

X1 Higher Secondary Exam. March 2024

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Mathematics (Science)

1. i)  $2^{3 \times 2} = 2^6 =$

ii)  $\frac{x}{3} + 1 = -\frac{2}{3} \Rightarrow \frac{x}{3} + \frac{3}{3} = -\frac{2}{3}$

$\Rightarrow x + 3 = -2$

$\therefore x = -2 + 3 = -5$

$y - \frac{2}{3} = \frac{2}{3} \Rightarrow \frac{3y}{3} - \frac{2}{3} = \frac{2}{3}$

$3y = 2 + 2 = 4$

$y = \frac{4}{3}$

2) i)  $\cos(x+y) + \cos(x-y) = 2 \cos x \cos y$

ii) LHS =  $\cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{3\pi}{4} - x\right)$

$= 2 \cos \frac{3\pi}{4} \cdot \cos x$

$= 2 \times \frac{-1}{\sqrt{2}} \cos x$

$= -\sqrt{2} \cos x$

$= \text{RHS}$

$\cos \frac{3\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\cos \frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$

3) i)  $x + \frac{x}{2} + \frac{x}{3} \leq 10 + \frac{x}{6}$

$6x + 3x + 2x \leq 60 + x$

$11x - x \leq 60$

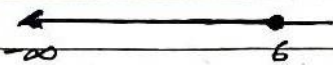
$10x \leq 60$

$x \leq \frac{60}{10}$

$x \leq 6$

$\therefore x = (-\infty, 6]$

ii)



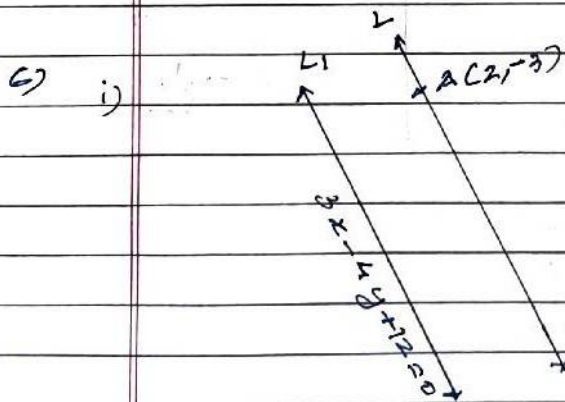
4) i)  ${}^n C_r = \frac{n!}{r!(n-r)!}$

ii) No. of selections =  ${}^7 C_3 \times {}^5 C_2$   
 $= 35 \times 10 = 350$

5) i) Sum of Coe. =  $2^n$

ii)  $(\frac{x}{3} + \frac{3}{x})^4 = (\frac{x}{3})^4 + 4C_1 (\frac{x}{3})^3 \cdot \frac{3}{x} +$   
 $4C_2 (\frac{x}{3})^2 (\frac{3}{x})^2 + 4C_3 (\frac{x}{3}) (\frac{3}{x})^3 + (\frac{3}{x})^4$   
 $= \frac{x^4}{81} + 4 \cdot \frac{x^3}{27} \cdot \frac{3}{x} + 6 \frac{x^2}{9} \cdot \frac{9}{x^2} +$   
 $4 \cdot \frac{x}{3} \cdot \frac{27}{x^3} + \frac{81}{x^4}$

$= \frac{x^4}{81} + \frac{4}{9} x^2 + 6 + \frac{36}{x^2} + \frac{81}{x^4}$



Eqn of L is  $3x - 4y + k = 0$  (1)

(1) passes through  $(2, -3)$

$3(2) - 4(-3) + k = 0$

$6 + 12 + k = 0 \Rightarrow k = -18$

in (1) :  $3x - 4y - 18 = 0$

$$\begin{aligned} \text{ii)} \quad \text{distance} &= \left| \frac{12}{\sqrt{3^2 + (-4)^2}} \right| \\ &= \frac{12}{5} \text{ units} \end{aligned}$$

$$\begin{aligned} \text{7)} \quad x^2 &= 12y \\ \Rightarrow x^2 &= 4(3)y \text{ is an upward parabola.} \end{aligned}$$

