1 i) $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y = 2x - 1\}$

$R = \{(1,1), (2,3), (3,5)\}$

Domain $= \{1, 2, 3\}$

Range $= \{1, 3, 5\}$

ii) Reflexive, $(a,a) \in R, \forall a \in A$

$(2,2) \notin R$

$\therefore$ R is not reflexive

R is not an equivalence relation

2 $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ $5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

3. i) $f(x) = 2x+3$

$f(1) = 2\times1+3 = 5$

$\lim_{x\to1} f(x) = \lim_{x\to1} 2x+3 = 2\times1+3 = 5$

ie $\lim_{x\to1} f(x) = f(1)$

$\therefore f$ is continuous at $x=1$

ii) $f(x) = \begin{cases} Kx+1, & x\le5 \\ 3x-5, & x>5 \end{cases}$

L.H.L $\lim_{x\to5^-} f(x) = \lim_{x\to5^-} Kx+1 = 5k+1$

R.H.L $\lim_{x\to5^+} f(x) = \lim_{x\to5^+} 3x-5 = 10$

L.H.L = R.H.L

$5k+1 = 10$

$5k = 9$

$k = \frac{9}{5}$

4 i) $\sin^{-1}\left(\frac{1}{2}\right)$

$\sin\frac{\pi}{6} = \frac{1}{2}, \frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

ii) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\,\pi/6\right)\right]$

$= \tan^{-1}\left[2\cos\pi/3\right]$

$= \tan^{-1}\left[2\cdot\frac{1}{2}\right]$

$= \tan^{-1}(1)$

$= \frac{\pi}{4}$

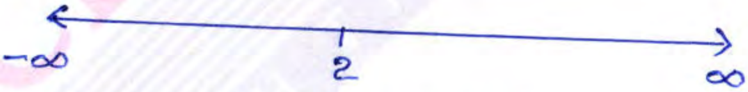
5 i) D - $\log x$

ii) $f(x) = x^2 - 4x + 6$

$f'(x) = 2x - 4$

$= 2(x-2)$

$f'(x) = 0 \Rightarrow x = 2$



$f'(x) > 0$ in $(2, \infty)$

$\therefore f(x)$ is strictly increasing $(2, \infty)$

$f'(x) < 0$ in $(-\infty, 2)$

$\therefore f(x)$ is strictly decreasing $(-\infty, 2)$

a) $(2, \infty)$ b) $(-\infty, 2)$

6. i) $\theta = \frac{\pi}{4}$

ii) $\vec{a} = \hat{i} - \hat{j} \qquad \vec{b} = \hat{i} + \hat{j}$.

$|\vec{b}| = \sqrt{2}$

$\vec{a} \cdot \vec{b} = (\hat{i} - \hat{j}) \cdot (\hat{i} + \hat{j}) = 1 - 1 = 0$

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{0}{\sqrt{2}} = 0$

7 i) Since A and B are independent events,

$P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4 = 0.12$

ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.12$

$= 0.58$

iii) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3$

8 $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1} \, dx$

$t = x^5 + 1$, when $x = -1, t = 0$

$\frac{dt}{dx} = 5x^4 \qquad x = 1, t = 2$

$dt = 5x^4 dx$

$$\int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx = \int_{0}^{2} \sqrt{t}\,dt$$

$$= \frac{2}{3}\left[t^{3/2}\right]_0^2$$

$$= \frac{2}{3}\left[2\sqrt{2}-0\right]$$

$$= \frac{4\sqrt{2}}{3}$$

9 i) 4

ii) $A = R-\{3\}$, $B = R-\{1\}$

$$f(x) = \frac{x-2}{x-3}$$

One-one

Let $x_1, x_2 \in R-\{3\}$

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$$

$$\Rightarrow -3x_1 + 2x_1 = -3x_2 + 2x_2$$

$$\Rightarrow -x_1 = -x_2$$

$$\Rightarrow x_1 = x_2$$

$\therefore f$ is one-one

Onto

Let $y \in B$ such that $f(x) = y$

$$\Rightarrow \frac{x-2}{x-3} = y \Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x - xy = 2 - 3y$$

$$\Rightarrow x(1-y) = 2 - 3y$$

$$\Rightarrow x = \frac{2-3y}{1-y}$$

$$f(x) = f\left(\frac{2-3y}{1-y}\right)$$

$$= \frac{\frac{2-3y}{1-y} - 2}{\frac{2-3y}{1-y} - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1}$$

$$f(x) = y$$

For each $y \in B$, there exists $x = \frac{2-3y}{1-y} \in A$

such that $f(x) = y$

$\therefore f$ is onto.

