4

Division Methods

Division and multiplication

How do we distribute 20 pieces of candy equally among 5 kids? We need only 5 candies to give one to each.

If one more is given to each?

Then $5 \times 2 = 10$ will be used; and there are some pieces left. Even giving one more each will mean $5 \times 3 = 15$ used; still some left.

If we give 4 to each, all the $5 \times 4 = 20$ pieces would be finished.

If we forget about the candy and the kids and think just about the numbers, the question is this:

If 20 is split into 5 equal parts, how much would be each part?

In the language of math, this is shortened further:

What is the number got on dividing 20 by 5 ?

We can shorten this by using math symbol.

What is $20 \div 5$?

How did we find the answer?

If the 5 parts are

- 1 each, then total is 5
- 2 each, then total is two times of 5. $5 \times 2 = 10$.
- 3 each, then total is three times of 5. $5 \times 3 = 15$.
- 4 each, then total is four times of 5. $5 \times 4 = 20$.

So the questions on parts and division above can be rewritten as questions on times and multiplication.

Parts and Division	Times and Multiplication
If 20 is split into 5 equal parts, how much would each part be?	How many times of 5 makes 20 ?
What number do we get on dividing 20 by 5 ?	By what number should 5 be multiplied to get 20 ?
$20 \div 5 = ?$	$5 \times ? = 20$

Now look at this problem:

40 litres of water was filled in 8 bottles of the same size. How many litres of water will be there in each bottle?

This can be written as 'parts' in ordinary language and as 'division' in math language, as follows.

Parts and Division	Times and Multiplication
If 40 is split into 8 equal parts, how much would be each part ?	
What number do we get on dividing 40 by 8 ?	
$40 \div 8 = ?$	

Can you rewrite these in terms of times and multiplication and find the answer?

Another problem:

6 persons equally shared 48 kilograms of rice. How many kilograms did each get?

Write this first as a question on parts and then division, then rewrite in terms of times and multiplication, and find the answer:

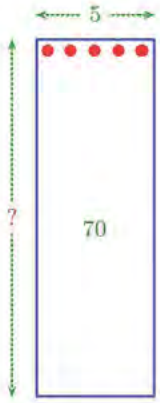
Parts and Division	Times and Multiplication

Rectangle division

Look at this problem:

A rectangle is to be made with 75 dots, 5 dots in each row. How many rows can be made?

After one row is made, 70 dots are left:

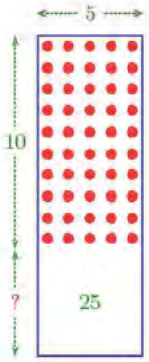


When one more row is added, 65 dots are left.
Instead of placing rows one by one and checking, why don't we place 10 rows at a time and see how much is left?
How many dots are used then?

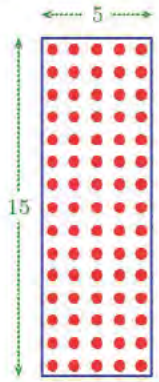
$$5 \times 10 = 50$$

How many more to be placed?

$$75 - 50 = 25$$



How many more rows can be made?



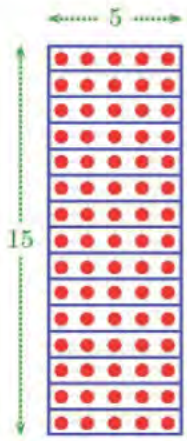
What actually did we compute here?

75 is the 15 times of 5.

That is, $5 \times 15 = 75$

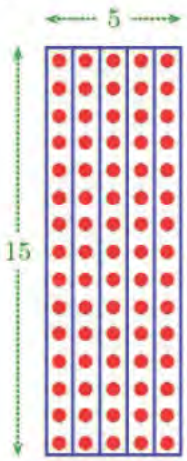
How do we say this in terms of parts?

If 75 is divided into parts of 5, there would be 15 parts:



This can also be stated as follows:

If 75 is divided into 5 equal parts, each part would be 15.



And in terms division?

$$75 \div 5 = 15$$

Money problem

An amount of 96 rupees which comprises of 9 ten rupee notes and 6 one rupee coins needs to be equally divided among 8 persons. How will you do that?

First give one ten rupee note to each. If the remaining note is changed to one rupee coins, then these together with the six already there, will make sixteen coins in all. Two each can be distributed, to the eight persons.

