7

NEGATIVE NUMBERS

Measures and numbers

We've seen in class 8, how we use negative numbers to denote temperatures below zero. We set zero degree celsius, written 0°C, as the temperature at which water freezes to solid ice. To denote temperatures colder than this, we have to use notations like −1°C, −20.5°C.

We've also seen how negative numbers are used to denote points in some games and as scores in some exams. We also noted some general results on calculations with such numbers.

For example, look at this question:

If temperature dropped 7°C from 3°C, what would it be?

To answer this we introduced the operation

$$3 - 7 = -4$$

and through other examples like this, came up with the general law for subtracting a small number from a larger number.

Through such examples of operations needed in different contexts, we formulated three general laws about operations with negative numbers:

For any two positive numbers x, y

(i) If $x < y$, then $x - y = -(y - x)$

(ii) $-x + y = y - x$

(iii) $-x - y = -(x + y)$

Using these ideas, can't you do the following?

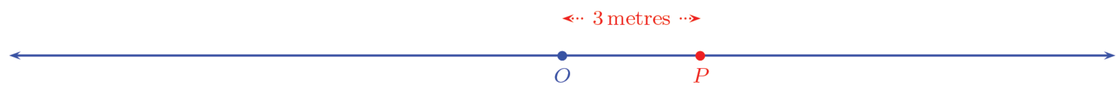
(i) $6 - 8$ (ii) $-6 + 8$ (iii) $-6 - 8$

(iv) $2\frac{1}{2} - 3\frac{1}{2}$ (v) $-2\frac{1}{2} + 3\frac{1}{2}$ (vi) $-2\frac{1}{2} - 3\frac{1}{2}$

We next look at another context in which negative numbers are used.

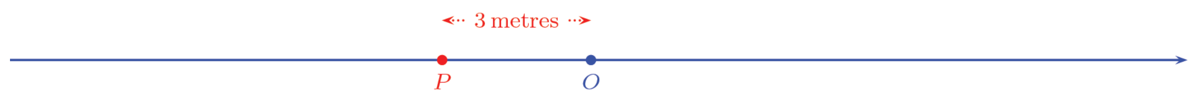
Position and number

Imagine a point moving along a straight line. To indicate its position, we can specify its distance (measured in some unit) from a fixed point:



The picture shows the position of the moving point *P* at a distance of 3 metres to the right of the fixed point *O*.

P can also be at 3 metres to the left of *O*.

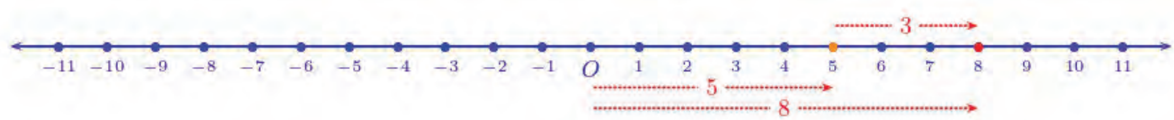


So to know the exact position of *P*, distance alone will not do; we'll have to specify the direction also.

One way to overcome this, is to specify distances to the left as negative numbers. Once we have decided on a unit of length (such metre or centimetre), we can denote all positions, except *O*, on the line using positive and negative numbers.

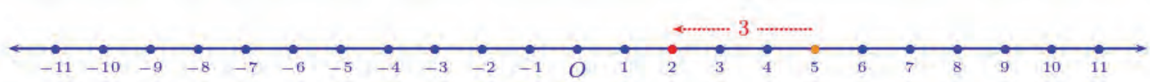
The position of the fixed point *O* can be denoted by 0.

Now suppose that the moving point shifts from the position 5 to the position 8. We can say that the displacement (change in position) is 8 – 5 = 3:



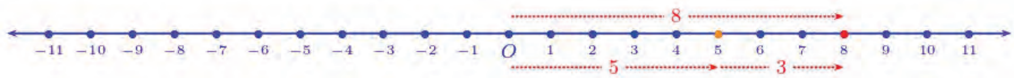
On the other hand, if we say that the point shifts 3 places from 5, can we say that its current position is 8 ?

