 0.2$

$= 0.7$

(ii) let E_1: Speaks truth

E_2: Speaks lie.

A: Gets 5 on die, B: not 5

$P(E_1) = \frac{4}{5}$, $P(E_2) = \frac{1}{5}$, $P(A) = \frac{1}{6}$, $P(B) = \frac{5}{6}$.

$P(E_1/A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A/E_2)}$

$$\therefore P(E_1/A) = \frac{\frac{4}{5}\times\frac{1}{6}}{\frac{4}{5}\times\frac{1}{6}+\frac{1}{5}\times\frac{5}{6}}$$

$$= \frac{\frac{4}{30}}{\frac{4}{30}+\frac{5}{30}} = \frac{\left(\frac{4}{30}\right)}{\left(\frac{9}{30}\right)} = \frac{4}{9}$$

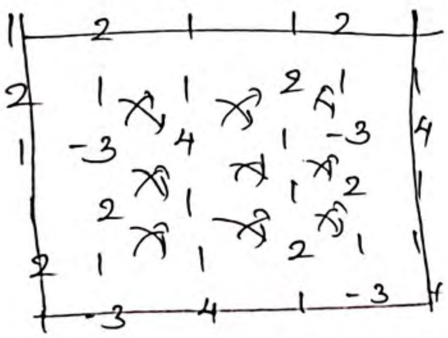
17) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & -3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 18 \\ 5 \\ 3 \end{bmatrix}$

$X = A^{-1}B$, $A^{-1} = \frac{\operatorname{adj}A}{|A|}$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & -3 & 4 \end{vmatrix} = 1(4+3) - 2(8-1) + 1(-6-1)$$

$$= 1(7) - 2(7) + 1(-7)$$

$$= 7 - 14 - 7 = -14$$



$$\text{cofactor}(A) = \begin{bmatrix} 4+3 & 1-8 & -6-1 \\ -3-8 & 4-1 & 2+3 \\ 2-1 & 2-1 & 1-4 \end{bmatrix} = \begin{bmatrix} 7 & -7 & -7 \\ -11 & 3 & 5 \\ 1 & 1 & -3 \end{bmatrix}$$

$$\operatorname{adj}A = \begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}$$

$$X = \frac{-1}{14}\begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 3 \end{bmatrix} = \frac{-1}{14}\begin{bmatrix} 126-55+3 \\ -126+15+3 \\ -126+25-9 \end{bmatrix} = \frac{-1}{14}\begin{bmatrix} 74 \\ -108 \\ -110 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore x = \frac{-74}{14},\quad y = \frac{-108}{-14},\quad z = \frac{-110}{-14}$$

$$x = \frac{-37}{7},\quad y = \frac{54}{7},\quad z = \frac{55}{7}$$

18) (i) $y = \sqrt{\sin x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\sin x}} \times \frac{d}{dx}(\sin x)$$

$$\frac{dy}{dx} = \frac{\cos x}{2\sqrt{\sin x}}$$

(ii) $x = a(t - \sin t)$, $y = a(1 + \cos t)$

$$\frac{dx}{dt} = a(1 - \cos t)$$

$$\frac{dy}{dt} = a(-\sin t)$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-a\sin t}{a(1-\cos t)} = \frac{-\sin t}{1-\cos t}$$

(iii) $y = x^x$.

taking log on both sides

$\log y = x \cdot \log x$.

diff wrt x

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \frac{1}{x} + \log x \cdot 1$$

$$\frac{dy}{dx} = y\left[1 + \log x\right]$$

$$= x^x\left[1 + \log x\right]$$

19) (i) $I = \int e^x \sin x \, dx = \int \sin x \cdot e^x \, dx = \sin x \int e^x dx - \int \cos x \times e^x \cdot dx$

$$= \sin x \, e^x - \left[\cos x \cdot e^x - \int -\sin x \, e^x \, dx\right]$$

$$= \sin x \, e^x - \left[e^x \cos x + \int \sin x \, e^x \, dx\right]$$

$$I = \sin x \, e^x - e^x \cos x - I$$

$$2I = \sin x \, e^x - e^x \cos x$$

$$I = \frac{e^x[\sin x - \cos x]}{2} + C$$

(ii)

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\, dx \longrightarrow (1)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)}\, dx \qquad \because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$= \cancel{\int_0^{\frac{\pi}{2}} \frac{\cos^3\left(\frac{\pi}{2} - x\right)}{\cos^3(\pi}}$$

