HIGHER SECONDARY FIRST TERMINAL EXAMINATION – AUGUST 2023

Part–III

Mathematics (Science)

Answers

1. (a) (2,2)

   (b) $3x - y = 0 \Rightarrow y = 3x$

   $\therefore$ R = {(1,3),(2,6),(3,9),(4,12)}

   $1 \in A$ but $(1,1) \notin R$ $\therefore$ R is not reflexive

   Here $(1,3) \in R$ but $(3,1) \notin R$ $\therefore$ R is not symmetric.

   Here $(1,3), (3,9) \in R$ but $(1,9) \notin R$ $\therefore$ R is not transitive

2. (i) (C) f is one-one but not onto

   (ii) Clearly, $f(2) = 2^2 = 4$ and $f(-2) = (-2)^2 = 4$

   ie, the elements 2 and -2 in the domain have the same image 4. $\therefore$ f is not one one.

   x^2 is always a non-negative real number.

   $\therefore$ There exists elements like -1,-2,... in the Co-domain, R of the function which is not the image of any value from the domain of the function.

   $\therefore$ f is not onto.

3. $\tan^{-1}\left(2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right)$

   $= \tan^{-1}\left(2\cos\left(2.\frac{\pi}{6}\right)\right) = \tan^{-1}\left(2\cos\left(\frac{\pi}{3}\right)\right)$

   $= \tan^{-1}\left(2.\frac{1}{2}\right)$

   $= \tan^{-1} 1$

   $= \frac{\pi}{4}$

4. (i) We have, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

   Given, $a_{ij} = i - j$

   $\therefore$ $a_{11} = 1 - 1 = 0$

   $a_{12} = 1 - 2 = -1$

   $a_{21} = 2 - 1 = 1$

   $a_{22} = 2 - 2 = 0$

   $\therefore A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

   (ii) $A' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -A$

   $\therefore$ A is skew symmetric

5. (a) 3

(b) A.(adj A) $= \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6-5 & -15+15 \\ 2-2 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6. Let , $(x_1, y_1) = (-2, -3)$

$(x_2, y_2) = (3, 2)$

$(x_3, y_3) = (-1, -8)$

Area of the triangle , $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$

$= \frac{1}{2}\{(-2)(2+8) + 3(3+1) + 1(-24+2)\}$

$= \frac{1}{2}\{(-2)(10) + 3(4) + 1(-22)\}$

$= \frac{1}{2}\{-20+12-22\} = \frac{1}{2}(-30) = -15$

= 15 sq. Units

7. Since the given function is continuous at x = 5,

$\lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x) = f(5)$

$\lim_{x\to5^-} f(x) = \lim_{x\to5^-} (3x-8) = 3x5 - 8 = 7$

$\lim_{x\to5^+} f(x) = \lim_{x\to5^+} (2k) = 2k$

f(5) = 3x5 – 8 = 7

Equating we get, 2k = 7

ie, k = 7/2

8. (a) (iii) $a^x \log a$

(b) $\frac{d}{dx}(\sin(\tan^{-1}(e^{-x}))) = \cos(\tan^{-1}(e^{-x}))\frac{d}{dx}(\tan^{-1}(e^{-x}))$

$= \cos(\tan^{-1}(e^{-x}))\frac{1}{1+e^{-2x}}\frac{d}{dx}(e^{-x})$

$= \cos(\tan^{-1}(e^{-x}))\frac{1}{1+e^{-2x}}(-e^{-x})$

$= \frac{-e^{-x}\cos(\tan^{-1}(e^{-x}))}{1+e^{-2x}}$

9. Put $x = \tan\theta \therefore \theta = \tan^{-1} x$

$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right)$ $[1+\tan^2\theta = \sec^2\theta]$

$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$ $\quad [1-\cos\theta = 2\sin^2\frac{\theta}{2}]$

$= \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$ $\quad [\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}]$

$= \tan^{-1}\tan\frac{\theta}{2}$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