It's correct if the change in position is to the right; what if it is to the left ?

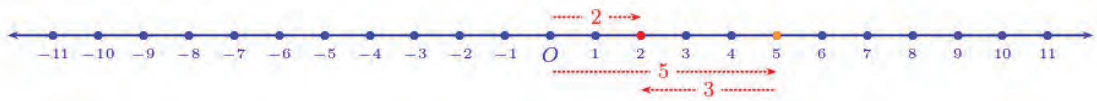


So it's not enough to know the amount of displacement, we should also know whether it is to the right or left.

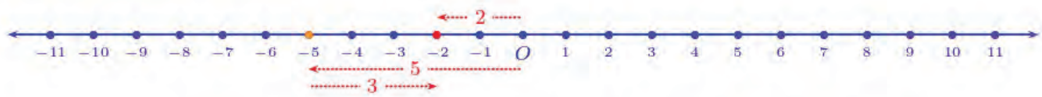
That is, if the point shifts 3 places to the right from 5, then it is at 8.



If the shift is to the left, the position is 2:



What if the point shifts 3 places to the right from –5 instead?



Is there a general method by which we can compute the final position, if we know the initial (first) position and the displacement with direction ?

For this, let's tabulate the various possibilities. First we consider only displacements to the right:

Initial Position	Displacement	Final Position
5	3 right	8
3	5 right	8
–5	3 right	–2
–3	5 right	

Write the last number also by drawing a picture (or by visualizing in the mind).

Let's write just the numbers for the displacements:

Initial Position	Displacement	Final Position
5	3	8
3	5	8
–5	3	–2
–3	5	2

Is there any relation between the numbers in each row?

We can see at a glance that in the first two rows, the third number is the sum of the first two numbers.

What about the next line?

We've seen earlier that

$$-5 + 3 = 3 - 5 = -2$$

And the last line?

We've also seen that

$$-3 + 5 = 5 - 3 = 2$$

So in every row of the table, the last number is the sum of the first two numbers:

Initial Position	Displacement	Final Position
5	3	8 = 5 + 3
3	5	8 = 3 + 5
–5	3	–2 = –5 + 3
–3	5	2 = –3 + 5

Now let's look at displacements to the left:

Initial Position	Displacement	Final Position
5	3 left	2
3	5 left	–2
–5	3 left	–8
–3	5 left	–8

As in the case of positions, let's write displacements to the left also as negative numbers:

Initial Position	Displacement	Final Position
5	–3	2
3	–5	–2
–5	–3	–8
–3	–5	–8

As in the first table, is the last number in each row, the sum of the first two in this also ?

To check this for the first row here, we must compute the sum 5 + (–3). But we haven't seen such a sum before.

Let's think like this: for two positive numbers, even if we change the order of summation, the sum doesn't change; for example

$$5 + 3 = 3 + 5 \qquad \frac{1}{2} + \frac{1}{4} = \frac{1}{4} + \frac{1}{2} \qquad \sqrt{2} + \sqrt{3} = \sqrt{3} + \sqrt{2}$$

So here also, we can take

$$5 + (-3) = -3 + 5$$

and in this, we've already seen that

$$-3 + 5 = 5 - 3 = 2$$

Thus we define

$$5 + (-3) = 5 - 3 = 2$$

Then in the first line of this table also, we'll get the last number as the sum of the first two numbers.

By the same token, if we take

$$3 + (-5) = -5 + 3 = 3 - 5 = -2$$

then the relation between the numbers in the second row would also be the same:

Initial Position	Displacement	Final Position
5	–3	2 = 5 + (–3)
3	–5	–2 = 3 + (–5)
–5	–3	–8
–3	–5	–8

What about the remaining two rows ?