All these calculations can be shown in terms of just numbers, as below:

	1	2
	Ten	One
8	9	6
	8	
		16
		16

This can be further shortened like this:

$$8 \overline{)96} \quad 12$$
$$8$$
$$16$$
$$16$$
$$0$$

Let's write our method of computation without drawing dots and using only numbers:

5
75

	5
10	50
	25

	5
10	50
5	25

	5
15	75

Another problem:

A rectangle is to be made with 96 dots, 6 dots in each row. How many rows can be made?

We can think like this

- 10 rows would use up 60 dots; 36 left
- Since $6 \times 6 = 36$, we can have 6 more rows
- Altogether, $10 + 6 = 16$ rows

We can draw boxes and write the calculations like this:

6
96

	6
10	60
	36

	6
10	60
6	36

	6
16	96

We can just draw one box and do the calculations one by one within that:

6		
96		

6		
	96–	
10	60	$6 \times 10 = 60$
	36	

	6	
	96–	
10	60	$6 \times 10 = 60$
	36–	
6	36	$6 \times 6 = 36$
$96 \div 6 = 16$	0	$6 \times 16 = 96$

At the end, if you don't remember $36 = 6 \times 6$, you can do like this:

	6	
	96–	
10	60	$6 \times 10 = 60$
	36–	
5	30	$6 \times 5 = 30$
	6–	
1	6	$6 \times 1 = 6$
$96 \div 6 = 16$	0	$6 \times 16 = 96$

Here's another problem:

If 192 rupees is shared equally between 8 persons, how much will each get?

How do we think about this?

- If each takes 10 rupees, that makes 80 rupees;

  $192 - 80 = 112$ rupees. Here 112 rupees is remaining.
- If another 10 rupees is given to each of them, 32 rupees will remain.

  $112 - 80 = 32$ rupees.
  - * If each takes 1 more rupee, only 8 rupees is taken.
  - * 2 rupees each means only 16 rupees is taken.
  - * 3 rupees each means 24 rupees is taken.
  - * 4 rupees each means the entire 32 rupees are used up.

Each gets $10 + 10 + 4 = 24$ rupees.

Another money problem

We can think about the problem of dividing 1 hundred rupee note, 9 ten rupee notes and 2 one rupee coins equally among 8 persons, in a slightly different way.

So, in all, the hundred rupee note changed into 10 ten rupee notes and the 9 ten rupee notes would make 19 ten rupee notes. If 2 of these are given to each, 3 notes would be left. If these are changed to one rupee coins, then together with the 2 coins already in hand, there will be 32 coins. These remaining 32 coins can be shared by giving 4 to each.

All these computations can be condensed as before:

		2	4
	Hundred	Ten	One
8	1	9	2
		19	2
		16	
		3	2
			32
			32
			0

This can be compressed further into this:

```
   24
8 | 192
    16
   ---
    32
    32
   ---
     0
```

We can write these steps like this:

	8	
	192–	
10	80	$8 \times 10 = 80$
	112–	
10	80	$8 \times 10 = 80$
	32–	
4	32	$8 \times 4 = 32$
$192 \div 8 = 24$	0	$8 \times 24 = 192$

While doing it, if we recall $8 \times 2 = 16$ and $8 \times 20 = 160$, we can reduce the number of steps:

	8	
	192–	
20	160	$8 \times 20 = 160$
	32–	
4	32	$8 \times 4 = 32$
$192 \div 8 = 24$	0	$8 \times 24 = 192$

Let's try changing the numbers in the same problem.

If 3000 rupees is shared equally among 12 persons, how much will each get?

If as before, we think of each person taking 10 rupees, 20 rupees and so on, the total amount would be reduced by 120 rupees, 240 rupees and so on. But these numbers are much smaller than the total 3000.

Let's consider each of them taking 100 rupees, 200 rupees and so on.

The total would then be reduced by 1200 rupees, 2400 rupees and so on.

So, let's start like this:

	12	
	3000–	
200	2400	$12 \times 200 = 2400$
	600	

Now there's no enough money for each of them to take 100 rupees more.

If this amount is shared as 10 rupees, 20 rupees and so on, it would be reduced by 120 rupees, 240 rupees and so on, as seen before.