$$\text{Focus : } S(0, a) = (0, 3)$$

$$\text{Equation of directrix : } y = -a$$

$$y = -3 \Rightarrow y + 3 = 0$$

$$\text{Length of LR} = 4a = 4 \times 3 = 12.$$

$$\text{8) i) } n a^{n-1}$$

$$\text{ii) } n \cdot 2^{n-1} = 32 = 2^5 = 4 \cdot 2^{4-1}$$

$$\Rightarrow n = 4$$

$$\text{9) i) } A \cap A' = \phi$$

$$\text{ii) } A \cap B = \{2, 3, 4\}$$

$$\text{LHS} = (A \cap B)' = \{1, 5, 6\} \quad (1)$$

$$A' = \{1, 5, 6\}$$

$$B' = \{1, 5\}$$

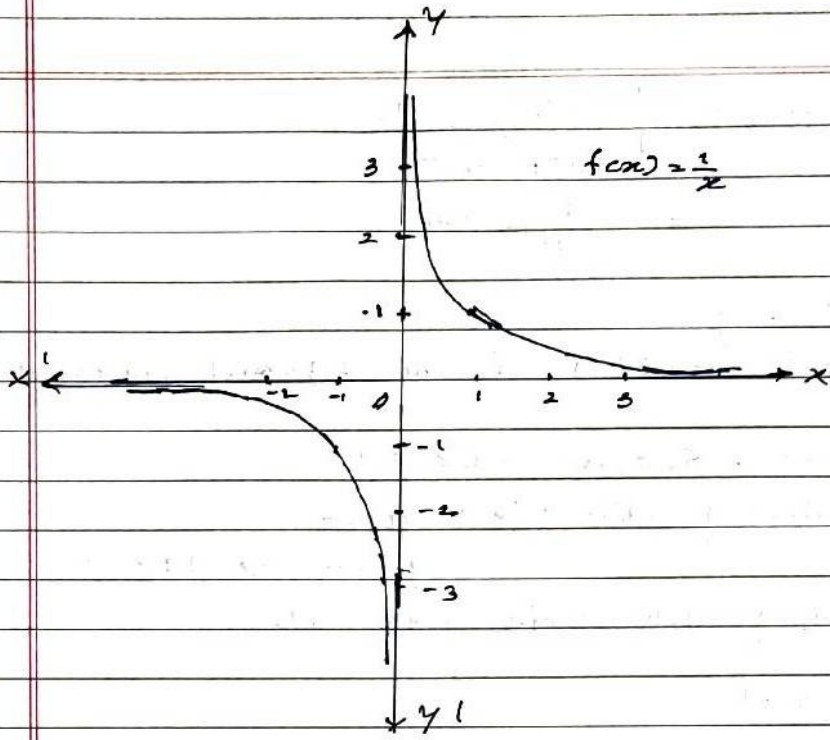
$$\text{RHS} = A' \cup B'$$

$$= \{1, 5, 6\} \quad (2)$$

From (1) & (2)

$$\text{LHS} = \text{RHS}$$

i)



ii)

$$f(x) = \sqrt{9-x^2}$$

To find the domain:

$$9-x^2 \geq 0$$

$$9 \geq x^2 \Rightarrow x^2 \leq 9 \Rightarrow x \leq \pm 3$$

$$\text{Domain} = [-3, 3]$$

To find the range:

$$y = \sqrt{9-x^2}$$

$$y^2 = 9-x^2$$

$$x^2 = 9-y^2$$

$$x = \sqrt{9-x^2}$$

$$\Rightarrow 9-x^2 \geq 0$$

$$9 \geq x^2$$

$$\Rightarrow x^2 \leq 9$$

$$\Rightarrow x \leq \pm 3$$

But,  $y \geq 0$

$$\therefore \text{Range} = [0, 3]$$



$$\text{ii) } (1-i)^6 = 1 + {}^6C_1 i + {}^6C_2 i^2 + {}^6C_3 i^3 + {}^6C_4 i^4 + {}^6C_5 i^5 + i^6$$

$$= 1 - 6(i) + 15(-1) - 20(-i) + 15(1) - 6(i) + (-1)$$

$$= 1 - 6i - 15 + 20i + 15 - 6i - 1$$

$$= 0 + 20i - 12i$$

$$= 8i$$

$$= 0 + i^8$$

$$\text{ii) Let } z = \frac{1-i}{1+i} \quad (\text{Deleted})$$

$$= \frac{1-i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{(1-i)^2}{(1+i)(1-i)}$$