10 i) $A' = -A$

ii) $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

$$A + A^T = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

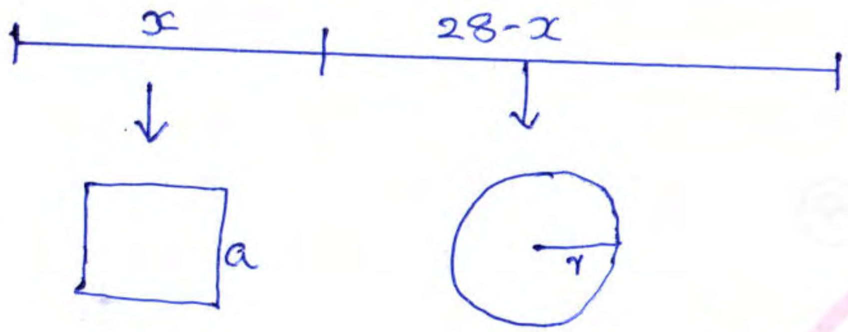
$$Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

$$P + Q = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= A$$

11



$$4a = x \qquad\qquad 28 - x = 2\pi r$$

$$a = \frac{x}{4} \qquad\qquad r = \frac{28-x}{2\pi}$$

$$A_1 = \frac{x^2}{16} \qquad\qquad A_2 = \pi\left(\frac{28-x}{2\pi}\right)^2$$

$$= \frac{(28-x)^2}{4\pi}$$

$$A = A_1 + A_2$$

$$A = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$$

$$\frac{dA}{dx} = \frac{2x}{16} - \frac{2(28-x)}{4\pi}$$

$$\frac{dA}{dx} = 0 \Rightarrow \frac{2x}{16} = \frac{2(28-x)}{4\pi}$$

$$\Rightarrow \frac{x}{4} = \frac{28-x}{\pi}$$

$$\Rightarrow x = \frac{112}{\pi+4}$$

$$\frac{d^2A}{dx^2} = \frac{2}{16} + \frac{2}{4\pi} > 0$$

At $x = \frac{112}{\pi+4}$, $\frac{d^2A}{dx^2} > 0$

Thus A is minimum

Length of square piece $= \frac{112}{\pi+4}$

Length of circular piece $= 28 - \frac{112}{\pi+4}$

$$= \frac{28\pi}{\pi+4}$$

12 i) order $= 2$

degree is not defined

ii) $x\frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2y}{x} = x$$

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2}{x}, \quad Q = x$$

Integrating Factor $IF = e^{\int p\,dx}$

$$= e^{\int \frac{2}{x}dx}$$

$$= e^{2\log x} = e^{\log x^2}$$

$$= x^2$$

iii) Solution is

$$y \cdot IF = \int Q \cdot IF\, dx$$

$$y \cdot x^2 = \int x \cdot x^2\, dx$$

$$y \cdot x^2 = \frac{x^4}{4} + c$$

13. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

i) $\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a} - \vec{b} = -\hat{j} - 2\hat{k}$

ii) $(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$

$$= -2\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right| = \sqrt{(-2)^2+4^2+(-2)^2}$$

$$= \sqrt{24} = 2\sqrt{6}$$

A vector which is perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is given by

$$\frac{-2\hat{i}+4\hat{j}-2\hat{k}}{2\sqrt{6}}$$

$$= -\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$$

14. $\vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$

$\vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

$\vec{a}_1 = \hat{i}+\hat{j}$, $\vec{b}_1 = 2\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2 = 2\hat{i}+\hat{j}-\hat{k}$, $\vec{b}_2 = 3\hat{i}-5\hat{j}+2\hat{k}$