How much money should each of them take for 600 rupees to be used up?

So, let's write as follows:

	12	
	3000–	
200	2400	$12 \times 200 = 2400$
	600–	
50	600	$12 \times 50 = 600$
$3000 \div 12 = 250$	0	$12 \times 250 = 3000$

Here, if it's difficult to see $12 \times 50 = 600$, you can repeatedly subtract 120 till all of 600 is used up.

Or, subtract 240 two times and 120 once:

	12	
	3000–	
200	2400	$12 \times 200 = 2400$
	600–	
20	240	$12 \times 20 = 240$
	360–	
20	240	$12 \times 20 = 240$
	120–	
10	120	$12 \times 10 = 120$
$3000 \div 12 = 250$	0	$12 \times 250 = 3000$

Now let's try these:

1. Total price of 7 pens of equal rate is 98 rupees. What's the price of a pen?
2. If 168 rupees is shared equally among 8 persons, how much will each get?
3. The school store needs 1825 notebooks. How many packs of 25 notebooks, are to be bought?

Remainders

A small problem:

How can 20 pens be packed into packets of 5 ?

How did you find the answer?

You have seen how this can be written in two ways:

$$20 \div 5 = 4$$

$$5 \times 4 = 20$$

How can 21 pens be packed into packets of 5 ?

There will be 4 packets and one pen left over.

In the language of math, we say it like this

21 divided by 5 gives quotient 4 and remainder 1

In this, the quotient shows the number of parts, with 5 in each, into which we can split 21 and the remainder is the number left over.

This we can write in terms of multiplication also:

$$21 = (5 \times 4) + 1$$

Like this,

$$22 = (5 \times 4) + 2$$

$$23 = (5 \times 4) + 3$$

$$24 = (5 \times 4) + 4$$

Can we write other numbers also as so many times 5 and a remainder?

What about 20 ?

$$20 = (5 \times 4) + 0$$

This means, 20 divided by 5 gives quotient 4 and remainder 0

Can we write numbers less than 5 like this ?

For example, if we take 4,

$$4 = (5 \times 0) + 4$$

That is, 4 divided by 5 gives quotient 0, remainder 4 itself.

Thus any number can be written as so many times 5 and a remainder; and the remainder in all cases will be one of the numbers 0, 1, 2, 3 or 4.

What about numbers other than 5 ?

Let's take 4:

$0 = (4 \times 0) + 0$	$4 = (4 \times 1) + 0$	$8 = (4 \times 2) + 0$
$1 = (4 \times 0) + 1$	$5 = (4 \times 1) + 1$	$9 = (4 \times 2) + 1$
$2 = (4 \times 0) + 2$	$6 = (4 \times 1) + 2$	$10 = (4 \times 2) + 2$
$3 = (4 \times 0) + 3$	$7 = (4 \times 1) + 3$	$11 = (4 \times 2) + 3$

We can continue this as much as we wish. For example, taking 135, we have

	4	
	135–	
30	120	$30 \times 4 = 120$
	15–	
3	12	$3 \times 4 = 12$
Times 33	3	$(33 \times 4) + 3 = 135$
	Remainder	

So, any number can be written as so many times a number other than zero, and a remainder; and the remainder will be less than the second number:

Now let's try these problems:

1. What are the numbers which leave remainder 0 on division by 2 ? And those that leave remainder 1 ?

2. What are the possible remainders on dividing a number by 3 ? Write the pattern of numbers leaving the same remainder for each of these.

3. See the way numbers are arranged below:

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

   i. Is there any relation between the quotients on dividing the numbers in any one row by 5 ? And the remainders?

   ii. What if numbers in any one column are divided by 5 ?

   iii. What are the first and last numbers of the 10^{th} row?

   iv. What is the 4^{th} number in the 12^{th} row?

   v. Which column and row does the number 123 belong to?

4. Write each of the numbers as certain times 9 and a remainder

   (a) 11 (b) 111 (c) 1111

   i. Can you guess how 1111 is written like this? Check your guess by actual division.

   ii. Write out this number pattern.

5. Write each of the numbers as certain times 8 and a remainder

   (a) 9 (b) 98 (c) 987

   i. Can you guess how 9876 is written like this? Check your guess by actual division.

   ii. Write out this number pattern.