# XII HSE KEY 2024 MARCH

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1. i) $R = \{(1,1), (2,3), (3,5)\}$

Domain $= \{1, 2, 3\}$

Range $= \{1, 3, 5\}$

ii) Since $(2,2) \in R \Rightarrow R$ is not reflexive

$(2,3) \in R$ but $(3,2) \notin R \Rightarrow R$ is not symmetric

$(2,3) \in R, (3,5) \in R$ but $(2,5) \notin R$

$\therefore$ R is not transitive

$\therefore$ R is not an equivalence relation.

2. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9+-1 & 3+2 \\ -3+-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$ ; $7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

LHS $= A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$, proved.

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3 i) $f(x) = 2x+3$ at $x=1$

Since it is a polynomial fun,
it is continuous everywhere

$\therefore f(x)$ is continuous at $x=1$

ii)

Since $f(x)$ is continuous at $x=5$

LHL = RHL

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x)$$

$$\lim_{x \to 5^-} (kx+1) = \lim_{x \to 5^+} (3x-5)$$

$$k(5)+1 = 3(5)-5$$

$$5k+1 = 15-5$$

$$5k = 10-1$$

$$k = \frac{9}{5}$$

4) i) $\frac{\pi}{6}$

ii) The expn $= \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right)$$

$$= \tan^{-1}\left(2\times\frac{1}{2}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

5 i) d) $\log x$

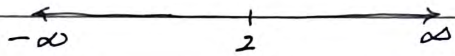
ii) $f(x) = x^2 - 4x + 6$

$f'(x) = 2x - 4$

$f'(x) = 0 \Rightarrow 2x - 4 = 0$

$2x = 4$

$x = 2$



when $x \in (-\infty, 2)$; say $x = 0$, $f'(0) = 2(0) - 4$

$= -4 < 0 \downarrow$

when $x \in (2, \infty)$, say $x = 3$, $f'(3) = 2(3) - 4$

$= 6 - 4$

$= 2 > 0 \uparrow$

a) $f(x)$ is increasing on $(2, \infty)$

b) $f(x)$ is decreasing on $(-\infty, 0)$

6 i) $|\bar{a} \cdot \bar{b}| = |\bar{a} \times \bar{b}|$

$ab\cos\theta = ab\sin\theta$

$\cos\theta = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

ii) $\bar{a} \cdot \bar{b} = (1)(1) + (-1)(1)$

$= 0$

$|\bar{b}| = \sqrt{1+1} = \sqrt{2}$

Projection of $\bar{a}$ on $\bar{b}$ $= \frac{\bar{a} \cdot \bar{b}}{|\bar{b}|} = \frac{0}{\sqrt{2}} = 0.$

7 i) $P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 0.12$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.12$

$= 0.7 - 0.12 = 0.58$

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iii) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.12}{0.4} = 0.3$

8) $I = \int_{-1}^{1} 5x^4 \sqrt{x^5+1}\, dx$

Put $x^5+1 = t$

$5x^4 . dx = dt$

when $x = -1$, $t = (-1)^5+1 = -1+1 = 0$

when $x = 1$, $t = 1^5+1 = 2$

$\therefore I = \int_0^2 \sqrt{t} . dt$

$= \frac{2}{3} \left[ t^{3/2} \right]_0^2$

$= \frac{2}{3} \left[ 2^{3/2} - 0^{3/2} \right]$

$= \frac{2}{3} (\sqrt{2})^3 - 0$

$= \frac{2}{3} . 2\sqrt{2} = \frac{4\sqrt{2}}{3}$

9)

i) 4

ii) $f(x) = \dfrac{x-2}{x-3}$

one-one : $f(x_1) = f(x_2)$

$\dfrac{x_1-2}{x_1-3} = \dfrac{x_2-2}{x_2-3}$

$(x_1-2)(x_2-3) = (x_1-3)(x_2-2)$

$\cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1 x_2} - 2x_1 - 3x_2 + \cancel{6}$

$-3x_1 + 2x_1 = -3x_2 + 2x_2$

$-x_1 = -x_2 \rightarrow x_1 = x_2$

$\therefore f$ is one-one.