In those also, we should give meaning to the sum of the first two numbers. In the first two rows, we wrote the addition of a negative number as a difference, such as

$$5 + (-3) = 5 - 3 = 2$$

$$3 + (-5) = 3 - 5 = -2$$

How about doing this for the last two rows as well ?

$$(-5) + (-3) = -5 - 3$$

$$(-3) + (-5) = -3 - 5$$

We've already seen subtractions as in the right of these equations:

$$-5 - 3 = -(5 + 3) = -8$$

$$-3 - 5 = -(3 + 5) = -8$$

Thus if we define

$$(-5) + (-3) = -5 - 3 = -8$$

$$(-3) + (-5) = -3 - 5 = -8$$

then the relation between the numbers in the last two rows would also be the same as that in the first two rows:

Initial Position	Displacement	Final Position
5	–3	2 = 5 + (–3)
3	–5	–2 = 3 + (–5)
–5	–3	–8 = –5 + (–3)
–3	–5	–8 = –3 + (–5)

Now let's combine our two tables for displacements to the right and left:

Initial Position x	Displacement y	Final Position $x + y$
5	3	8 = 5 + 3
3	5	8 = 3 + 5
–5	3	–2 = (–5) + 3
–3	5	2 = (–3) + 5
5	–3	2 = 5 + (–3)
3	–5	–2 = 3 + (–5)
–5	–3	–8 = (–5) + (–3)
–3	–5	–8 = (–3) + (–5)

Here's another table like this, giving different initial positions and displacements:

Initial Position	Displacement	Final Position
7	3 right	
3	7 right	
–7	3 right	
–3	7 right	
7	3 left	
3	7 left	
–7	3 left	
–3	7 left	

Draw pictures to find the final positions and write in the table. Then make another table with the displacements as just numbers, positive or negative and check if in each row, the last number is the sum of the first two numbers.

Now to make the last number in each row, the sum of the first two numbers, we gave meaning to some new operations, such as

$$3 + (-5) = 3 - 5 = -2$$

$$5 + (-3) = 5 - 3 = 2$$

$$(-5) + (-3) = -5 - 3 = -8$$

$$(-3) + (-5) = -3 - 5 = -8$$

We note that in all these equations, the left side is the addition of a negative number; and it is defined as subtraction of a positive number in the right side.

We use algebra to make this more precise:

For any number x and any positive number y,

$$x + (-y) = x - y$$

Using this, we see that in our position problem, the displacement added to the initial position gives the final position, whatever be the kind of numbers these are.

That is, if we denote the initial position by x, displacement by y and final position by z, we can use the single equation

$$z = x + y$$

to find the final position, whatever be the kind of numbers x, y, z are.

Here we could write the relation connecting x, y, z in a single equation only because we could take them as positive or negative numbers.

Just think how many equations we would need, if we use only positive numbers with left or right directions specified in words.

One fact, many equations

What if we try to write the connection between positions and displacement in algebra, without using negative numbers? We can denote the distances from O by x and z and the distance between them by y. Since these are all to be positive numbers as distances, we will also have to specify the directions left or right. So we will have to write the relation between them in different ways depending on the situation:

For x and y in the same direction
$z = x + y$ in the same direction

For x and y in different directions

if $x > y$

then $z = x - y$ in the direction of x

if $x < y$

then $z = y - x$ in the direction of y

(1) Complete the table below:

x	y	$x + y$
6	−10	
−6	10	
−6	−10	
−6	6	
6	−6	

(2) Find two pairs of numbers x and y satisfying each of the following conditions:

(i) x positive, y negative with, $x + y = 1$

(ii) x negative, y positive with, $x + y = 1$

(iii) x positive, y negative with $x + y = -1$

(iv) x negative, y positive with $x + y = -1$

(3) Complete the table below:

x	y	z	$(x + y) + z$	$x + (y + z)$
2	4	−5		
2	−4	5		
−2	4	−5		
2	−4	−5		
−2	4	5		
−2	−4	5		
−2	−4	−5		

A different difference

An easy way to subtract 9 from 17 is to subtract 9 from 10 and add 7:

$$17 - 9 = (10 + 7) - 9$$
$$= (10 - 9) + 7$$
$$= 1 + 7 = 8$$

We can also do using addition of negative numbers.

$$17 - 9 = (10 + 7) - 9$$
$$= 10 + (7 - 9)$$
$$= 10 + (-2)$$
$$= 10 - 2 = 8$$

In the same way,

$$57 - 29 = (50 - 20) + (7 - 9)$$
$$= 30 + (-2)$$
$$= 30 - 2 = 28$$

Displacement

We can have a different problem on positions of a moving point: to move from the position 4 to the position 7, how many places should the point move?

