3

Multiplication Methods

Product difference

You know how to add and multiply numbers, don't you?

Here's a simple question:

How much is 15 + 8 ?

And 18 + 5 ?

Next try multiplication:

How much is 15 × 8 ?

And 18 × 5 ?

Why do we get the same answer on adding?

Now let's try as follows:

$$15 + 8 = 10 + 5 + 8$$

$$18 + 5 = 10 + 8 + 5$$

Eight added to five and five added to eight both give thirteen, right?

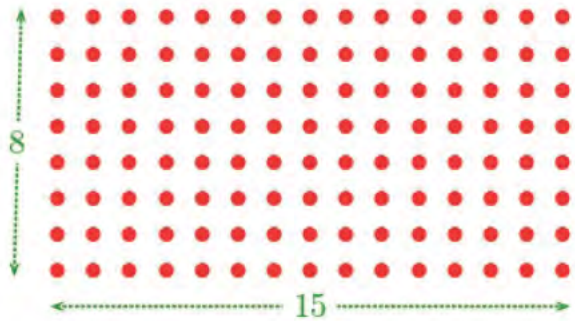
What about multiplication?

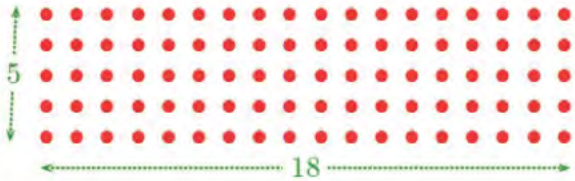
$$15 \times 8 = (10 + 5) \times 8 = (10 \times 8) + (5 \times 8)$$

$$18 \times 5 = (10 + 8) \times 5 = (10 \times 5) + (8 \times 5)$$

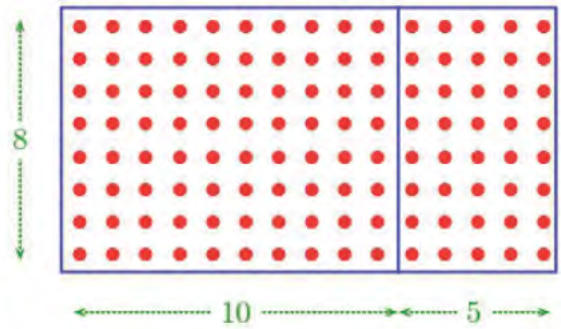
Here also five times eight and eight times five give the same product, forty. But the remaining products are different.

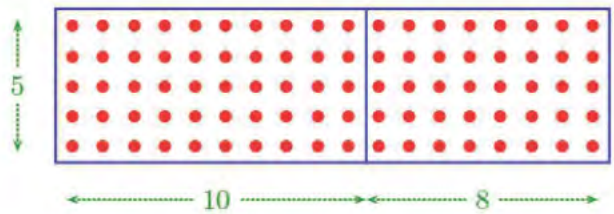
We can see this by drawing pictures also:





In each picture, let's mark off rows of ten dots:





Now in both pictures, the right blocks have 40 dots

But the left blocks have 80 and 50, right?

By looking at the products like this, we also see some other things:

- 15×8 is larger than 18×5
- $80 - 50 = 30$ more

So, can we say which of 16×9 or 19×6 is larger, without multiplying?

How do we think about this?

$$16 \times 9 = (10 + 6) \times 9 = (10 \times 9) + (6 \times 9)$$

$$19 \times 6 = (10 + 9) \times 6 = (10 \times 6) + (9 \times 6)$$

Which is larger? How much more?

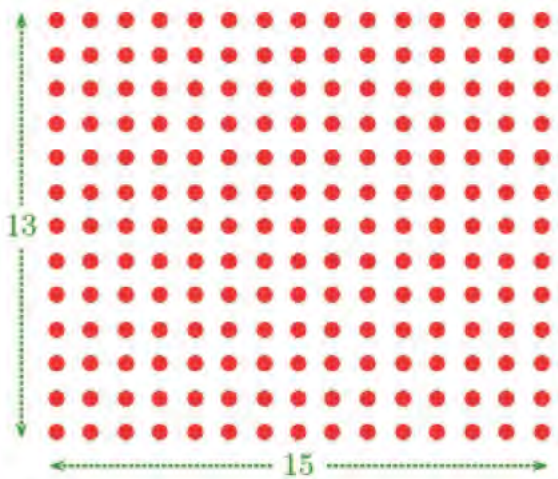
Now in each of the pairs of products given below, can you figure out in your head which is larger and how much more?