onto :

$$f(x) = \frac{x-2}{x-3} \quad \text{———} (1)$$

$$\Rightarrow y = \frac{x-2}{x-3}$$

$$y(x-3) = (x-2)$$

$$yx - 3y = x - 2$$

$$yx - x = -2 + 3y$$

$$x(y-1) = 3y - 2$$

$$\therefore x = \frac{3y-2}{y-1}$$

$$(1) \Rightarrow f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$$

$$= \frac{3y-2-2(y-1)}{3y-2-3(y-1)}$$

$$= \frac{3y - \cancel{2} - 2y + \cancel{2}}{\cancel{3y} - 2 - \cancel{3y} + 3}$$

$$= \frac{y}{1} = y$$

$\therefore$ f is onto.
Hence verified.

10) i) $A' = -A$

ii) $$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

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$$P = \frac{1}{2}(A + A')$$

$$= \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$P' = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = P, \text{ Symmetric}$$

$$Q = \frac{1}{2}(A - A')$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$$Q' = \frac{1}{2}\begin{bmatrix} 0 & 1 & 5 \\ -1 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = -Q, \text{ Skew-Symmetric}$$

Now $P + Q = \frac{1}{2}\begin{bmatrix} 4 & -4 & -8 \\ -2 & 6 & 8 \\ 2 & -4 & -6 \end{bmatrix}$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$= A$, a square matrix.

Hence verified.

11) Let the length of one piece be $x$ m.
then other piece of length $= (28-x)$ m.

Circumference of circle, $2\pi r = x$

$$r = \frac{x}{2\pi} \text{ m}$$

$\therefore$ Area of the circle, $A_1 = \pi r^2$

$$= \pi \cdot \left(\frac{x}{2\pi}\right)^2$$

$$= \frac{x^2}{4\pi}$$

Perimeter of square, $4a = (28-x)$

$$a = \frac{28-x}{4} \text{ m}$$

Area, $A_2 = a^2 = \frac{(28-x)^2}{16}$

Combined area, $A = A_1 + A_2$

$$= \frac{x^2}{4\pi} + \frac{(28-x)^2}{16}$$

$$\frac{dA}{dx} = \frac{2x}{4\pi} + \frac{2(28-x)}{16} \times (-1)$$

$$= \frac{x}{2\pi} - \frac{28-x}{8}$$

For min. area, $\frac{dA}{dx} = 0$

$$\frac{x}{2\pi} - \frac{28-x}{8} = 0$$

$$\frac{8x - 2\pi(28-x)}{16\pi} = 0$$

$$\frac{4x - 28\pi + \pi x}{8\pi} = 0$$

$$(\pi + 4)x = 28\pi$$

$$\therefore x = \frac{28\pi}{\pi+4}$$

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$$\frac{d^2A}{dx^2} = \frac{1}{2\pi} - \frac{0-1}{8} = \frac{1}{2\pi} + \frac{1}{8} > 0$$

$\therefore$ A is minimum.

$\therefore$ The wire must be cut at a distance of $\frac{28\pi}{\pi+4}$ m from one end.

Hence the length of the two pieces are
$\frac{28\pi}{\pi+4}$ m and $28 - \frac{28}{\pi+4} = \frac{112}{\pi+4}$ m.

12 i) order $- 2$
degree $-$ Not defined ( Not a polynomial)

ii) $$\frac{dy}{dx} + \frac{2}{x}y = x$$

$$P = \frac{2}{x}$$

$$\int P\,dx = \int \frac{2}{x}dx = 2\int \frac{1}{x}dx = 2\log x = \log x^2.$$

$$\therefore IF = e^{\int Pdx} = e^{\log x^2} = x^2.$$

iii) $$\int Q e^{\int Pdx} dx = \int x \cdot x^2 dx$$

$$= \int x^3 dx = \frac{x^4}{4} + c$$

$\therefore$ Soln is $y e^{\int Pdx} = \int Q e^{\int Pdx} \cdot dx + c$

$$y \cdot x^2 = \frac{x^4}{4} + c$$

(or) $$y = \frac{x^2}{4} + \frac{c}{x^2}.$$

13 i) $\bar{a} = \hat{i} + \hat{j} + \hat{k}$ ; $\bar{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\bar{a} + \bar{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\bar{a} - \bar{b} = 0\hat{i} - \hat{j} - 2\hat{k}$

ii) $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$

$= \hat{i}(-6 - 4) - \hat{j}(-4 - 0) + \hat{k}(-2 - 0)$

$= \hat{i}(-6 + 4) - \hat{j}(-4) + \hat{k}(-2)$

$= -2\hat{i} + 4\hat{j} - 2\hat{k}$

$|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})| = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$