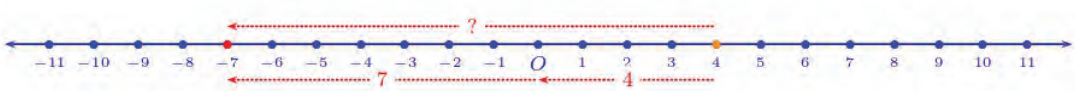
It is not enough to say 3 places; if the shift is towards the left from 4, it would reach 1, right ? So to move from 4 to 7, we must say 3 places to the right.

How about moving from 7 to 4 ?

Again 3 places, but now to the left.

How many places to shift to move from 4 to −7 ?

Let's draw a picture:



Let's calculate the displacements for various initial and final positions, to the left and right, and make a table as before. First we write displacements as positive numbers with the qualifications left or right:

Initial Position	Final Position	Displacement
4	7	3 right
7	4	3 left
4	−7	11 left
−7	4	
−4	7	
7	−4	
−4	−7	
−7	−4	

Next let's write displacements also as just numbers, with those to the left negative:

Initial Position	Final Position	Displacement
4	7	3
7	4	−3
4	−7	−11
−7	4	11
−4	7	11
7	−4	−11
−4	−7	−3
−7	−4	3

Now let's see in this table also, what the relation between the three numbers in each row is.

The first displacement was actually calculated as $7 - 4 = 3$.

Can we do this in the second row also?

$$4 - 7 = -3$$

So in this row also, we get the third number by subtracting the first from the second. What about the next row?

We've seen that

$$-7 - 4 = -(7 + 4) = -11$$

Thus this calculation works here also.

To check whether this holds in the fourth row, we have to say what the operation $4 - (-7)$ means.

Let's think like this. Subtraction of positive numbers can be described in many ways. One way of interpreting $10 - 6$ is, finding what should be added to 6 to make 10; then from $6 + 4 = 10$, we get $10 - 6 = 4$

Similarly we can think of the operation $4 - (-7)$ as finding what should be added to -7 to make 4. This can be done in two steps. 7 added to -7 makes 0. To get 4 from 0, we must add 4 more. Thus we have to add $7 + 4 = 11$ to get 4 from -7.

According to this interpretation,

$$4 - (-7) = 7 + 4 = 11$$

It is better to write this as

$$4 - (-7) = 4+7 = 11$$

for easy recall later.

Thus in the fourth row also, the third number is got by subtracting the first from the second. Thinking along the same lines, we can also see that

$$7 - (-4) = 7 + 4 = 11$$

This makes the fifth row of the table also conform to our rule.

As for the sixth row we have

$$-4 - 7 = -(4 + 7) = -11$$

as seen from earlier definitions.

Let's write our computations so far in the table:

Initial Position	Final Position	Displacement
4	7	$3 = 7 - 4$
7	4	$-3 = 4 - 7$
4	-7	$-11 = -7 - 4$
-7	4	$11 = 4 - (-7)$
-4	7	$11 = 7 - (-4)$
7	-4	$-11 = -4 - 7$
-4	-7	-3
-7	-4	3

To continue this for the next line, we should say what (–7) – (−4) means. Recall that we defined

$$7 - (-4) = 7 + 4$$

earlier in this problem itself. Similarly, we define

$$(-7) - (-4) = -7 + 4$$

We've already seen

$$-7 + 4 = 4 - 7 = -3$$

Thus we have

$$(-7) - (-4) = -7 + 4 = -3$$

which makes the seventh row also right.

