10 R

Quarterly Examination - 2023
MATHEMATICS

Time : 3.00 Hrs.

Marks : 100

I.   **Answer all of the following questions.**   **14 x 1 = 14**

1. If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is
    a) 3 b) 2 c) 4 d) 8

2. If { (a, 8), (6, b) }represents an identity function, then the value of a and b respectively
    a) (8, 6) b) (8, 8) c) (6, 8) d) (6, 6)

3. The sum of the exponents of the prime factors in the prime factorization of 1729 is
    a) 1 b) 2 c) 3 d) 4

4. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120?
    a) 6 b) 7 c) 8 d) 9

5. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$ then the value of k is
    a) 3 b) 5 c) 6 d) 8

6. The square root of $\dfrac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to   a) $\dfrac{16}{5}\left|\dfrac{x^2z^4}{y^2}\right|$ b) $16\left|\dfrac{y^2}{x^2z^4}\right|$ c) $\dfrac{16}{5}\left|\dfrac{y}{xz^2}\right|$ d) $\dfrac{16}{5}\left|\dfrac{xz^2}{y}\right|$

7. A function $f : R \rightarrow R$ define by $f(x) = C, \forall x \in R$ called a...........
    a) The identity function b) Quadratic function c) Constant function d) Reciprocal function

8. If in triangles ABC and EDF, $\dfrac{AB}{DE} = \dfrac{BC}{FD}$ then they will be similar, when
    a) $\angle B = \angle E$ b) $\angle A = \angle D$ c) $\angle B = \angle D$ d) $\angle A = \angle E$

9. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the y axis. The path travelled by the man is
    a) x = 10 b) y = 10 c) x = 0 d) y = 0

10. If slope of the line PQ is $\dfrac{1}{\sqrt{3}}$ then slope of the perpendikcular bisector of PQ is

    a) $\sqrt{3}$ b) $-\sqrt{3}$ c) $\dfrac{1}{\sqrt{3}}$ d) 0

11. The point of intersection of 3x – y = 4 and x + y = 8 is
    a) (5, 3) b) (2, 4) c) (3, 5) d) (4, 4)

12. $\tan\theta\,\csc^2\theta - \tan\theta$ is equal to
    a) $\sec\theta$ b) $\cot^2\theta$ c) $\sin\theta$ d) $\cot\theta$

13. $a\cot\theta + b\csc\theta = p$ and $b\cot\theta + a\csc\theta = q$ then $p^2 - q^2$ is equal to
    a) $a^2 - b^2$ b) $b^2 - a^2$ c) $a^2 + b^2$ d) $b - a$

14. $x^2 - 24x + \ldots\ldots\ldots$is a perfect square, then the value of $\ldots\ldots\ldots$is
    a) 64 b) 16 c) 144 d) 81

II. **Answer any 10 questions. 28th question is a compulsory one.**   **10 x 2 = 20**

15. If B x A = { (–2, 3), (–2, 4), (0, 3), (0, 4), (3, 3), (3, 4) } find A and B.

16. Let X = {1, 2, 3, 4} and Y = {2, 4, 6, 8, 10} and R = { (1, 2), (2, 4), (3, 6), (4, 8) }. Show that R is a function and find its domain, co-domain and range?

17. Determine whether the graph given belkow functions. Given reason for your answers concerning each graph.

    i)

    ii)
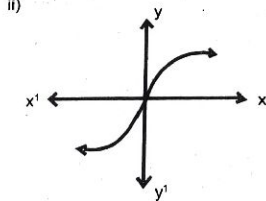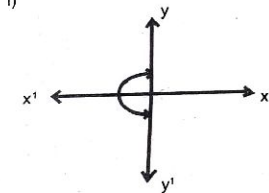
18. If $13824 = 2^a \times 3^b$ then find a and b.

19. If $3 + k$, $18 - k$, $5k + 1$ are in A.P then find K.

20. Find the LCM of the following : $8x^4 y^2$, $48x^2y^4$

21. Reduce the following rational expression to its lowerst form : $\dfrac{9x^2 + 81x}{x^3 + 8x^2 - 9x}$

22. Determine the quadratic equations, whose sum and product of roots are -9, 20.

23. In $\triangle ABC$, D and E are points on the sides AB and AC respectively such that DE $\parallel$ BC ; If $AD = 8x - 7$, $DB = 5x - 3$, $AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

24. Prove that $(cosec\theta - sin\theta)(sec\theta - cos\theta)(tan\theta + cot\theta) = 1$

25. Find the equation of a line whose inclination is 30^0 and making an intercept -3 on the y axis.

26. Find the area of the triangle formed by the points (i) $(1, -1)$, $(-4, 6)$ and $(-3, -5)$

27. The line r passes through the points $(-2, 2)$ and $(5, 8)$ and the lines s passes through the points $(-8, 7)$ and $(-2, 0)$. Is the line r perpendicular to s.

