

MATHEMATICS

1. The number of roots of $z^3 + \bar{z} = 0$ is
- A. 2
B. 3
C. 4
D. 5
2. If z_1 and $z_2 (\neq 0)$ are the complex numbers such that $\left| \frac{z_1 - z_2}{z_1 + z_2} \right| = 1$, then
- A. $z_2 = kz_1, k \in R$
B. $z_2 = ikz_1, k \in R$
C. $z_1 = z_2$
D. None of these.
3. If z is a point in the argand plane satisfying $|z - a| - |z - b| = c < |a - b|$, where a, b, c are fixed real numbers, then the locus of z is a
- A. parabola
B. ellipse
C. hyperbola
D. line-segment.
4. If z_1 and z_2 are two non - zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg(z_1) - \arg(z_2)$ is equal to
- A. $-\pi$
B. $-\frac{\pi}{2}$
C. $\frac{\pi}{2}$
D. 0
5. If $1, \omega, \omega^2, \dots, \omega^{n-1}$ are the n^{th} roots of unity, then $(2 - \omega)(2 - \omega^2) \dots (2 - \omega^{n-1})$ equals
- A. $2^n - 1$
B. $2^n + 1$
C. $2^{n+1} - 1$
D. ${}^n C_1 + {}^n C_2 + \dots + {}^n C_{n-1} + 1$
6. Suppose three real numbers a, b, c are in geometric progression. Let $z = \frac{a + ib}{c - ib}$ then
- A. $z = i \frac{b}{c}$
B. $z = i \frac{a}{c}$
C. $z = 0$
D. None of these.

14. If the equations $ax^2 + bx + c = 0$ and $x^2 + 2x + 2 = 0$ has a common root, then $a : b : c =$
- A. 2:1:1
B. 1:2:2
C. 2:1:2
D. 1:1:1
15. The number of real roots of the equation $(\sqrt{2} + 1)^x + (\sqrt{2} - 1)^x = 1$ is
- A. 1
B. 2
C. 0
D. 3
16. The range of the function $f(x) = |x-1| + |x-2| + |x-3|$ is
- A. $[3, \infty)$
B. $[2, \infty]$
C. $[2, \infty)$
D. $[1, \infty)$
17. If $x^{11} + x + 1$ is divided by $x^2 + x + 1$ the remainder is
- A. 1
B. $x + 1$
C. $11x + 1$
D. 0
18. If the point $(3, 4)$ is rotated in the coordinate plane about origin through an angle $\frac{\pi}{2}$, the coordinates of the new point is
- A. $(-4, 3)$
B. $(4, 3)$
C. $(4, -3)$
D. $(-3, -4)$
19. The real values of x satisfying $x^2 + 6|x| - 16 < 0$ is given by
- A. $-8 < x < 2$
B. $-8 < x < 8$
C. $-2 < x < 8$
D. $-2 < x < 2$
20. If a^2, b^2, c^2 are in A.P., then $b + c, c + a, a + b$ are in
- A. H.P.
B. G.P.
C. A.P.
D. None of these.
21. A vector \vec{c} , directed along the internal bisector of the angle between the vectors $\vec{a} + 7\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = -2\hat{i} - \hat{j} + 2\hat{k}$ with $|\vec{c}| = 5\sqrt{6}$, is
- A. $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$
B. $\frac{5}{3}(5\hat{i} - 5\hat{j} + 2\hat{k})$
C. $\frac{5}{3}(\hat{i} - 7\hat{j} + 2\hat{k})$
D. $\frac{5}{3}(-5\hat{i} + 5\hat{j} + 2\hat{k})$

35. Which of the following function is not invertible
- A. $f : R \rightarrow R, f(x) = 4x + 5$ B. $f : R \rightarrow R^+ \cup \{0\}, f(x) = 2x^2$
 C. $f : R^+ \rightarrow R^+, f(x) = \frac{1}{x^2}$ D. None of these.
36. If $f : [0, \infty) \rightarrow [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then the function f is
- A. One-one and onto B. One-one but not onto
 C. Onto but not one-one D. Neither one-one nor onto.
37. $\lim_{x \rightarrow 0} \frac{x-2}{|x-2|}$ equals
- A. 2 B. 0
 C. -2 D. None of these.
38. $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$ is equal to
- A. $\frac{3}{2}$ B. $\frac{1}{2}$
 C. $\frac{2}{3}$ D. None of these.
39. Let $f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 3-ax^2 & \text{if } x < 1 \end{cases}$. The value of a for which $f(x)$ is continuous is
- A. 1 B. 2
 C. -1 D. -2
40. The number of points at which the function $f(x) = \frac{1}{\log|x|}$ is discontinuous is
- A. 1 B. 2
 C. 3 D. 4
41. For $x \in R, \lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2} \right)^x$ is equal to
- A. e B. e^{-1}
 C. e^{-5} D. e^5