10. (i) (A) $x \in [-\pi/2, \pi/2]$

(ii) $\sin^{-1}\sin\left(\frac{13\pi}{4}\right) = \sin^{-1}\sin(585)$

$= \sin^{-1}\sin(360+225)$

$= \sin^{-1}\sin(225)$

$= \sin^{-1}\sin(180+45)$

$= \sin^{-1}(-\sin(45))$

$=\sin^{-1}(-\frac{1}{\sqrt{2}})$

$= -\sin^{-1}(\frac{1}{\sqrt{2}}) = -\frac{\pi}{4}$

11. (i) Given, $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 9+(-8) & -6+4 \\ 12+(-8) & -8+4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$A^2 = kA - 2I \Rightarrow$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ie, $\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

ie, $\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

Equating corresponding elements, we can see

4 = 4k

ie, k = 1

(ii) a = 6

12. Given , $A = \begin{bmatrix} 4 & 0 & 3 \\ 5 & 1 & 2 \\ 6 & 2 & 7 \end{bmatrix}$

$\therefore\ A^T = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \\ 3 & 2 & 7 \end{bmatrix}$

Symmetric Part, P = $\frac{1}{2}\left(A + A^T\right)$

$\therefore\ P = \frac{1}{2}\left(\begin{bmatrix} 4 & 0 & 3 \\ 5 & 1 & 2 \\ 6 & 2 & 7 \end{bmatrix} + \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \\ 3 & 2 & 7 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 8 & 5 & 9 \\ 5 & 2 & 4 \\ 9 & 4 & 14 \end{bmatrix}$

$= \begin{bmatrix} 4 & 5/2 & 9/2 \\ 5/2 & 1 & 2 \\ 9/2 & 2 & 7 \end{bmatrix}$

Skew symmetric Part, Q = $\frac{1}{2}\left(A - A^T\right)$

$\therefore\ Q = \frac{1}{2}\left(\begin{bmatrix} 4 & 0 & 3 \\ 5 & 1 & 2 \\ 6 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \\ 3 & 2 & 7 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & 0 \\ 3/2 & 0 & 0 \end{bmatrix}$

We have , A = P + Q

ie, $\begin{bmatrix} 4 & 0 & 3 \\ 5 & 1 & 2 \\ 6 & 2 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 5/2 & 9/2 \\ 5/2 & 1 & 2 \\ 9/2 & 2 & 7 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & 0 \\ 3/2 & 0 & 0 \end{bmatrix}$

13. LHS = $(AB)^{-1}$

AB $= \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 + (-3) & -4 + 9 \\ 1 + 4 & -2 + (-12) \end{bmatrix}$

$= \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$

$|AB| = \begin{vmatrix} -1 & 5 \\ 5 & -14 \end{vmatrix}$ = 14 – 25 = -11

Adj (AB) = $\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$

$\therefore\ (AB)^{-1} = \frac{1}{|AB|}$ adj (AB) = $\frac{1}{-11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

RHS = $B^{-1}A^{-1}$

$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix}$ = -8 – 3 = -11

Adj A = $\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$ adj A $= \frac{1}{-11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix}$ = 3 – 2 = 1

Adj B = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}$ adj B $= \frac{1}{1}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} . \frac{1}{-11}\begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} . \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$

$= \frac{1}{-11}\begin{bmatrix} -12+(-2) & -9+4 \\ -4+(-1) & -3+2 \end{bmatrix}$

$= \frac{1}{-11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$

$= \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$

ie, LHS = RHS

14. (a) 0

(b) Given, A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

LHS = |3A|

3A = $3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

$\therefore$ |3A| $= \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$

$= 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 6 \\ 0 & 12 \end{vmatrix} + 3\begin{vmatrix} 0 & 3 \\ 0 & 0 \end{vmatrix}$

= 3(36 – 0) – 0(0 – 0) + 3(0 – 0)

= 3(36) – 0 + 0

= 108

RHS = 27 |A|

|A| $= \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 2 \\ 0 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

= 1(4 – 0) – 0(0 – 0) + 1(0 – 0)