In the same way we define

$$(-4) - (-7) = -4 + 7 = 3$$

and then everything is fine with the last row also and thus with the whole table:

Initial Position x	Final Position y	Displacement $y - x$
4	7	$3 = 7 - 4$
7	4	$-3 = 4 - 7$
4	–7	$-11 = -7 - 4$
–7	4	$11 = 4 - (-7)$
–4	7	$11 = 7 - (-4)$
7	–4	$-11 = -4 - 7$
–4	–7	$-3 = (-7) - (-4)$
–7	–4	$3 = -4 - (-7)$

Now to make this right, we've made some new definitions of subtraction:

$$4 - (-7) = 4 + 7 = 11$$

$$7 - (-4) = 7 + 4 = 11$$

and

$$(-7) - (-4) = -7 + 4 = -3$$

$$(-4) - (-7) = -4 + 7 = 3$$

In general, we defined the subtraction of a negative number as removing the negative sign and adding.

This we can write in algebra as below:

For any number x and any positive number y

$$x - (-y) = x + y$$

Suppose we take $x = 0$ and $y = 3$ in this equation. We get

$$0 - (-3) = 0 + 3 = 3$$

We have $0 - 3 = -3$. Similarly if we write $0 - (-3)$ simply as $-(-3)$, then using the above equation we get,

$$-(-3) = 0 - (-3) = 0 + 3 = 3$$

This is a new definition, that the meaning of the negative of negative of a positive number is the number itself.

For any positive number x.

$$-(-x) = 0 - (-x) = 0 + x = x$$

Using this, we can extend the first equation we gave at the beginning of this lesson. This is the equation we mean:

For any two positive numbers x and y, if $x < y$, then $x - y = -(y - x)$

What if $x > y$ in this equation ?

For example, let's take $x = 10$ and $y = 3$.

$$x - y = 10 - 3 = 7$$

$$y - x = 3 - 10 = -7$$

$$-(y - x) = -(-7) = 7$$

Here also, we get $x - y = -(y - x)$

In general, if $x < y$ in this equation, then $y - x$ is a positive number and $x - y$ is its negative

If $x > y$, then $y - x$ is negative and $x - y$ is its negative

Meaning of negative

We introduced negative numbers through situations requiring numbers less than zero. Later we saw that they could be used to distinguish between opposite directions. And we made some new definitions of addition and subtraction to facilitate this interpretation. Continuing with these, when we defined the negative of negative, the process of taking the negative becomes a mathematical operation.

Since we have defined the negative of the negative of a positive number as that number itself, the equation we are considering holds in the second case also.

So we can remove the condition in the equation and generalize it:

For any numbers x and y

$$x - y = -(y - x)$$

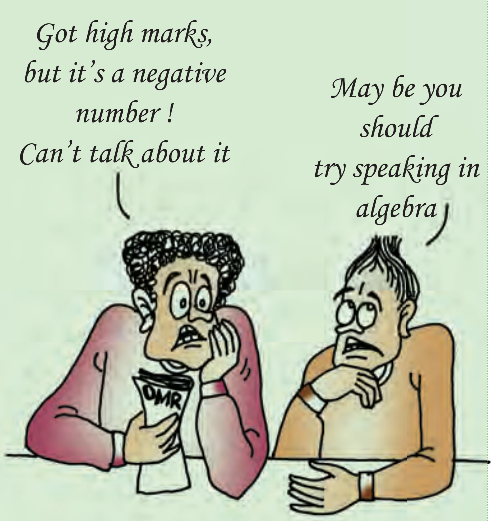
(1) Take different numbers, positive and negative, as x, y, z and calculate $x - (y - z)$ and $(x - y) + z$. Are both the same number in all cases ?