Unit vector ⊥r to both the vectors

$$= \frac{(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})}{|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})|}$$

$$= \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{2\sqrt{6}}$$

$$= \frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}$$

$$= \frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$$

14) $\bar{a}_1 = \hat{i} + \hat{j}$ $\quad \bar{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$a_2 = 2\hat{i} + \hat{j} - \hat{k}$ $\quad \bar{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$

$= \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3)$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

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$|\bar{b_1} \times \bar{b_2}| = \sqrt{9+1+49} = \sqrt{59}$

$\bar{a_2} - \bar{a_1} = \hat{i} + 0\hat{j} - \hat{k}$

$(\bar{b_1} \times \bar{b_2}) \cdot (\bar{a_2} - \bar{a_1}) = (3)(1) + (-1)(0) + (-7)(-1)$
$= 3 + 0 + 7 = 10$

$\therefore SD = \left| \dfrac{(\bar{b_1} \times \bar{b_2}) \cdot (\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|} \right|$

$= \dfrac{10}{\sqrt{59}} = \dfrac{10\sqrt{59}}{59}$ units.

15) Let $E_1, E_2$ and $E_3$ be the events that ~~a factor~~ bolts produced by machines A, B and C resp.

$P(E_1) = \dfrac{25}{100}$ ; $P(E_2) = \dfrac{35}{100}$ , $P(E_3) = \dfrac{45}{100}$

Let E be the event a bolt is defective.

$P(E/E_1) = \dfrac{5}{100}$

$P(E/E_2) = \dfrac{4}{100}$

$P(E/E_3) = \dfrac{2}{100}$

Req. probability $= P(E_2/E)$

$= \dfrac{P(E_2) \cdot P(E/E_2)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)}$

$= \dfrac{\frac{35}{100} \times \frac{4}{100}}{\frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{45}{100} \times \frac{2}{100}}$

$$= \frac{35 \times 4}{25 \times 5 + 35 \times 4 + 45 \times 2}$$

$$= \frac{140}{125 + 140 + 90}$$

$$= \frac{140}{355} = \frac{28}{71}$$

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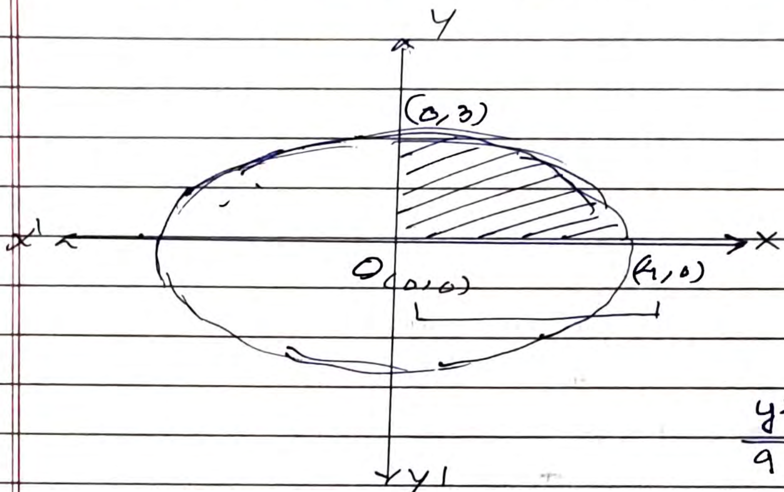