$\vec{a}_2-\vec{a}_1 = \hat{i}-\hat{k}$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i}-\hat{j}-7\hat{k}$$

$$|\vec{b}_1\times\vec{b}_2| = \sqrt{3^2+(-1)+(-7)^2} = \sqrt{59}$$

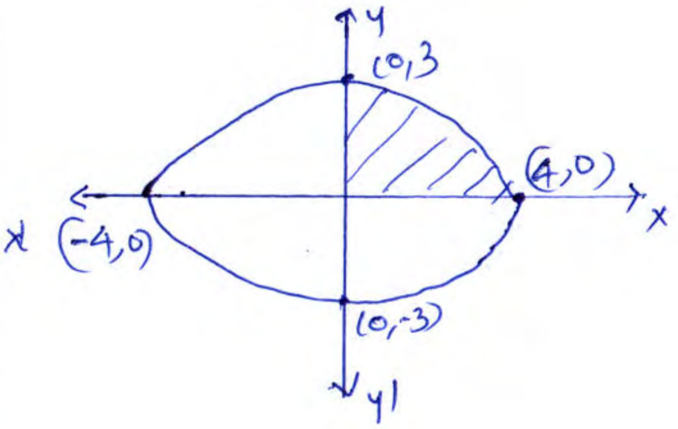
$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k})$$

$$= 10$$

$$S.D = \frac{10}{\sqrt{59}}$$

15. Wrong question

16 $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$



$\frac{y^2}{9}=1-\frac{x^2}{16}$

$=\frac{16-x^2}{16}$

$y^2=\frac{9}{16}(16-x^2)$

$y=\frac{3}{4}\sqrt{16-x^2}$

Required Area $=4\int_0^4 y\cdot dx$

$=4\cdot\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$

$=3\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

$=3\left[8\sin^{-1}1-0\right]$

$=3\times 8\times\frac{\pi}{2}$

$=12\pi$ sq.unit

17 $2x-3y+5z=11$

$3x+2y-4z=-5$

$x+y-2z=-3$

$$A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}\quad B=\begin{bmatrix}11\\ -5\\ -3\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$X=A^{-1}\cdot B$$

$$|A|=\begin{vmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{vmatrix}=-1$$

$$\text{co-factor matrix of } A=\begin{bmatrix}0 & 2 & 1\\ -1 & -9 & -5\\ 2 & 23 & 13\end{bmatrix}$$

$$\text{adj } A=\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}.$$

$$A^{-1}=\frac{1}{|A|}\text{adj} A=-\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$= -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= -\begin{bmatrix} 0+5-6 \\ 22+45-69 \\ 11+25-39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1,\ y = 2,\ z = 3$

18. i) $\sin x + \cos y = xy$

diff both sides w.r.t x

$$\cos x - \sin y \frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\cos x - y = x\frac{dy}{dx} + \sin y \frac{dy}{dx}$$

$$= \frac{dy}{dx}(x + \sin y)$$

$$\frac{dy}{dx} = \frac{\cos x - y}{x + \sin y}$$

ii) $x = a\cos^3 t$ $\qquad$ $y = a\sin^3 t$

$$\frac{dx}{dt} = -3a\cos^2 t\sin t \qquad \frac{dy}{dt} = 3a\sin^2 t\cos t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t}$$

$$= -\tan t$$

iii $\quad y = (\sin^{-1}x)^2$

$$\frac{dy}{dx} = 2\sin^{-1}x\,\frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = 2\sin^{-1}x$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$

diff both sides w.r.t x

$$(1-x^2)\,2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 x - 2x = 4\frac{dy}{dx}$$

$\div 2\frac{dy}{dx}$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

19. i) $\displaystyle\int \frac{x-1}{(x-2)(x-3)}\,dx$

$$\frac{x-1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$= \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$$

$$x-1 = A(x-3)+B(x-2)$$

put $x=2 \Rightarrow 1=-A \Rightarrow A=-1$

$x=3 \Rightarrow 2=B \Rightarrow B=2$

$$\int \frac{x-1}{(x-2)(x-3)}\,dx = \int \frac{-1}{x-2}\,dx + \int \frac{2}{x-3}\,dx$$

$$= -\log|x-2| + 2\log|x-3| + c$$

$$= -\log|x-2| + \log|(x-3)^2| + c$$

$$= \log\left|\frac{(x-3)^2}{x-2}\right| + c$$

19 ii) $\int_0^{-\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2 \rightarrow$ Incorrect question