49. $f(x) = xe^{(1-3x)}$, then $f(x)$ is
- A. increasing on $\left[-\frac{1}{2}, 1\right]$ B. decreasing on R
C. increasing on R D. decreasing $\left[-\frac{1}{2}, 1\right]$
50. If $f(x) = \int e^x(x-1)(x-4) dx$, then f decreases in the interval
- A. $(-\infty, -4)$ B. $(-4, -1)$
C. $(1, 4)$ D. $(4, +\infty)$
51. The function $f(x) = x^2 e^{-2x}$, $x > 0$. Then the maximum value of $f(x)$ is
- A. $\frac{1}{e}$ B. $\frac{1}{2e}$
C. $\frac{1}{e^2}$ D. None of these.
52. If the line $ax + by + c = 0$ is a normal to the curve $xy = 2$, then
- A. $a < 0, b > 0$ B. $a > 0, b > 0$
C. $a < 0, b < 0$ D. None of these.
53. If $x^3 y' = 72$, then $\frac{dy}{dx}$ at $(2, 3)$ is given by
- A. $-\frac{9 + \log 729}{4 + \log 64}$ B. $\frac{9 + \log 729}{4 + \log 64}$
C. $-\frac{3 + \log 9}{2 + \log 8}$ D. $\frac{3 + \log 9}{2 + \log 8}$
54. If $f(x) = \log_e(\log_e x)$, then $f'(e)$ is equal to
- A. e^{-1} B. e
C. 1 D. 0

61. If $I = \int_{-2}^2 |1-x^4| dx$, then I equals
- A. 6
B. 8
C. 12
D. 21
62. The degree of the differential equation satisfying $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ is
- A. 1
B. 2
C. 3
D. None of these.
63. The differential equation of all ellipses centred at the origin is
- A. $y^2 + xy_1^2 - yy_1 = 0$
B. $xyy_2 + xy_1^2 - yy_1 = 0$
C. $yy_2 + xy_1^2 - xy_1 = 0$
D. None of these.
64. A particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, $y(0) = 0$ is
- A. $e^{3x} + 3e^{-4y} = 4$
B. $4e^{3x} - 3e^{-4y} = 3$
C. $3e^{3x} + 4e^{-4y} = 7$
D. $4e^{3x} + 3e^{-4y} = 7$
65. The orthogonal trajectories of the families of curve $y = cx^k$ are given by
- A. $x^2 + cy^2 = \text{constant}$
B. $x^2 + ky^2 = \text{constant}$
C. $kx^2 + y^2 = \text{constant}$
D. $x^2 - ky^2 = \text{constant}$
66. The curve $\frac{dy}{dx} = \frac{(1+y^2)}{(1+x^2)}$, $x=0$, $y=1$ represents
- A. an ellipse
B. a parabola
C. a hyperbola
D. None of these.
67. Which one of the following is not a solution of the equation $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$
- A. $x^2 + 4y = 0$
B. $y = x + 1$
C. $y + x = 1$
D. $y^2 - 4x + 0$

75. The matrix $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 8 & 4 \end{bmatrix}$ satisfies the equation

- A. $x^3 - 7x^2 + 22x + 16 = 0$ B. $x^3 + 7x^2 - 4x - 16 = 0$
C. $x^3 + 7x^2 + 22x + 16 = 0$ D. $x^3 - 7x^2 - 10x + 16 = 0$

76. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, then the matrix A^4 is

- A. $\begin{bmatrix} 290 & 634 \\ 199 & 435 \end{bmatrix}$ B. $\begin{bmatrix} 634 & 290 \\ 199 & 435 \end{bmatrix}$
C. $\begin{bmatrix} 634 & 290 \\ 435 & 199 \end{bmatrix}$ D. $\begin{bmatrix} 199 & 435 \\ 290 & 634 \end{bmatrix}$

77. The eigen values of $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ are

- A. 6 and 4 B. 3 and 2
C. 1 and 2 D. None of these.

78. In an election, candidate A has 0.4 chance of winning. B has 0.3 chance. C has 0.2 chance and D has 0.1 chance. Just before the election C withdraws. Then the chances of winning of A, B and D are respectively

- A. 0.45, 0.35, 0.15 B. 0.35, 0.6, 0.3
C. 0.5, 0.375, 0.125 D. None of these.

79. If m is a natural number such that $m \leq 5$, then the probability that the quadratic equation $x^2 + mx + \frac{1}{2} + \frac{m}{2} = 0$ has real roots is

- A. $\frac{1}{5}$ B. $\frac{2}{3}$
C. $\frac{3}{5}$ D. $\frac{1}{5}$

84. A bag contains 10 identical white balls, 10 identical black balls and 10 identical red balls. Nine balls are drawn at random from the bag and are put into a box. The probability that the box contains 3 balls of each colour is

- A. $\frac{3!}{10!}$ B. $\frac{3^3}{10^3}$
C. $\frac{({}^{10}C_3)^3}{{}^{30}C_9}$ D. $\frac{1}{55}$

85. The number $\tan 15^\circ$ is

- A. an integer B. a rational number but not an integer
C. an irrational number D. Not a real number

86. The period of $\sin n\theta$ is

- A. $\frac{2\pi}{n}$ B. $\frac{\pi}{n}$
C. $\frac{\pi}{2n}$ D. None of these.