(2) Find two pairs of numbers x and y, satisfying each of the conditions below:

(i) x positive, y negative, $x - y = 1$

(ii) x negative, y positive, $x - y = -1$

(3) (i) Take four consecutive natural numbers or their negatives as a, b, c, d and calculate $a - b - c + d$



(ii) Explain using algebra why it is zero for all such numbers.

(iii) What do we get if we calculate $a + b - c - d$ instead of $a - b - c + d$?

(iv) What about $a - b + c - d$?

Time and speed

We saw how we can write the relations between the positions and displacement of a point moving along a straight line as general algebraic equations using positive and negative numbers, even if the positions and displacement are in different directions.

For the point to be displaced, it must move; and to describe motion in terms of numbers, we must take into account not only displacement, but time also.

If we are told that an object travelled 50 metres in 5 seconds, we get an idea of its speed. If the speed is also known to be steady with no increase or decrease during the journey, we can say that the speed is 10 metres per second. We can also calculate that if it continues like this, it will cover 100 metres in 10 seconds (We can also remember that the fastest time so far in 100 metre sprint is 9.58 seconds).

For a point moving along a line with steady speed in either direction, if we know its speed and the time of travel, then we can calculate the distance travelled. For example if the speed is 10 metres per second (written 10 m/sec), then in 4 seconds it travels a distance of $10 \times 4 = 40$ metres. But to fix its final position, we must know the direction of travel.

As before, let us imagine a point travelling along a straight line, moving either left or right with respect to a fixed point O. As usual, we mark positions on the line using distances from the point O, positive numbers for positions to the right of O and negative numbers for positions to the left of O. Let's suppose that the point starts from O. Also, fix the unit of distances as metre, unit of time as second and the unit of speed as metre per second.

If the point starts from O and travels to the right at the speed of 10 m/s for 3 seconds, it would reach the position $3 \times 10 = 30$.

If the direction of travel is to the left ? The position is $-(3 \times 10) = -30$

Speed math

The fastest time in 100 metre sprint is currently 9.58 seconds, set by Usain Bolt of Jamaica in 2009 at Berlin.

He didn't run at a steady speed throughout this time. The time he took to cover every 20 metres has been recorded:

Distance Matres	Time Seconds
0 – 20	2.89
20 – 40	1.75
40 – 60	1.67
60 – 80	1.61
80 – 100	1.66

So to calculate the position, if the travel is to the right, we multiply the speed by time; if it is to the left we take the negative of this product.

To state this in algebra, we denote the time by t, speed by v, and position by s. Then

$$s = tv \text{ if moving to the right}$$

$$s = -(tv) \text{ if moving to the left}$$

To combine them into a single equation, we can try denoting speeds to the right by positive numbers and that to the left by negative numbers.

For example if the travel is to the right at 10 m/s, we take $v = 10$ and if it is to the left, $v = -10$.

But then, there's a problem: for a point starting from O and moving to the left at 10 m/s for 3 seconds, if we want to calculate the position as a product, we will have to define what we mean by $3 \times (-10)$.

We can think about this like this: for positive numbers, multiplication is computing so many times something. For example,

$$3 \times 10 = 3 \text{ times } 10 = 10 + 10 + 10 = 30$$

How about defining $3 \times (-10)$ also like this?

$$3 \times (-10) = (-10) + (-10) + (-10) = -30$$

This definition of multiplying a negative number by a natural number we can extend to all numbers.

So denoting the time by t, the speed by v and the position by s, we can say the relation between them by the general equation, $s = tv$

Now let's change the context a bit. Suppose a point is moving along the line at a steady speed of 10 m/s and we start observing it only from the time it reaches O.

If the journey is towards from left to right, at 2 seconds after we start watching, its position would be at 20.

What about the position at 2 seconds before we start watching?

It needs 2 more seconds to reach O. So it's at a distance of 20 metres to the left of O, that is at the position -20.

And if the journey is from right to left?