$$\frac{y^2}{9} = 1 - \frac{x^2}{16}$$

$$y^2 = \frac{9}{16}(16 - x^2)$$

$$y = \frac{3}{4}\sqrt{16 - x^2}$$

$$\text{Area} = 4 \times \int_0^4 y \, dx$$

$$= 4 \int_0^4 \frac{3}{4}\sqrt{4^2 - x^2} \, dx$$

$$= 3\left[\frac{x}{4}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$= 3\left[\cancel{\frac{4}{4}\sqrt{16-16}} + \frac{16}{2}\sin^{-1}(1) - (0+0)\right]$$

$$= 3 \times \frac{16}{2} \times \frac{\pi}{2}$$

$$= 12\pi \text{ sq. units}$$

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17) $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ ; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ; $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$|A| = 2(-4+4) + 3(-6+4) + 5(3-2)$

$= 2(0) + 3(-2) + 5(1)$

$= -6+5 = -1 \neq 0$

$\therefore A^{-1}$ exists.

$A_{11} = -4+4 = 0$

$A_{12} = -(-6+4) = 2$

$A_{13} = 3-2 = 1$

$A_{21} = -(6-5) = -1$

$A_{22} = -4-5 = -9$

$A_{23} = -(2+3) = -5$

$A_{31} = 12-10 = 2$

$A_{32} = -(-8+15) = 23$

$A_{33} = 4+9 = 13$

$\text{adj} A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \cdot \text{adj} A$

$= \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow x = 1, \; y = 2$ and $z = 3$

18 i) $\sin x + \cos y = xy$

diff wrt $x$

$$\cos x + -\sin y \frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$-\sin y \frac{dy}{dx} - x\frac{dy}{dx} = y - \cos x$$

$$\frac{dy}{dx}(-\sin y - x) = y - \cos x$$

$$\therefore \frac{dy}{dx} = \frac{y - \cos x}{-(\sin y + x)}$$

$$= \frac{\cos x - y}{\sin y + x}$$

ii) $x = a\cos^3 t \;;\; y = a\sin^3 t$

$$\frac{dx}{dt} = a \cdot 3\cos^2 t(-\sin t) = -3a\sin t\cos^2 t$$

$$\frac{dy}{dt} = a \cdot 3\sin^2 t(\cos t) = 3a\sin^2 t\cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$

$$= \frac{3a\sin^2 t\cos t}{-3a\sin t\cos^2 t} = \tan t.$$

iii) $y = (\sin^{-1} x)^2$

$$\frac{dy}{dx} = 2\sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}}$$

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$$\sqrt{1-x^2}\cdot\frac{dy}{dx} = 2\sin^{-1}x$$

diff wrt $x$

$$\sqrt{1-x^2}\cdot\frac{d^2y}{dx} + \frac{dy}{dx}\cdot\frac{-x}{\sqrt{1-x^2}} = 2\,\frac{1}{\sqrt{1-x^2}}$$

xing by $\sqrt{1-x^2}$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2,\quad \text{proved.}$$

19 i) $$I = \int\frac{x-1}{(x-2)(x-3)}\,dx$$

Let $$\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} \quad\text{(1)}$$

$$\Rightarrow \quad x-1 = A(x-3) + B(x-2)$$

Put $x=2$,

$$2-1 = A(2-3) + B(0)$$

$$1 = -A \Rightarrow A = -1$$

Put $x=3$

$$3-1 = A(0) + B(3-2)$$

$$2 = B$$

in (1) $$\frac{x-1}{(x-2)(x-3)} = \frac{-1}{x-2} + \frac{2}{x-3}$$

$$\therefore\ I = -1\times\int\frac{dx}{x-2} + 2\int\frac{dx}{x-3}$$

$$= -\log|x-2| + 2\log|x-3| + c$$

$$= \log(x-3)^2 - \log(x-2) + c$$

$$= \log\left|\frac{(x-3)^2}{x-2}\right| + c.$$

ii) $I = \int_0^{\pi/4} \cancel{\log(1+\tan x)dx}$ —— (1)

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\therefore I = \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$

$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$

$= \int_0^{\pi/4} \log\left[\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right]dx$

$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$

$= \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \cancel{\log(1+\tan x)dx}$ —— (2)

(1) + (2) $\Rightarrow$

$2I = \log 2 \int_0^{\pi/4} dx$

$= \log 2 \cdot (x)_0^{\pi/4}$

$= \log 2 \cdot \left(\frac{\pi}{4} - 0\right)$

$2I = \frac{\pi}{4}\log 2$

$\therefore I = \frac{\pi}{8}\log 2$, Proved.