(1) 12×8 ; 18×2 (2) 17×6 ; 16×7

(3) 13×9 ; 19×3 (4) 25×6 ; 26×5

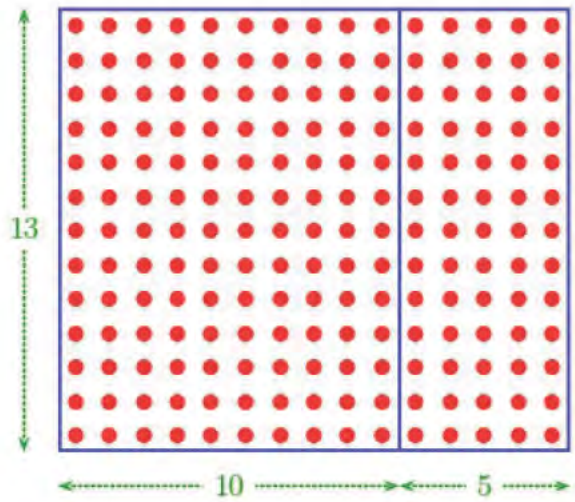
Rectangle multiplication

How many dots are there in this picture?



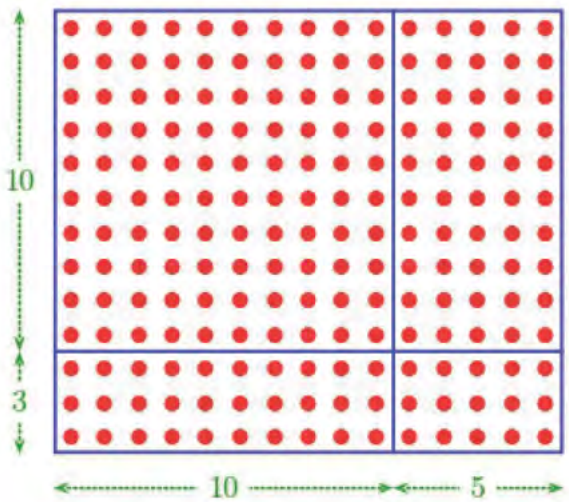
13 rows, each of 15 dots; 15 × 13 dots in all.

To compute this, first let's split each row into 10 and 5 dots:

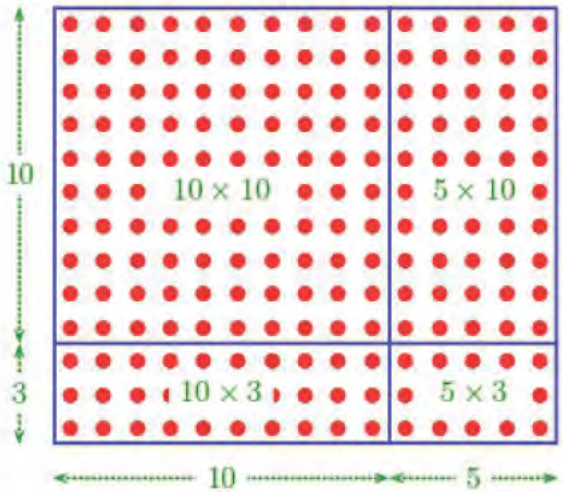


Now the left block has 10 × 13 dots and the right, 5 × 13.

To make the calculations still more simple, let's split each column into 10 and 3.



Now we can easily compute the number of dots in each of the four blocks separately:



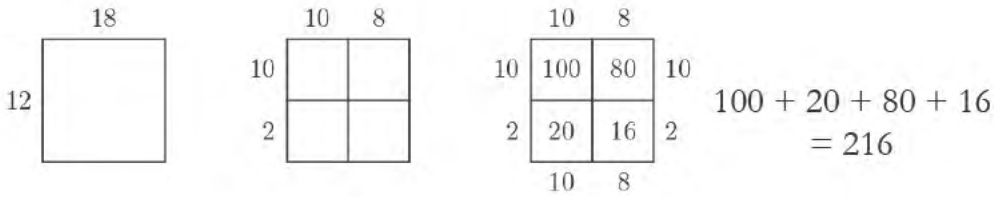
Thus we can calculate

$$15 \times 13 = (10 \times 10) + (5 \times 10) + (10 \times 3) + (5 \times 3)$$
$$= 100 + 50 + 30 + 15$$
$$= 195$$

Now can't you calculate such products without drawing dots?