Correct question is

$$\boxed{\int_0^{\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2}$$

$$I = \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$= \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} \log\left[\frac{1+\tan x+1-\tan x}{1+\tan x}\right]dx$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$I = \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$I = \log 2 \cdot \pi/4 - I$$

$$2I = \log 2 \cdot \pi/4$$

$$I = \frac{\pi}{8}\log 2$$

20. Maximise $Z = 60x + 15y$

Subject to $x + y \leq 50$

$3x + y \leq 90$

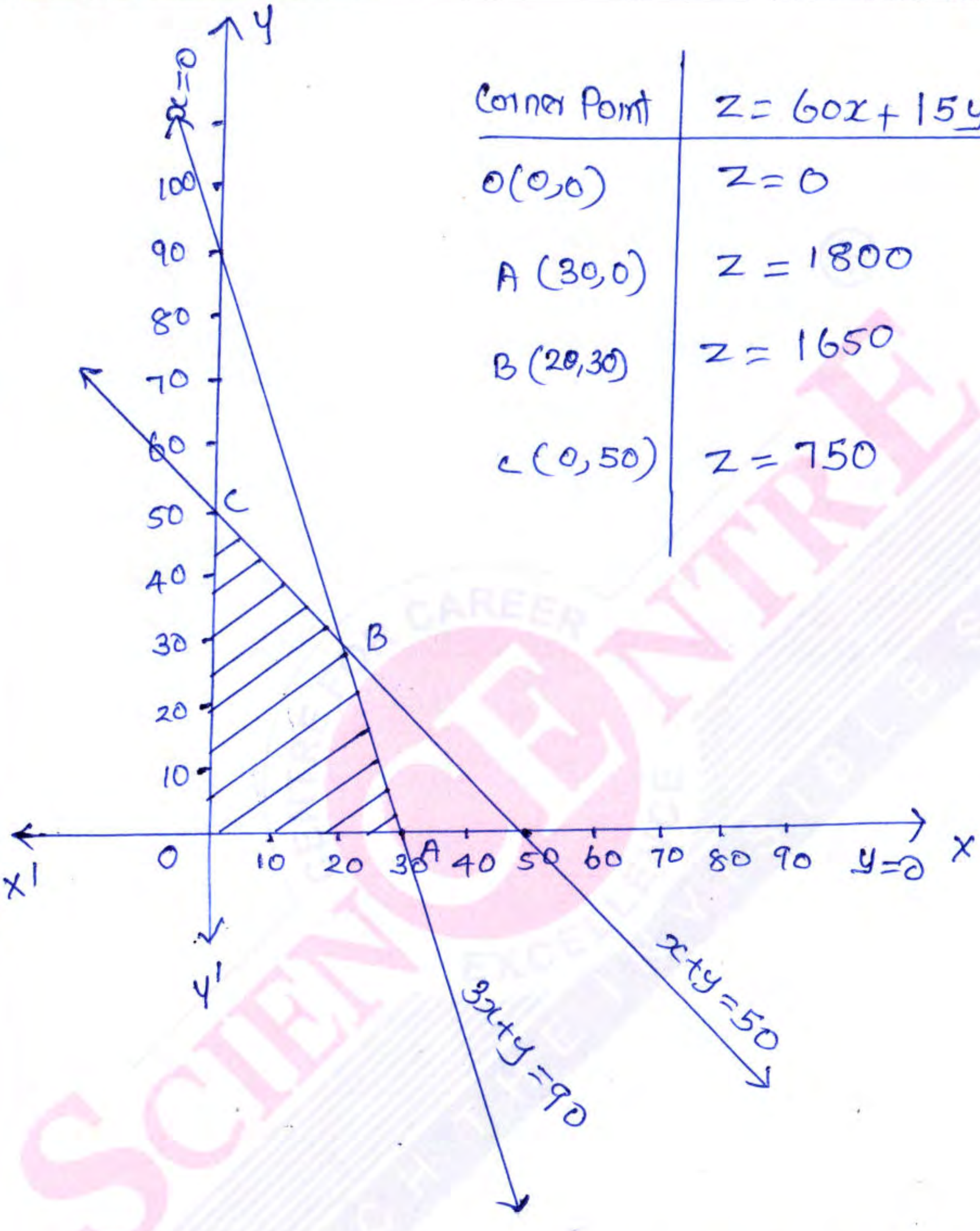
$x \geq 0, y \geq 0$

$x + y = 50$

x	y
0	50
50	0

$3x + y = 90$

x	y
0	90
30	0



Corner Point	$Z = 60x + 15y$
O(0,0)	$Z = 0$
A (30,0)	$Z = 1800$
B (20,30)	$Z = 1650$
C (0,50)	$Z = 750$

Maximum value of z is 1800 at (30,0)