$$= \frac{1-2i+i^2}{1-i^2}$$

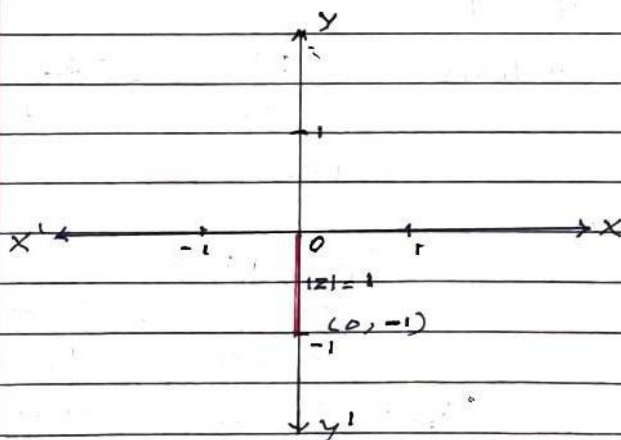
$$= \frac{1-2i-1}{1+1}$$

$$= \frac{-2i}{2} = -i$$

$$= 0 + i(-1)$$

$$= 0 + i(-1)$$

$$|z| = (-1)^2 = 1$$



$$12 \text{ i) } {}^n P_r = 840$$

$${}^n C_r = 35$$

$$\frac{n!}{r!(n-r)!} = 35$$

$$\frac{n!}{(n-r)!} = 35 \times r!$$

$${}^n P_r = 35 \times r!$$

$$\frac{840}{35} = r!$$

$$24 = r!$$

$$\Rightarrow r! = 24 = 1 \times 2 \times 3 \times 4 = 4!$$

$$\Rightarrow r = 4$$

ii) The word ATTITUDE has

$$A - 1$$

$$T - 4$$

$$I - 1$$

$$U - 1$$

$$D - 1$$

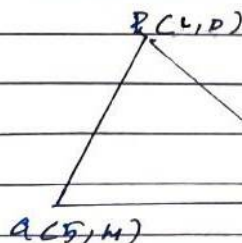
$$E - 1$$

$$\hline \text{Total} - 9$$

$$\therefore \text{No. of Permutations} = \frac{9!}{4!} = \frac{362880}{24}$$

$$= 15120.$$

13) i)



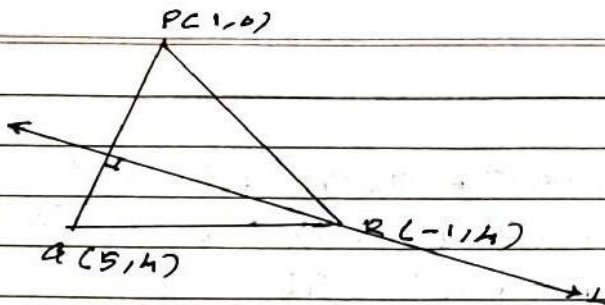
Eqn of PA is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

$$\frac{x-1}{5-1} = \frac{y-0}{4-0}$$

$$\frac{x-1}{4} = \frac{y}{4}$$

$$x - y - 1 = 0.$$



$$\begin{aligned} \text{Slope of } L &= -1 \\ &= \frac{-1}{\text{Slope of } PQ} \\ &= \frac{-1}{\frac{4-0}{5-1}} = \frac{-1}{1} = -1 (= m) \end{aligned}$$

Eq of L is  $y - y_1 = m(x - x_1)$

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x - 1$$

$$y - 4 + x + 1 = 0$$

$$x + y - 3 = 0.$$

14)  $c = 4$   $a = 5$

$$c^2 = a^2 - b^2$$

$$4^2 = 5^2 - b^2$$

$$b^2 = 5^2 - 4^2 = 9$$

$$b = \pm 3.$$

a) Length of minor axis =  $2b = 2(3) = 6$

b) Length of LR =  $\frac{2b^2}{a}$

$$= \frac{2 \times 3^2}{5} = \frac{18}{5}$$

$$e = \frac{c}{a} = \frac{4}{5}$$

iii) Eqn is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

15) i)  $(-1, 2, 3)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(-1 - 0)^2 + (6 - 7)^2 + (6 - 10)^2}$$

$$= \sqrt{1 + 1 + 16} = \sqrt{18}$$

$$BC = \sqrt{(6 - 3)^2 + 3^2 + 0^2}$$

$$= \sqrt{9 + 9} = \sqrt{18}$$

$$AC = \sqrt{(6 - 4)^2 + 2^2 + (6 - 4)^2}$$

$$= \sqrt{4 + 4 + 16} = \sqrt{24}$$

$$AB^2 + BC^2 = (\sqrt{18})^2 + (\sqrt{18})^2$$

$$= 18 + 18$$

$$= 36$$

$$= AC^2$$

$\therefore \Delta ABC$  is a right  $\Delta$ .

16) i)  $P(\text{all 3 are white}) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

ii)  $P(\text{all 3 are red}) = \frac{{}^8C_3}{{}^{13}C_3} = \frac{56}{286} = \frac{28}{143}$

iii)  $P(\text{one is red and two balls are white})$

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{8 \times 10}{286}$$

$$= \frac{40}{143}$$

17) i)  $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos 2\theta$

$$\therefore \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \cos 2(15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Ans: (b).



$$\begin{aligned}
 \text{ii)} \quad \text{LHS} &= \frac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x} \\
 &= \frac{2 \cos\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right)}{\cos 2x} \\
 &= \frac{2 \cos 2x \cdot \sin x}{\cos 2x} \\
 &= 2 \sin x \\
 &= \text{RHS.}
 \end{aligned}$$

$$\text{iii)} \quad \tan \theta = \frac{1}{2} \quad ; \quad \tan \phi = \frac{1}{3}$$

$$\text{Let } \theta + \phi = \frac{\pi}{4}$$

$$\begin{aligned}
 \tan(\theta + \phi) &= \tan \frac{\pi}{4} \\
 \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{LHS} &= \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} \\
 &= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} \\
 &= \frac{3+2}{6-1} = \frac{5}{5} = 1 = \text{RHS}
 \end{aligned}$$