= 1(4) – 0 + 0

= 4

$\therefore$ 27 |A| = 27 x 4 = 108

ie, LHS = RHS

|3A| = 27 |A|

15. (a) Given, $x^2 + xy + y^2 = 100$

Differentiating with respect to x,

$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(100)$

$2x + x\frac{dy}{dx} + y \cdot 1 + 2y\frac{dy}{dx} = 0$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x + 2y) = -2x - y$$

$$\therefore \frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

(b) Given, $y = x\cos x$

$$\frac{dy}{dx} = x\frac{d}{dx}(\cos x) + \cos x\frac{d}{dx}(x)$$

$$= -x\sin x + \cos x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(-x\sin x + \cos x)$$

$$= -\frac{d}{dx}(x\sin x) + \frac{d}{dx}(\cos x)$$

$$= -(x\cos x + \sin x.1) - \sin x$$

$$= -x\cos x - \sin x - \sin x$$

$$= -x\cos x - 2\sin x$$

16. Given, $y = x^x + x^{\sin x}$

Let $y = u + v$, where $u = x^x$ and $v = x^{\sin x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \rightarrow \langle 1 \rangle$$

We have, $u = x^x$

Taking log on both sides,

$\log u = x\log x$ [$\log p^q = q\log p$]

Diff. w.r.to x,

$$\frac{d}{dx}(\log u) = \frac{d}{dx}(x\log x)$$

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$

$$= x \cdot \frac{1}{x} + \log x. 1$$

$$= 1 + \log x$$

$$\frac{du}{dx} = u(1 + \log x)$$

$$= x^x(1 + \log x)$$

We have, $v = x^{\sin x}$

Taking log on both sides,

$\log v = \log(x^{\sin x})$

$\log v = \sin x\log x$ [$\log p^q = q\log p$]

Diff. w.r. to x,

$\frac{d}{dx}(\log v) = \frac{d}{dx}(\sin x \log x)$

$\frac{1}{v}\frac{dv}{dx} = \sin x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\sin x)$

$= \sin x \cdot \frac{1}{x} + \log x \cos x$

$= \frac{\sin x}{x} + \log x \cos x$

$\therefore \quad \frac{dv}{dx} = v\left(\frac{\sin x}{x} + \log x \cos x\right)$

$= x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$

$\langle 1 \rangle \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$= x^x(1 + \log x) + x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$

17. (a) For every $a \in Z$, 2 divides $a - a = 0$.

ie, $(a,a) \in R$ $\therefore$ R is reflexive

$(a, b) \in R \Rightarrow$ 2 divides $a - b$

$\Rightarrow$ 2 divides $b - a$, since $b - a = -(a - b)$

$\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric

If $(a,b) \in R$ and $(b,c) \in R \Rightarrow$ 2 divides $a - b$ and 2 divides $b - c$

$\Rightarrow$ 2 divides $a - c$, since $(a-c) = (a-b) + (b-c)$

$\Rightarrow (a,c) \in R$

$\therefore$ R is transitive

ie, R is reflexive, symmetric and transitive. $\therefore$ R is an equivalence relation.

(b) Given, $f(x) = 4x + 3$

$f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3$

$\Rightarrow 4x_1 = 4x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one one.

Let $y = f(x)$

ie, $y = 4x + 3$

ie, $4x = y - 3$

$\therefore x = \frac{y-3}{4}$

For every real number y in the co domain of the function, there exists another real number $\frac{y-3}{4}$ in the domain of the function such that $f\left(\frac{y-3}{4}\right) = y$.

$\therefore$ f is an onto function. $\therefore$ f is bijective.