28. Find k if fof $(K) = 5$ where $f(K) = 2K-1$

III. Answer any 10 questions. 42nd question is the compulsory one. $10 \times 5 = 50$

29. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$ show that $A \times A = (B \times B) \cap (C \times C)$

30. Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f : A \to B$ be a function given by $f(x) = 3x - 1$.
    Represent this function
    i) by arrow diagram   ii) in a table form   iii) as a set of ordered pairs   iv) in a graphical form

31. Consider the functions f(x), g(x), h(x) as given below, show that (fog) oh = fo(goh) .
    $f(x) = x - 1$, $g(x) = 3x + 1$, $h(x) = x^3$

32. Find the HCF of 396, 504, 636

33. Find the sum of all natural numbers between 602 and 902 which are not divisible by 4.

34. Rekha has 15 square colour papers of size 10cm, 11cm, 12cm...... 24 cm. How much area can be decorated with these colour papers.

35. Find the square root of the following polynomial by division method $x^4 - 12x^3 + 42x^2 - 36x + 9$.

36. Find $\dfrac{16x^2 - 2x - 3}{3x^2 - 2x - 1} \div \dfrac{8x^2 + 11x + 3}{3x^2 - 11x - 4}$

37. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

38. State and prove Thales theorem.

39. Find the area of the quadrilateral whose vertices are at $(-9, 0)$, $(-8, 6)$, $(-1, -2)$, $(-6, -3)$

40. Find the equation of the perpendicular bisector of the line joining the points $A(-4, 2)$ and $B(6, -4)$

41. Prove that $tan^2A - tan^2B = \dfrac{sin^2 A - sin^2 B}{cos^2 A \, cos^2 B}$

42. Find the sum of n terms of the series $5 + 55 + 555 + \ldots\ldots$

IV. Answer all the questions. $1 \times 8 = 8$

43. Construct a triangle similar to a given triangle PQR with its sides equal to $\dfrac{7}{4}$ of the corresponding sides of the triangle

PQR (scale factor $\dfrac{7}{4} > 1$)

(OR)

Construct a $\triangle PQR$ which the base $PQ = 4.5$ cm, $\angle R = 35^0$ and the median RG from R to PQ is 6cm.

44. Draw the graph of $xy = 24$, x, $y > 0$. Using the graph find i) x when $y = 6$ ii) y when $x = 3$.

(OR)

A bus is travelling at a uniform speed of 50 km/hr. Draw the distance - time graph and hence find
i) the constant of variation
ii) how far will it travel in 90 minutes?
iii) the time required to cover a distance of 300 km from the graph.

Quarterly Exam 2023 ①

I. 1. b. 2   8. e. $\underline{B} = \underline{D}$
2. a. (8, 6)   9. a. $x = 10$
3. c. 3   10. b. $-\sqrt{3}$
4. c. 8   11. e. (3, 5)
5. b. 5   12. d $\cot \theta$.
6. d $16/5 \left| \dfrac{xz^2}{y} \right|$ 13. b. $b^2 - a^2$
7. e Constant fune.   14. c. 144.

II. 15. $A = \{3, 4\}$ $B = \{-2, 0, 3\}$.

16.
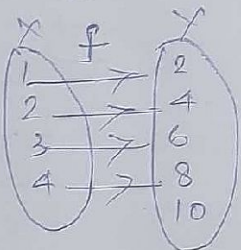
Func.
$D = \{1, 2, 3, 4\}$
$C \cdot D = \{2, 4, 6, 8, 10\}$
$R = \{2, 4, 6, 8\}$

17. (I) Not a func.
Vertical line touches the graph at 2 points.
(ii) Func.
Vertical line touches the graph at only one point.

18.
```
2 | 13824
  2 | 6912
    2 | 3456
      2 | 1728
        2 | 864
          | 432
```
```
2 | 432          9   3
  2 | 216      2 × 3
    2 | 108      a = 9
      2 | 54       b = 3
        3 | 27
          3 | 9
            3 | 3
              | 1
```

19.

$$3+k, \quad 18-k, \quad 5k+1$$

A.P

$$\therefore t_2 - t_1 = t_3 - t_2$$

$$18-k-(3+k) = (5k+1)-(18-k)$$

$$18-k-3-k = 5k+1-18+k$$

$$-2k+15 = 6k-17$$

$$-2k-6k = -17-15$$

$$-8k = -32$$

$$k = \frac{32}{8} = \underline{4}.$$

20.

$$8x^4y^2$$

$$48x^2y^4$$

$$48x^4y^4$$

$$\begin{array}{r|l} 2 & 8,48 \\ \hline 2 & 4,24 \\ \hline 2 & 2,12 \\ \hline 2 & 1,6 \\ \hline 3 & 1,3 \\ \hline & 1,1 \end{array}$$

21.

$$\frac{9x^2+81x}{x^3+8x^2-9x}$$

$$\frac{9x(x+9)}{x(x^2+8x-9)} = \frac{9(x+9)}{(x-1)(x+9)}$$
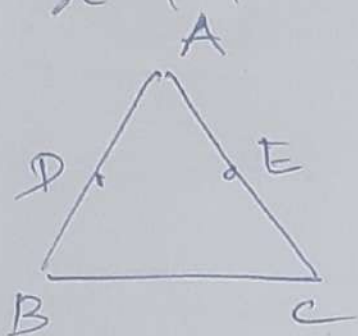
$$= \frac{9}{(x-1)}$$

22.