Let's calculate the positions for different directions of travel at times before and after we start observing, and make a table. To write down the positions using an actual picture or mental image, we first write speed and positions using the qualifications right and left:

Time	Speed	Position
2 seconds after	10 right	20 right
2 seconds after	10 left	20 left
2 seconds before	10 right	20 left
2 seconds before	10 left	20 right

Now let's write position and speed as only numbers, positive or negative:

Time	Speed	Position
2 seconds after	10	20
2 seconds after	–10	–20
2 seconds before	10	–20
2 seconds before	–10	20

To remove the qualifications before and after for time, we can write time after observation as positive and times before observation as negative:

Time	Speed	Position
2	10	20
2	–10	–20
–2	10	–20
–2	–10	20

In the first row of this table, the last number is got by multiplying the second number by the first.

In the second row also, according our new definition of multiplication, we have $2 \times (-10) = -20$. What about the third line ?

What is the meaning of $(-2) \times 10$?

We think like this. For positive numbers, changing the order of numbers in a multiplication doesn't change the product. For example

$$5 \times 3 = 3 \times 5 \qquad \frac{1}{2} \times \frac{1}{4} = \frac{1}{4} \times \frac{1}{2} \qquad \sqrt{2} \times \sqrt{3} = \sqrt{3} \times \sqrt{2}$$

So here also, we define $(-2) \times 10 = 10 \times (-2)$

And we've already seen that

$$10 \times (-2) = -20$$

Thus if we define

$$(-2) \times 10 = 10 \times (-2) = -20$$

Then the same relation, as in the first two rows, holds for the numbers in the third row of the table also.

What about the last row?

How do define the meaning of $(-2) \times (-10)$?

We have to define this as 20, if we want the relation between the numbers in each row.

We can give a purely mathematical justification for such a definition like this.

To multiply the sum of two positive numbers by a positive number, we need only multiply each and add. For example,

$$(5 + 2) \times 10 = (5 \times 10) + (2 \times 10)$$

In algebraic language,

$$(x + y)z = xz + yz \text{ for any positive numbers } x, y, z$$

Let's see what we'll have to do to make this true for negative numbers also. For example, let's take $x = 5$, $y = -2$, $z = -10$ and calculate the two sides of the above equation $(x + y)z = xz + yz$ separately.

$$(x + y)z = (5 + (-2)) \times (-10) = 3 \times (-10) = -30$$

$$xz + yz = (5 \times (-10)) + ((-2) \times (-10)) = -50 + ((-2) \times (-10))$$

If this products are to be equal, what should be $(-2) \times (-10)$?

The first product is -30 and the second product is some number added to -50

What is the number to be added to –50 to make it –30 ?

So, if we want the equation $(x + y)z = xz + yz$ to be true for $x = 5$, $y = –2$, $z = –10$ we'll have to define $(–2) \times (–10)$ as 20.

If we define like this, we get the last number in each row of our table as the product of the first two:

Time t	**Speed** v	**Position** tv
2	10	$20 = 2 \times 10$
2	–10	$–20 = 2 \times (–10)$
–2	10	$–20 = (–2) \times 10$
–2	–10	$20 = (–2) \times (–10)$

Thus we can give the relation between the time t, the speed v and the positions s can be given by the single equation

$$s = tv$$

The new definitions used to get this are these

For any two positive numbers x and y

$$(–x)y = x(–y) = –(xy)$$

$$(–x)\,(–y) = xy$$

(1) Take different numbers, positive and negative, as x, y, z and calculate $(x + y)\,z$ and $x\,z + yz$. Check if the equation

$(x + y)\,z = x\,z + yz$ holds in all cases.

(2) Prove that the equation
$(x+y)\,(u+v)= xu + xv + yu + yv$
holds if x, y, u, v are replaced by
$–x, –y, –u, –v$.

Oscillation

What is $(–1) \times (–1)$? By the definition of multiplying negative numbers, it is 1. What about $(–1) \times (–1) \times (–1)$?

The product of the first two –1's is 1. When this is multiplied by –1 again, we get –1. We can write these as

$$(–1)^2 = 1$$

$$(–1)^3 = –1$$

Compute some more powers of –1. Can you find a general pattern?