18. (a) $A^2 = A.A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1-6 & 3+12 \\ -2-8 & -6+16 \end{bmatrix} = \begin{bmatrix} -5 & 15 \\ -10 & 10 \end{bmatrix}$

$A^2 - 5A + 10I = \begin{bmatrix} -5 & 15 \\ -10 & 10 \end{bmatrix} - 5\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5 & 15 \\ -10 & 10 \end{bmatrix} - \begin{bmatrix} 5 & 15 \\ -10 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

ie, $A^2 - 5A + 10I = 0$

We have, $A^2 - 5A + 10I = 0$

Multiplying both sides by A^{-1}, we have,

$A^{-1}.(A^2 - 5A + 10I) = A^{-1}.\ 0$

ie, $A^{-1} A^2 - 5 A^{-1} A + 10 A^{-1} I = 0$

ie, $A - 5I + 10 A^{-1} = 0$

ie, $10A^{-1} = 5I - A$

$\therefore \quad A^{-1} = \frac{1}{10}(5I - A)$

$= \frac{1}{10}\left(5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\right)$

$= \frac{1}{10}\left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\right)$

$= \frac{1}{10}\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{5} & \frac{-3}{10} \\ \frac{1}{5} & \frac{1}{10} \end{bmatrix}$

(b) Given, $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \rightarrow \langle 1 \rangle$

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \rightarrow \langle 2 \rangle$

$\langle 1 \rangle + \langle 2 \rangle \Rightarrow 2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\therefore \quad X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$\langle 1 \rangle \Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - X$

$= \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

ie, $X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}, Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

$2X + Y = 2\begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 12 & 0 \\ 3 & 9 \end{bmatrix}$

19. Given, $3x - 2y + 3z = 8$

$2x + y - z = 1$

$4x - 3y + 2z = 4$

ie, $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

ie, AX = B, where A = $\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

B = $\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

We have , $X = A^{-1} B$

$A^{-1} = \frac{1}{|A|}$ (adj A)

$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = 3(2-3)+2(4+4)+3(-6-4)$

$= 3(-1) + 2(8) + 3(-10)$

$= -3 + 16 - 30$

$= -17$

To find adj A :–

Cofactor of 3, $A_{11} = \begin{vmatrix} 1 & -1 \\ -3 & 2 \end{vmatrix} = 2 - 3 = -1$

Cofactor of –2, $A_{12} = -\begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} = -(4+4) = -8$

Cofactor of 3, $A_{13} = \begin{vmatrix} 2 & 1 \\ 4 & -3 \end{vmatrix} = -6 - 4 = -10$

Cofactor of 2, $A_{21} = -\begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -(-4+9) = -5$

Cofactor of 1, $A_{22} = \begin{vmatrix} 3 & 3 \\ 4 & 2 \end{vmatrix} = 6 - 12 = -6$

Cofactor of –1, $A_{23} = -\begin{vmatrix} 3 & -2 \\ 4 & -3 \end{vmatrix} = -(-9+8) = 1$

Cofactor of 4, $A_{31} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix} = 2 - 3 = -1$

Cofactor of –3, $A_{32} = -\begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} = -(-3 - 6) = 9$

Cofactor of 2, $A_{33} = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 3 + 4 = 7$

adj A = $\begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}^T = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$\therefore \quad A^{-1} = \frac{1}{|A|}$ (adj A)

$= \frac{1}{-17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$X = A^{-1} B = \frac{1}{-17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

$= \frac{1}{-17} \begin{bmatrix} -8 - 5 - 4 \\ -64 - 6 + 36 \\ -80 + 1 + 28 \end{bmatrix} = \frac{1}{-17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

ie, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore$ x = 1, y = 2, z = 3

20. (a) We have, $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$

$$= \frac{\frac{d}{dt}[at^2]}{\frac{d}{dt}[2at]} = \frac{a\frac{d}{dt}[t^2]}{2a\frac{d}{dt}[t]}$$

$$= \frac{a.2t}{2a.1} = t$$

(b) Given, $y = \tan^{-1} x$

Diff. w.r. to x,

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Multiplying both sides by $(1+x^2)$, we get

$$(1+x^2)\frac{dy}{dx} = 1$$

Diff. again w.r.to x,

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{d}{dx}(1+x^2) = 0$$

$$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = 0$$

$(1+x^2)\,y_2 + 2xy_1 = 0$