Hence proved.

$$18 \text{ i)} \quad a = 2 \quad ; \quad r = 4$$

$$a_n = 32768$$

$$a r^{n-1} = 32768$$

$$2 \times 4^{n-1} = 32768$$

$$4^{n-1} = 16384$$

$$= 4^7$$

$$4^{n-1} = 4^{8-1}$$

$$\therefore n = 8$$

ii)  $a + ar + ar^2 = 14$  — (1)

$$ar^3 + ar^4 + ar^5 = 112$$
 — (2)

$$(2) \div (1) \Rightarrow$$

$$\frac{r^3(a + ar + ar^2)}{(a + ar + ar^2)} = \frac{112}{14}$$

$$r^3 = 8 = 2^3$$

$$\therefore r = 2$$

in (1)  $a + a(2) + a(2^2) = 14$

$$a + 2a + 4a = 14$$

$$7a = 14$$

$$a = 2$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, r > 1$$

$$= \frac{2(2^n - 1)}{2 - 1}$$

$$= \frac{2(2^n - 1)}{1}$$

$$= \underline{\underline{2(2^n - 1)}}$$

19) i)

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$\begin{array}{r} 2 \overline{) 16384} \\ \underline{2 \phantom{00000}} \\ 8192 \\ \underline{2 \phantom{00000}} \\ 4096 \\ \underline{2 \phantom{00000}} \\ 2048 \\ \underline{2 \phantom{00000}} \\ 1024 \\ \underline{2 \phantom{00000}} \\ 512 \\ \underline{2 \phantom{00000}} \\ 256 \\ \underline{2 \phantom{00000}} \\ 128 \\ \underline{2 \phantom{00000}} \\ 64 \\ \underline{2 \phantom{00000}} \\ 32 \\ \underline{2 \phantom{00000}} \\ 16 \\ \underline{2 \phantom{00000}} \\ 8 \\ \underline{2 \phantom{00000}} \\ 4 \\ \underline{2 \phantom{00000}} \\ 2 \end{array}$$

$$\begin{aligned}
 f(x+h) - f(x) &= \frac{1}{x+h} - \frac{1}{x} \\
 &= \frac{x - (x+h)}{x(x+h)} \\
 &= \frac{-h}{x(x+h)}
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \left[ \frac{1}{h} \times \frac{-h}{x(x+h)} \right] \\
 &= \frac{-1}{x(x+0)} = \frac{-1}{x^2}
 \end{aligned}$$

ii)  $f(x) = \frac{x^2+1}{x^2-1}$

$$\begin{aligned}
 f'(x) &= \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2} \\
 &= \frac{2x [x^2-1 - x^2-1]}{(x^2-1)^2} \\
 &= \frac{2x(-2)}{(x^2-1)^2} \\
 &= \frac{-4x}{(x^2-1)^2}
 \end{aligned}$$

20 i)

$x_i$	$ x_i - \bar{x} $
4	6
7	3
8	2
9	1
10	0
12	2
13	3
17	7
$\Sigma x_i = 80$	$\Sigma  x_i - \bar{x}  = 24$

$$\begin{aligned}
 * \bar{x} &= \frac{\Sigma x_i}{n} \\
 &= \frac{80}{8} = 10
 \end{aligned}$$

$$\begin{aligned}
 * MD(\bar{x}) &= \frac{\Sigma |x_i - \bar{x}|}{n} \\
 &= \frac{24}{8} = 3
 \end{aligned}$$

ii)	class	$x_i$	$f_i$	$u_i$	$f_i u_i$	$u_i^2$	$f_i u_i^2$
	4-8	4	3	-2	-6	4	12
	8-12	8	6	-1	-6	1	6
	12-16	$\frac{12}{2}$	4	0	0	0	0
	16-20	16	7	1	7	1	7
			$N=20$		$\sum f_i u_i = -5$		$\sum f_i u_i^2 = 25$

$$\begin{aligned} \text{Variance} &= \left[ \frac{\sum f_i u_i^2}{N} - \left( \frac{\sum f_i u_i}{N} \right)^2 \right] \times b^2 \\ &= \left[ \frac{25}{20} - \left( \frac{-5}{20} \right)^2 \right] 4^2 \\ &= \left[ \frac{5}{4} - \frac{1}{16} \right] \times 16 \\ &= \left( \frac{20}{16} - \frac{1}{16} \right) \times 26 \\ &= \frac{19}{16} \times 16 \\ &= 19 \end{aligned}$$

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