ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2024

PART-~~I/II~~/III

SUBJECT: MATHEMATICS (SCIENCE)

CODE NO: **SY 554** VERSION: Q

80 SCORES $2\frac{1}{2}$ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.		$f(x) = \cos x \qquad g(x) = 3x^2$		
		$g\circ f(x) = g[f(x)]$	½	
		$= g[\cos x]$	½	
		$= 3\cos^2 x$	½	3
		$f\circ g(x) = f[g(x)]$	½	
		$= f(3x^2)$	½	
		$= \cos(3x^2)$	½	
2.		$[a_{ij}]_{2\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$	1	
		$a_{ij} = 2i+j$		3
		$\therefore$ required matrix is $\begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$	2	
3.	(i)	(c) $k^3\|A\|$	1	
	(ii)	$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$		
		$3 - x^2 = 3 - 8 = -5$	1	3
		$x^2 = 3+5 = 8$	½	
		$x = \pm\sqrt{8} = \pm 2\sqrt{2}$	½	

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4.		f is continuous at $x=5$		
		$\therefore \lim_{x\to 5^-} F(x) = \lim_{x\to 5^+} f(x)$	1	
		$\lim_{x\to 5^-} kx+1 = \lim_{x\to 5^+} 3x-5$	½	3
		$5k+1 = 10$	½	
		$5k = 9$	1	
		$k = 9/5$		
5.	(i)	(c) $\dfrac{\cos x}{2\sqrt{\sin x}}$	1	
		(ii) $y + \sin y = \cos x$		
		Diff w.r.t x		
		$\dfrac{dy}{dx} + \cos y \dfrac{dy}{dx} = -\sin x$	1	3
		$(1+\cos y)\dfrac{dy}{dx} = -\sin x$	½	
		$\dfrac{dy}{dx} = \dfrac{-\sin x}{1+\cos y}$	½	
6	(i)	$f(x) = x^2 - 4x + 6$		
		$f'(x) = 2x - 4$	1	
	(ii)	$f'(x) > 0$ when f is increasing	1	3
		(ie) $2x - 4 > 0$	½	
		$2x > 4$	½	
		$x > 2$		
		f is increasing in the interval $(2, \infty)$		

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7.	(i)	(B) 2	1	
	(ii)	$\frac{dy}{dx} = (1+x^2)(1+y^2)$		
		$\frac{dy}{1+y^2} = (1+x^2)\,dx$	1	3
		Integrating		
		$\int \frac{dy}{1+y^2}\,dx = \int (1+x^2)\,dx$	½	
		$\tan^{-1} y = x + x^3/3 + c$	½	
		$y = \tan(x + x^3/3 + c)$		
8		$A(-1,0,2)$ $B(3,4,6)$		
		$\bar{a} = -\hat{i} + 2\hat{k}$ $\bar{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$	1	3
		required equation is		
		$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$	1	
		ie) $\bar{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$	1	
9	(i)	$f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3$	1	
		$\Rightarrow 2x_1 = 2x_2$	½	
		$\Rightarrow x_1 = x_2$	½	
		$\Rightarrow f$ is one-one		4
	(ii)	let $y = 2x+3$		
		$\therefore 2x = y - 3 \Rightarrow x = \frac{y-3}{2} \in R$	1	
		$\therefore f$ is onto		
		Hence f is invertible	1	
		$f^{-1}(x) = \frac{x-3}{2}$		

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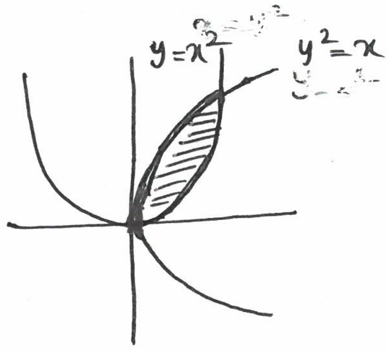
10	(i)	(A) $\frac{\pi}{3}$	1	
	(ii)	$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left[\frac{1/2 + 2/11}{1 - 1/2 \cdot 2/11}\right]$	1	
		$= \tan^{-1}\left[\frac{\frac{11+4}{22}}{\frac{22-2}{22}}\right]$	1/2	4
		$= \tan^{-1}\left[\frac{15}{20}\right]$	1/2	
		$= \tan^{-1}\frac{3}{4}$	1	
11.	(i)	$x = \log t \qquad y = \sin t$		
		$\frac{dx}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = \cos t$	1	
		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	1/2	
		$= \frac{\cos t}{1/t}$	1/2	
		$= \underline{\underline{t \cos t}}$		4
	(ii)	$y = 3\sin x + 2\cos x$		
		$\frac{dy}{dx} = 3\cos x - 2\sin x$	1	
		$\frac{d^2y}{dx^2} = -3\sin x - 2\cos x$	1	
		$= -y$		
		$\therefore \frac{d^2y}{dx^2} + y = 0$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
12.	(i)	(D) $\int_0^a f(a-x)\,dx$	1	
	(ii)	let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ —— ①		
		Also $I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)}+\sqrt{\cos(\pi/2 - x)}}\,dx$	½	
		$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ —— ②	½	
		① + ② $\Rightarrow$		
		$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$	½	4
		$= \int_0^{\pi/2} dx$	½	
		$= [x]_0^{\pi/2}$		
		$= \pi/2 - 0$	½	
		$2I = \frac{\pi}{2}$		
		$\therefore I = \frac{\pi}{4}$	½	
		$\therefore \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx = \frac{\pi}{4}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
13		$y = x^2$		
		$y^2 = x \Rightarrow y = \sqrt{x}$		
				