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x}\, dx. \rightarrow (2)$$

$$(1) + (2) \Rightarrow \quad 2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x}\, dx = \int_0^{\pi/2} dx.$$

$$2I = \left[x\right]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2}$$

$$\underline{I = \frac{\pi}{4}}.$$

20). $3x + 4y = 60$

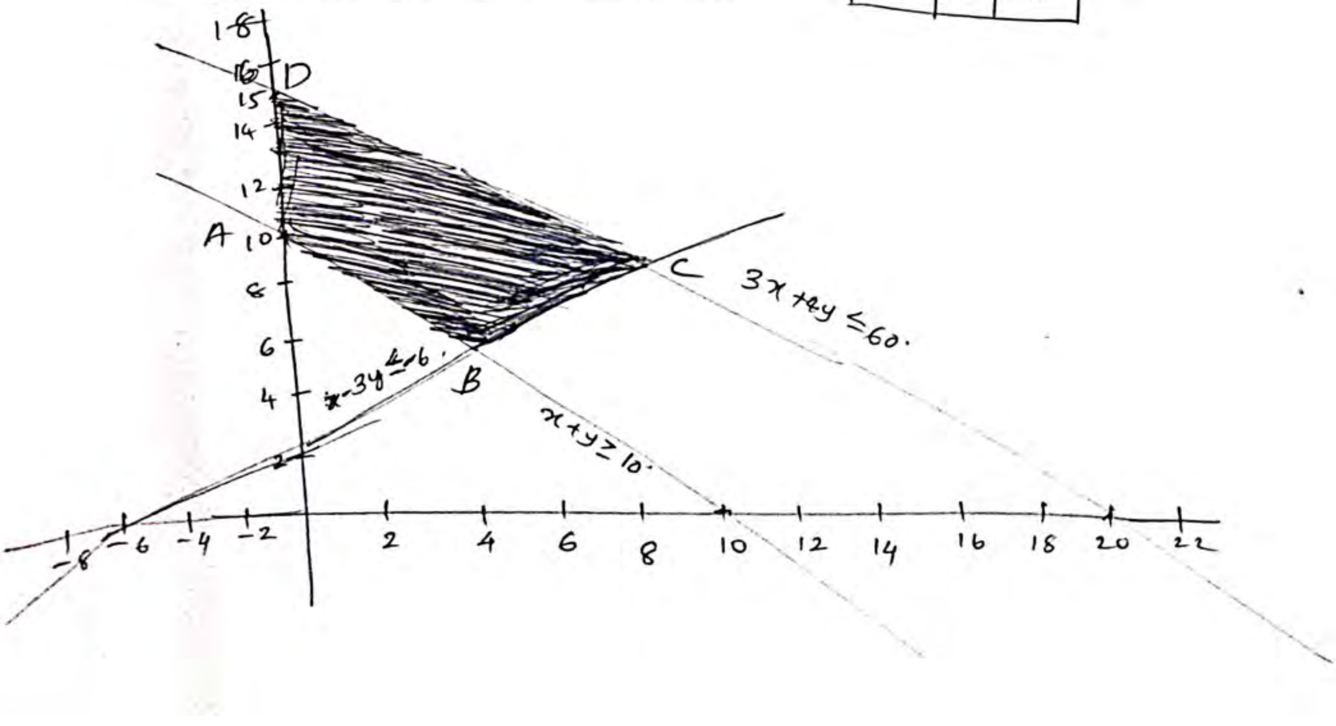
x	0	20
y	15	0

$x + y = 10$

x	0	10
y	10	0

$x - 3y = -6$

x	0	-6
y	2	0



To find co-ordinates of B

$x + y = 10 \rightarrow (1)$
$x - 3y = -6 \rightarrow (2)$

$(1) - (2) \Rightarrow 4y = 16$

$y = \frac{16}{4} = 4$

from (1), $x = 6$

$\therefore B = (6, 4)$.

Co-ordinates of C

$3x + 4y = 60 \rightarrow (3)$
$x - 3y = -6 \rightarrow (2)$

$(2) \times 3 \Rightarrow 3x - 9y = -18 \rightarrow (4)$
$3x + 4y = 60 \rightarrow (3)$

$(4) - (3) \Rightarrow -13y = -78$

$y = \frac{78}{13} = 6$

from (3) $3x + 24 = 60$

$3x = 36$

$x = 12$

$\therefore C = (12, 6)$

Corner pts	$Z = 3x + 7y$
$A(0,10)$	$Z = 70$
$B(6,4)$	$Z = 18 + 28 = 46$
$C(12,6)$	$Z = 36 + 42 = 78$
$D(0,15)$	$Z = 105$

$\therefore$ Max $Z = 105$ at $D(0,15)$.