87. If $0 < x < 2\pi$, then

- A. $\sin x < x$ B. $\sin x > x$
C. $\tan x > x$ D. None of these.

88. $\cos 20^\circ \cos 40^\circ \cos 80^\circ =$

- A. $\frac{1}{2}$ B. $\frac{1}{4}$
C. $\frac{1}{6}$ D. $\frac{1}{8}$

89. In a triangle ABC, the maximum value of $\cos A + \cos B + \cos C =$

- A. $\frac{3\sqrt{3}}{2}$ B. $\frac{3}{2}$
C. 3 D. None of these.

97. If the points $(x_i, y_i), i = 1, 2, 3$ are vertices of a triangle with area 12, then the area of the triangle with vertices $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right), \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$ and $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$ is
- A. 4
B. 6
C. 12
D. 3
98. Tangents at two points A and B on a parabola $(x - 2y)^2 = 4(2x + y) + 2$ meet in a point P. If the line AB passes through its focus, P lies on the
- A. axis
B. directrix
C. tangent at the vertex
D. None of these.
99. If a variable circle touches two fixed circles externally. Then the locus of the centre of the variable circle is a
- A. parabola
B. hyperbola
C. ellipse
D. circle.
100. If the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 8x - xy + c = 0$ touch each other externally, then the value of c is
- A. 16
B. 9
C. 1
D. cannot be determined.
101. The number of common tangents to the circles $(x - 1)^2 + (y - 2)^2 = 9$ and $(x - 2)^2 + (y - 5)^2 = 1$ is
- A. 1
B. 4
C. 3
D. 2
102. Let P be a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci F_1 and F_2 . If A is the area of the triangle PF_1F_2 , then the maximum value of A is ...
- A. $2abc$
B. $2ab$
C. abc
D. None of these.
103. The equation $x - y = 4$ and $x^2 + 4xy + y^2 = 0$ represent the sides of
- A. an equilateral triangle
B. a right angled triangle
C. an isosceles triangle
D. None of these.

111. A point P moves so that the sum of the squares of its distances from n fixed points is given. The locus of P
- A. is a straight line
B. is a circle
C. depends on n
D. None of these.
112. The length of the tangent from (5, 1) to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is
- A. 81
B. 29
C. 7
D. 21
113. A line makes equal angle θ with x and y axes. If θ is acute, the minimum and maximum values of θ are (in 3-dimensional space)
- A. $\frac{\pi}{6}, \frac{\pi}{3}$
B. $\frac{\pi}{3}, \frac{\pi}{2}$
C. $\frac{\pi}{4}, \frac{\pi}{2}$
D. $\frac{\pi}{6}, \frac{\pi}{2}$
114. In 3 - dimensional space, the equation $y^2 + z^2 = 0$ represents
- A. origin
B. y - z plane
C. x - axis
D. None of these.
115. The foot of the perpendicular from (0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ is
- A. (-2, 3, 4)
B. (2, -1, 3)
C. (2, 3, -1)
D. (3, 2, -1)
116. If $L: \frac{x+1}{3} = \frac{y+1}{2} = \frac{z-3}{-1}$ is a line and $P: x-2y-z=0$ be a plane, then
- A. L is parallel to P
B. L is perpendicular to P
C. L lies in the plane P
D. None of these.
117. If $\omega \neq 1$ is a cube root of unity and $(\omega + x)^n = 1 + 12\omega + 69\omega^2 + \dots$ the values of n and x are respectively
- A. 36, 1
B. 12, 2
C. $24, \frac{1}{2}$
D. $18, \frac{1}{3}$

118. The lines $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-2}{7}$ and $\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{7}$ are
- A. parallel
B. intersecting
C. skew
D. perpendicular
119. The radius of the circle given by the equations $x^2 + y^2 + z^2 = 9$ and $x + y + z = 3$ is
- A. $\sqrt{3}$
B. $\sqrt{6}$
C. 3
D. 6
120. The remainder when 2^{2003} is divided by 17 is
- A. 1
B. 2
C. 8
D. None of these.
121. The co-efficient of x^7 in the expansion of $(1 - 2x + x^3)^6$ is
- A. 28
B. 360
C. -450
D. -280
122. If every element of a group G is its own inverse then G is
- A. finite
B. infinite
C. cyclic
D. abelian
123. Let G denote the set of all $n \times n$ non-singular matrices with rational numbers as entries ($n \geq 2$) then under multiplication
- A. G is a subgroup
B. G is a finite abelian group
C. G is infinite non-abelian group
D. G is infinite abelian group
124. In a group $(G, *)$ for some a of G , $a^2 = e$ where e is the identity element then
- A. $a = \sqrt{e}$
B. $a = a^{-1}$
C. $a = e$
D. None of these.
125. The set of fourth roots of unity forms a group of order
- A. 4 under addition
B. 4 under multiplication
C. 4 under division
D. None of these.