If the exponent is an odd number, then the power is –1; if exponent is even, the power is 1.

The algebraic form of this is

$$(–1)^{2n-1} = –1$$

$$(–1)^{2n} = 1$$

for any natural number n.

Now take different natural numbers as n and compute $1+ (–1)^n$.

What about $\frac{1}{2}(1+(-1)^n)$?

(3) In each of the equations below, find y when x is the given number:

(i) $y = x^2$, $x = -1$ (ii) $y = x^2 + 3x + 2$, $x = -1$

(iii) $y = x^2 + 3x + 2$, $x = -2$ (iv) $y = (x + 1)(x + 2)$, $x = -1$

(v) $y = (x + 1)(x + 2)$, $x = -2$

(4) In the equation $y = x^2 + 4x + 4$, take different numbers, positive and negative, as x and calculate y. Why is y positive or zero in all cases?

(5) Natural numbers, their negatives and zero can be together called integers.

(i) How many pairs of integers (x, y) can you find, satisfying the equation

$x^2 + y^2 = 25$

(ii) How many pairs of integers (x, y) can you find satisfying $x^2 - y^2 = 25$?

(6) $1 \times 2 \times 3 \times 4 \times 5 = 120$, what is $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$?

(7) What is $(1 \times 2 \times 3 \times 4 \times 5) + [(-1) \times (-2) \times (-3) \times (-4) \times (-5)]$?

Negative division

For positive numbers, division is defined in terms of multiplication. For example, the meaning of $6 \div 2$ is finding that number which on multiplication by 2 gives 6,

Since $2 \times 3 = 6$, write

$$6 \div 2 = 3$$

Since $3 \times 2 = 6$ also, we have

$$6 \div 3 = 2$$

According to this, what is $(-6) \div 2$?

Since $2 \times (-3) = -6$, we have

$$-6 \div 2 = (-3)$$

Also, since $(-3) \times 2 = -6$, we have

$$-6 \div (-3) = 2$$

Similarly, since $(-3) \times (-4) = 12$, we have

$$12 \div (-3) = -4$$

and

$$12 \div (-4) = -3$$

We can calculate other divisions using negative numbers in the same way.

In algebra, we usually write $x \div y$ as $\frac{x}{y}$. So, the general rules of negative division can be stated like this:

For any positive numbers x and y

$$\frac{-x}{y} = \frac{x}{-y} = -\left(\frac{x}{y}\right)$$

$$\frac{-x}{-y} = \frac{x}{y}$$

Unification of equations

An object thrown upwards reaches a certain height and starts falling down. This can be mathematically described.

If thrown straight up, the speed decreases at the rate of 9.8 m/s. Continuously decreasing thus, when the speed becomes zero, it starts falling down. Throughout the fall, the speed increases at the rate of 9.8 m/s.

Suppose it is thrown up with a speed of 49 m/s. At 5 seconds, the speed becomes $49 - (5 \times 9.8) = 0$. So after 5 seconds, the journey is downwards with increasing speed. So, at 7 seconds, the speed is $0 + (2 \times 9.8) = 19.6$ m/s

So we need two equations to describe this journey algebraically:

$$v = 49 - 9.8t, \text{ if } t < 5$$

$$v = 9.8t - 49, \text{ if } t > 5$$

If we denote the speed upwards by positive numbers and the speed downwards by negative numbers, we can describe the motion using a single equation:

$$v = 49 - 9.8t$$

(1) Calculate y when x is taken as $2, -2, \frac{1}{2}, -\frac{1}{2}$ in the equation

$$y = \frac{1}{x}$$

(2) Calculate y when $x = -2$ and $x = -\frac{1}{2}$ in the equation

$$y = \frac{1}{x-1} + \frac{1}{x+1}$$

(3) Calculate z when x and y are taken as the numbers below in the equation $z = \frac{x}{y} - \frac{y}{x}$

(i) $x = 10, y = -5$ (ii) $x = -10, y = 5$ (iii) $x = -10, y = -5$