For example, let's do 18×12

We can do the computation like this:



$100 + 20 + 80 + 16$
$= 216$

Short version

We can also write 18 × 12 like this :

	8	10	
2	16	20	36
10	80	100	180
			216

Some of these calculations can be done in the head and the whole multiplication can be written in this shortened form:

$$\begin{array}{r} 18 \times \\ 12 \\ \hline 36 \\ 180 \\ \hline 216 \end{array}$$

If the numbers to be multiplied are written only on the top and the left side, the sum of each row can be written on the right and then add them to get the final product:

	10	8	
10	100	80	180
2	20	16	36

	10	8	
10	100	80	180
2	20	16	36
			216

Now try computing the products below like this:

i. 13 × 15 *ii.* 17 × 16 *iii.* 18 × 19 *iv.* 14 × 18 *v.* 15 × 15

We can also compute different products like this. For example, let's try 24 × 17:

	20	4
10	200	40
7	140	28

	20	4	
10	200	40	240
7	140	28	168
			408

What about 35 × 29 ?

	30	5	
20	600	100	700
9	270	45	315
			1015

How about 345 × 26 ?

We will have to draw more cells:

	300	40	5
20			
6			

	300	40	5
20	6000	800	100
6	1800	240	30

	300	40	5	
20	6000	800	100	6900
6	1800	240	30	2070
				8970

Now try these problems on your own:

1. Compute the products below:
   i. 12×34 *ii.* 23×45
   iii. 75×75 *iv.* 123×45
   v. 320×78
2. Given that $36 \times 15 = 540$; compute the following products in your head:
   i. 36×16 *ii.* 37×15
   iii. 36×14 *iv.* 35×15
3. A number multiplied by 16 gave 1360.
   i. What will be the product, if the next number is multiplied by 16 ?
   ii. What will be the product if the number just before this is multiplied by 16 ?

Stacked products

We can write calculations for 345×26 like this:

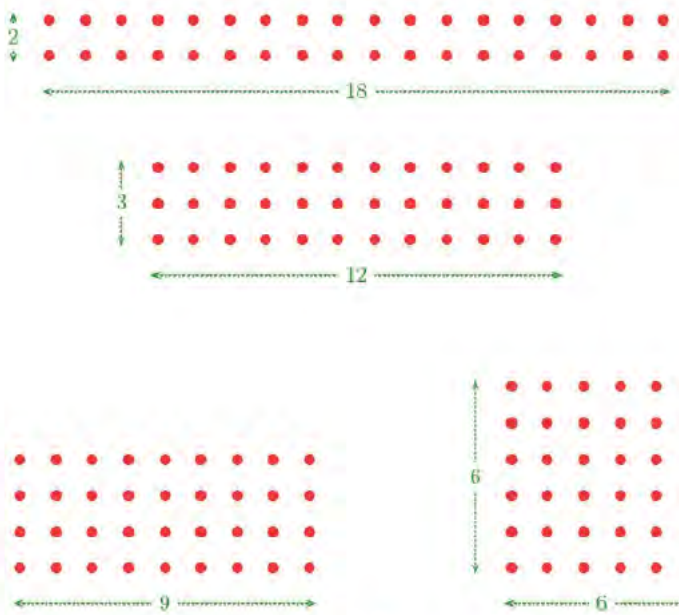
	5	40	300	
6	30	240	1800	2070
20	100	800	6000	6900
				8970

We can condense this by first writing the numbers one below another, then do the multiplication in each row above in head, and finally add these intermediate products by writing them one below the other:

$$\begin{array}{r} 345 \times \\ 26 \\ \hline 2070 \\ 6900 \\ \hline 8970 \end{array}$$

Square numbers

What are the different ways to arrange 36 dots into rectangles?



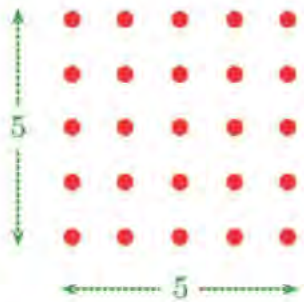
Now we can turn around the first three rectangles. One of them is a special kind of rectangle, isn't it? Can we make a square with any number of dots?

Can we make a square with less than 36 dots?

How many dots are placed in rows and columns to create a square with 36 dots?

Now can't you make a few more squares?