		$\therefore x^2 = \sqrt{x}$	1	
		$x^4 = x$		
		$x^4 - x = 0 \Rightarrow x(x^3-1) = 0$		
		$\therefore x = 0$ or 1		
		points of intersection of curves is $(0,0)$ and $(1,1)$	½	
		Required Area $= \int_a^b y_2 - y_1 \, dx$	½	
		$= \int_0^1 \sqrt{x} - x^2 \, dx$	½	
		$= \left[\dfrac{x^{3/2}}{3/2} - \dfrac{x^3}{3}\right]_0^1$	½	4
		$= \dfrac{1}{3/2} - \dfrac{1}{3}$		
		$= 2/3 - 1/3$	½	
		$= 1/3$ sq. units	½	
		Area of the region bounded by the two parabola is $1/3$ sq. units.		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
14	(i)	$\frac{dy}{dx} + \frac{y}{x} = x^2$		
		$P = \frac{1}{x}$ $\quad Q = x^2$	½	
		$\int P dx = \log x$		
		Integrating factor (IF) $= e^{\int P dx}$		
		$= e^{\log x}$	½	4
		$= x$		
	(ii)	general solution is		
		$y \times IF = \int Q \times IF \, dx$	1	
		$yx = \int x^2 \times x \, dx$	1	
		$xy = \frac{x^4}{4} + c$	1	
15.		$\bar{a}_1 = i + j$ $\quad \bar{b}_1 = 2i - j + k$		
		$\bar{a}_2 = 2i + j - k$ $\quad \bar{b}_2 = 3i - 5j + 2k$	1	
		$\bar{a}_2 - \bar{a}_1 = i - k$	½	
		$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$	½	
		$= 3i - j - 7k$	½	
		$\lvert \bar{b}_1 \times \bar{b}_2 \rvert = \sqrt{3^2 + 1^2 + 7^2}$	½	
		$= \sqrt{59}$		