20) $x + y = 50$

| | | |
|---|---|---|
| $x$ | 0 | 50 |
| $y$ | 50 | 0 |

$3x + y = 90$

| | | |
|---|---|---|
| $x$ | 0 | 30 |
| $y$ | 90 | 0 |

$x = 0$

$y = 0$

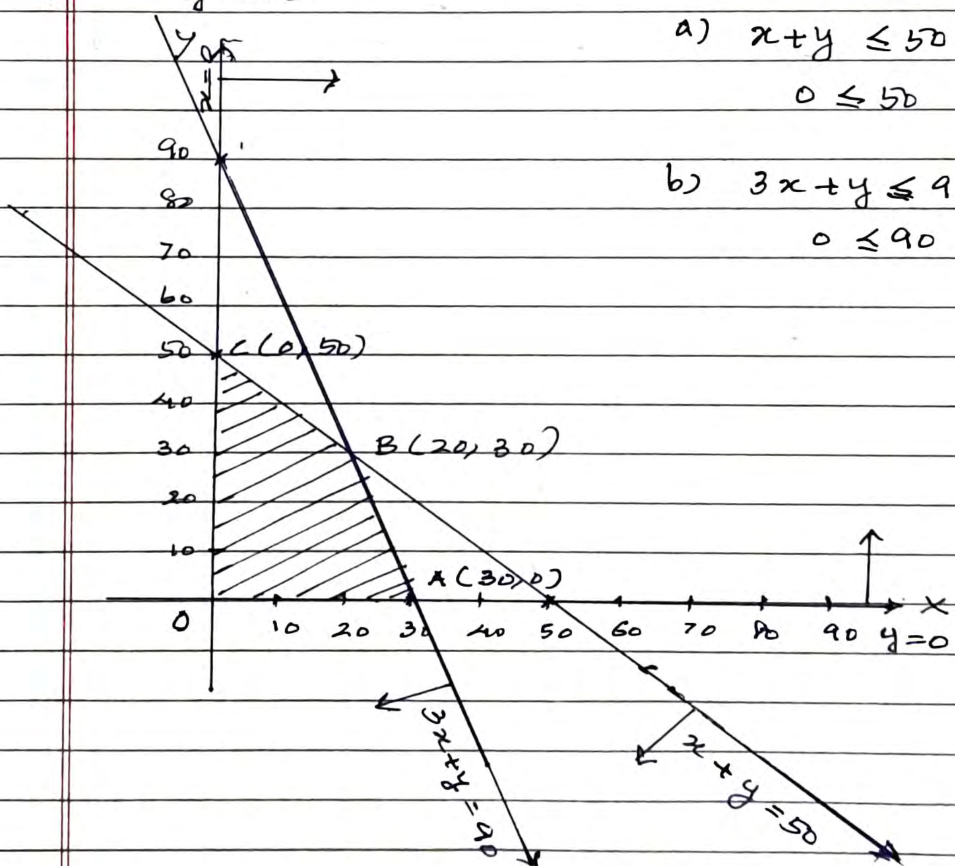
Feasible region:

a) $x + y \leq 50$

$0 \leq 50$ .T.

b) $3x + y \leq 90$

$0 \leq 90$ .F.




Corner points: O(0,0), A(20,0),
B(20,30), C(0,50)

| Point | $Z = 60x + 15y$ |
|---|---|
| $O(0,0)$ | $Z = 60(0) + 15(0) = 0$ |
| $A(30,0)$ | $Z = 60(30) + 15(0) = 1800$ |
| $B(20,30)$ | $Z = 60(20) + 15(30)$ <br> $= 1200 + 450 = 1650$ |
| $C(0,50)$ | $Z = 60(0) + 15(50) = 750$ |

$\therefore Z_{max} = 1800$ at $A(30,0)$