What can you say about the number of dots in a square?



$$36 = 6 \times 6$$

$$25 = 5 \times 5$$

$$16 = 4 \times 4$$

$$9 = 3 \times 3$$

$$4 = 2 \times 2$$

Such numbers are called *square numbers* or simply *squares.*

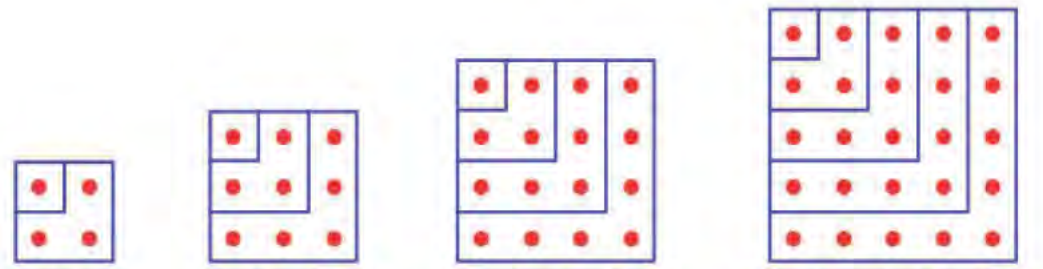
Thus squares are numbers got by multiplying a number with itself.

1 is also included among squares.

You know that $1 = 1 \times 1$.

Which is the next square after 36 ? Which one comes after it?

Now look at the pictures:



Which number is to be added to 25, to get the next square?

To get square 36, how many odd numbers, starting with 1, must be added?

At which position is 36 if squares are ordered 1, 4, 9, ... ?

Odd numbers $1 + 3 + 5 + 7 + \ldots$

$\downarrow \quad \downarrow \quad \downarrow$

Squares $1 \quad 4 \quad 9 \quad 16 \quad \ldots$

Now try these problems:

1. *i.* Calculate 11×11 and 111×111

ii. Can you guess what 1111×1111 would be?
Check whether your guess is correct.

iii. Write the pattern of such products.

2. Look at the following calculations:

$1 + 3 = 4$

$4 + 5 = 9$

$9 + 7 = 16$

Continue this to compute the squares up to 100

3. *i.* How many odd numbers like 1, 3, 5, are to be added to get 400 ?

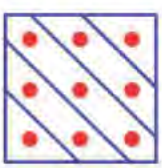
ii. Which is the last odd number added in this?

4. *i.* Which is the fiftieth odd number?

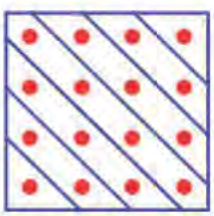
ii. What is the sum of the odd numbers from 1 to this number?

5. Look at these pictures:

$4 = 1 + 2 + 1$



$9 = 1 + 2 + 3 + 2 + 1$



$16 = 1 + 2 + 3 + 4 + 3 + 2 + 1$

i. How do we write 25 as a sum like this?

ii. How about 36 ?

iii. Can you write 100 as such a sum?

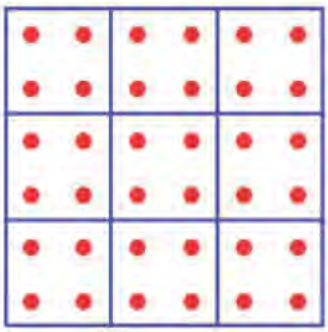
Square product

4 and 9 are squares. What about their product?

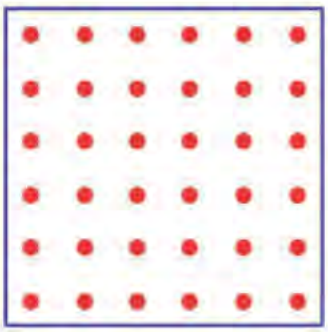
$$4 \times 9 = 36$$

36 also is a square

$$36 = 6 \times 6$$



4×9



6×6

Shall we consider two other squares?

Is 25×16 a square?

Before checking it by actual multiplication, let's think a bit about it:

$$25 = 5 \times 5$$
$$16 = 4 \times 4$$

So,

$$25 \times 16 = 5 \times 5 \times 4 \times 4$$

Changing the order of the numbers to multiply does not affect the product, right? So, we can calculate 25×16 in another way:

$$25 \times 16 = (5 \times 4) \times (5 \times 4) = 20 \times 20$$

Thus we see that 25×16 is also a square.

In the same way, can't we see that the product of any two squares is again a square?

1. Rewrite each of the products below as the product of a number by itself:

   i. 9×16 *ii.* 16×36 *iii.* 36×49 *iv.* 49×64 *v.* 81×25

2. Calculate each of the products below in your head:

   i. 25×4 *ii.* 25×16 *iii.* 25×36 *iv.* 25×64