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		$(\bar{a}_2 - \bar{a}_1)\cdot(\bar{b}_1 \times \bar{b}_2) = 1\times 3 + 1\times 7 = 10$		
		$SD = \left\lvert \dfrac{(\bar{a}_2-\bar{a}_1)\cdot(\bar{b}_1\times\bar{b}_2)}{\lvert \bar{b}_1\times\bar{b}_2\rvert} \right\rvert$	½	4
		$= \dfrac{10}{\sqrt{59}}$	½	
16	(i)	$\bar{a}\times\bar{b} = \begin{vmatrix} i & j & k \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$	1	
		$= 20i + 5j - 5k$	1	4
	(ii)	Area of parellelogram $= \lvert \bar{a}\times\bar{b}\rvert$	1	
		$= \sqrt{20^2+5^2+5^2}$		
		$= \sqrt{450}$ sq.units	1	
17	(i)	$\bar{r}\cdot(i+j-k) = 2$		
		Cartesian equation is		
		$x + y - z = 2$	2	
	(ii)	Distance $= \left\lvert \dfrac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}} \right\rvert$	1	4
		$= \left\lvert \dfrac{6x_1 - 3y_1 + 2z_1 - 4}{\sqrt{6^2+3^2+2^2}} \right\rvert$	½	
		$= \left\lvert \dfrac{6\times 2 - 3\times 5 + 2\times -3 - 4}{\sqrt{49}} \right\rvert = \dfrac{13}{7}$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
18	(i)	$P(A \text{ and } B) = P(A\cap B)$		
		$= P(A).\,P(B)$	½	
		$= 0.3 \times 0.6 = 0.18$	½	
	(ii)	$P(A \text{ or } B) = P(A\cup B)$		
		$= P(A) + P(B) - P(A\cap B)$	1	
		$= 0.3 + 0.6 - 0.18 = 0.72$	1	4
	(iii)	$P(\text{neither } A \text{ nor } B) = P(A'\cap B')$		
		$= P(A\cup B)'$		
		$= 1 - P(A\cup B)$	½	
		$= 1 - 0.72 = 0.28$	½	
19.	(i)	$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$		
		$A^2 = A \times A$	½	
		$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	1	
		$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$	1	
		$= \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = 0$	½	
	(ii)	let $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ $\therefore A^T = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$	½	
		$A + A^T = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$	½	
		let $P = \frac{A + A^T}{2} = \frac{1}{2}\begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 11/2 \\ 11/2 & 7 \end{bmatrix}$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$A - A^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\frac{1}{2}$	
		let $Q = \frac{A - A^T}{2}$		
		$= \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$	$\frac{1}{2}$	6
		$A = P + Q$		
		$\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 11/2 \\ 11/2 & 7 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$	$\frac{1}{2}$	
		where P is symmetric and Q is skew-symmetric		
20	(i)	let $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$		
		$B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$		
		$AX = B$		
		$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
	(ii)	$A_{11} = -1$ $A_{12} = -8$ $A_{13} = -10$		
		$A_{21} = -5$ $A_{22} = -6$ $A_{23} = 1$	1	
		$A_{31} = -1$ $A_{32} = 9$ $A_{33} = 7$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\text{Co-factor matrix} = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$	½	
		$\text{Adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	½	
	(iii)	$A^{-1} = \frac{1}{\lvert A \rvert} (\text{Adj } A)$		
		$\lvert A \rvert = 3 \times -1 + -2 \times -8 + 3 \times -10$		
		$= -3 + 16 - 30 = -17$	½	
		$A^{-1} = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	½	6
		$X = A^{-1} B$	½	
		$= \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	½	
		$= \frac{-1}{17} \begin{bmatrix} -8 - 5 - 4 \\ -64 - 6 + 36 \\ -80 + 1 + 28 \end{bmatrix}$	½	
		$= \frac{-1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$		
		$\therefore x = 1, \; y = 2, \; z = 3$	½	

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21	(i)	(a) $y = x^3 - x$		
		$\frac{dy}{dx} = 3x^2 - 1$	1	
		$\left[\frac{dy}{dx}\right]_{x=2} = 11$ (m)	½	
		$\therefore$ slope of tangent $= 11$	½	
		(b) equation of tangent at $x=2$		
		$x = 2 \Rightarrow y = 2^3 - 2 = 6$	½	
		Equation is $y - y_1 = m(x - x_1)$	½	
		$y - 6 = 11(x-2)$	½	
		$y - 6 = 11x - 22$		
		$11x - y - 16 = 0$	½	6
	(ii)	$y = \sqrt{x}$, $x = 36$, $\Delta x = 0.6$	½	
		$\Delta y = \sqrt{x + \Delta x} - \sqrt{x}$	½	
		$= \sqrt{36.6} - \sqrt{36}$		
		$= \sqrt{36.6} - 6$	½	
		$\therefore \sqrt{36.6} = 6 + \Delta y$		
		$\Delta y \simeq dy \simeq \left(\frac{dy}{dx}\right)\Delta x$		
		$= \frac{1}{2\sqrt{x}} \times 0.6 = \frac{0.6}{12} = 0.05$	½	
		$\therefore \sqrt{36.6} = 6 + 0.05 = 6.05$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22	(i)	$\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$		
		put $\tan^{-1}x = u$	1	
		$= \int e^u . du$ $\frac{1}{1+x^2}\,dx = du$		
		$= e^u + c = e^{\tan^{-1}x} + c$	1	
	(ii)	$\int \frac{1}{x^2-6x+13}\,dx$		
		$= \int \frac{1}{(x-3)^2+4}\,dx$	1	
		$= \int \frac{1}{(x-3)^2+2^2}\,dx$	½	6
		$= \frac{1}{2}\tan^{-1}\frac{x-3}{2} + c$	½	
	(iii)	$\int x \log x\,dx$		
		$= \int \log x . x\,dx$	½	
		$= \log x . x^2/2 - \int 1/x . x^2/2\,dx$	½	
		$= x^2/2 \log x - 1/2 \int x\,dx$	½	
		$= \frac{x^2}{2}\log x - \frac{x^2}{4} + c$	½	

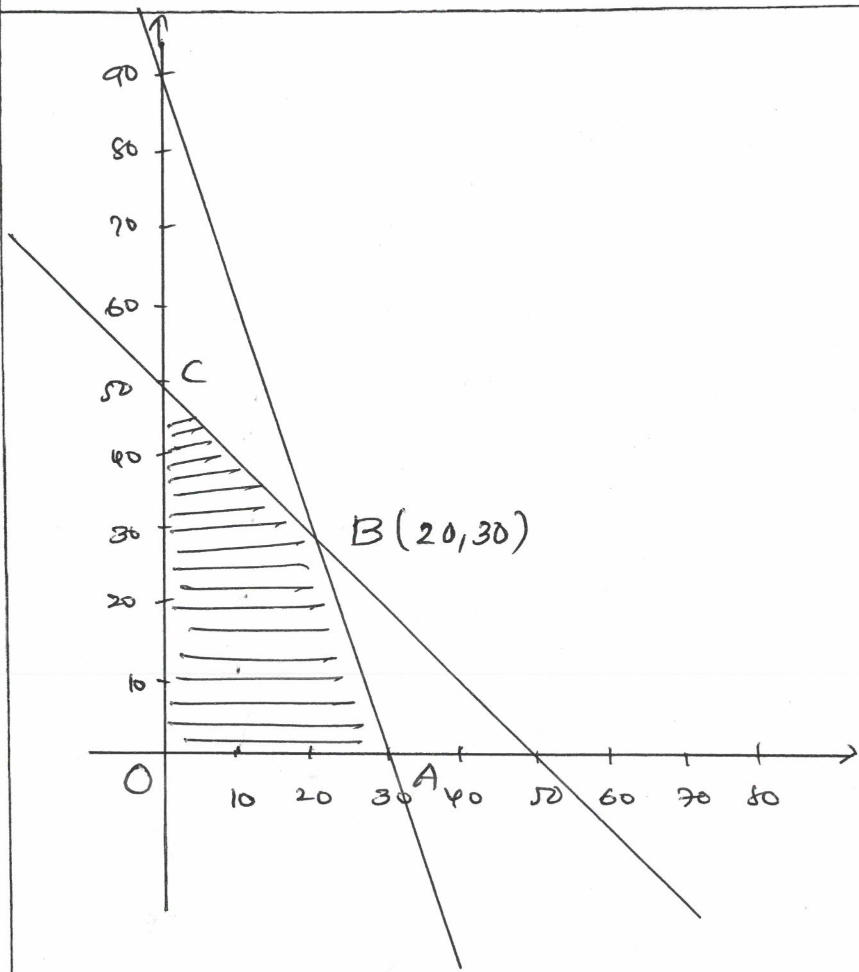
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
23	(i)	$\bar{a}\cdot\bar{b} = 1\times1 + 1\times -1 + -1\times1$	1	
		$= 1-1-1 = -1$	1	
	(ii)	$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{\lvert\bar{a}\rvert\lvert\bar{b}\rvert}$	1/2	
		$= \dfrac{-1}{\sqrt{3}\sqrt{3}}$	1/2	
		$= \dfrac{-1}{3}$	1/2	6
		$\theta = \cos^{-1}\left[\dfrac{-1}{3}\right]$	1/2	
	(iii)	projection of $\bar{a}$ on $\bar{b}$		
		$= \dfrac{\bar{a}\cdot\bar{b}}{\lvert\bar{b}\rvert}$	1	
		$= \dfrac{-1}{\sqrt{3}}$	1	
24.		$x+y=50$		
		$3x+y=90$	1	

$x+y=50$

x	0	50
y	50	0

$3x+y=90$

x	0	30
y	90	0

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
			3	

Corner point	$Z = 4x + y$	Score
O (0,0)	0	
A (30,0)	120 ←	2
B (20,30)	110	
C (0,50)	50	

Maximum value of $Z = 120$

at $A(30,0)$

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
25	(i)	X: No of heads		
		X can take values 0, 1, 2, 3		
		$P(X=0) = P(head) \times P(Head) \times P(Head)$		
		$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$		
		$P(X=1) = 3\left[P(Head) \times P(Tail) \times P(Tail)\right]$		
		$= 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$	1	
		$P(X=2) = \frac{3}{8}$		
		$P(X=3) = \frac{1}{8}$		6
		X: 0, 1, 2, 3; P(X): $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$, $\frac{1}{8}$	1	
	(ii)			

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\frac{3}{2}$	3

Score: 2

Mean of $X = \Sigma x_i p_i$ — ½

$= \frac{3}{2}$ — ½

Variance of $X = \Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$ — ½

$= 3 - \frac{9}{4}$ — ½

$= \frac{3}{4}$