$-9, 20 : x^2 - (\text{Sum of the roots})x + \text{product of the roots} = 0$

$x^2 - (-9)x + 20 = 0.$

$x^2 + 9x + 20 = 0.$

23.

$\dfrac{AD}{DB} = \dfrac{AE}{EC}.$

$\dfrac{8x-7}{5x-3} = \dfrac{4x-3}{(3x-1)}$

$(8x-7)(3x-1) = (5x-3)(4x-3)$
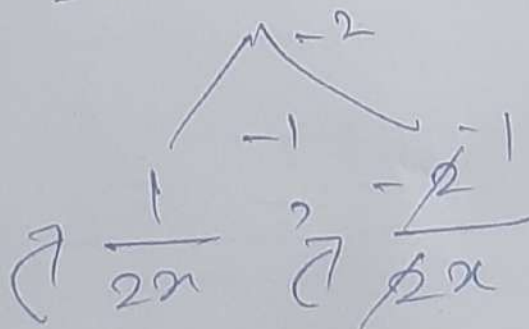
$24x^2 - 8x - 21x + 7 = 20x^2 - 15x - 12x + 9$

$24x^2 - 29x + 7 = 20x^2 - 27x + 9$

$24x^2 - 29x + 7 - 20x^2 + 27x - 9 = 0$

$4x^2 - 2x - 2 = 0$

$\div 2, \qquad 2x^2 - x - 1 = 0.$

$(2x+1)(x-1) = 0$

$2x + 1 = 0 \qquad \text{or} \quad x - 1 = 0$

$2x = -1 \qquad\qquad x = 1.$

$2x = -1$

$$x = -\frac{1}{2} \quad \text{impossible}$$

$$\therefore \quad x = \frac{1}{2}$$

24. LHS ⓧ eg 6.9

$$(cosec\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$\frac{1 - \sin^2\theta}{\sin\theta} \times \frac{1 - \cos^2\theta}{\cos\theta} \times \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\sin^2\theta + \cos^2\theta$$

$$= 1$$

$$= RHs$$

$$\therefore LHS = RHS.$$

25. $\theta = 30°$ $m = \tan\theta$

$$\text{slope } m = \tan 30° = \frac{1}{\sqrt{3}}.$$

y intercept $C = -3$

$$\therefore \quad y = mx + c$$

$$y = \frac{1}{\sqrt{3}}x - 3$$

$$\sqrt{3}y - x + 3\sqrt{3} = 0$$

$$x - 3\sqrt{3}y - 3\sqrt{3} = 0$$

26.

Area of △

$$\frac{1}{2}\begin{Bmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{Bmatrix} \text{ sq.units}$$

$$\frac{1}{2}\begin{Bmatrix} +1 & -4 & -3 & 1 \\ -1 & 6 & -5 & -1 \end{Bmatrix}$$

$$\frac{1}{2}\{(6+20+3)-(4-18-5)\}$$

$$\frac{1}{2}\{29-(-19)\}$$

$$\frac{1}{2}\{29+19\} = \frac{1}{2}\{48\} = \underline{24 \text{ sq.units}}$$

27.

slope $= \dfrac{y_2-y_1}{x_2-x_1}$ $(-2,2),(5,8)$

$$m_1 = \frac{8-2}{5+2} = \underline{\frac{6}{7}}$$

$(-8,7),(-2,0)$

$$m_2 = \frac{0-7}{-2+8} = \underline{\frac{-7}{6}}$$

$$m_1 \times m_2 = \frac{6}{7} \times \frac{-7}{6} = -1$$

$$m_1 \times m_2 = -1$$

$\Rightarrow$ The line $r \perp$ lar to s.

$$f \circ f(k) = 5 \quad ; \quad f(k) = 2k - 1$$

$$f \circ f(k) = f(f(k))$$
$$= 2(2k - 1) - 1$$
$$= 4k - 2 - 1$$
$$= 4k - 3.$$

$$\therefore \quad 4k - 3 = 5$$
$$4k = 5 + 3 = 8$$
$$4k = 8$$
$$k = 8/4 = 2.$$

III
29. LHS.
$$A \times A = \{5, 6\} \times \{5, 6\}$$
$$\{(5, 5), (5, 6), (6, 5), (6, 6)\} \quad —① $$

RHS $(B \times B) \cap (C \times C)$

$$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$$
$$\{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5),$$
$$(5, 6), (6, 4), (6, 5), (6, 6)\}$$

$$(C \times C) = \{5, 6, 7\} \times \{5, 6, 7\}$$
$$\{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6),$$
$$(6, 7), (7, 5), (7, 6), (7, 7)\}$$

$$(B \times B) \times (C \times C)$$
$$(5, 5), (5, 6), (6, 5), (6, 6) —②$$

From ① & ②.
proved.

30.

$A = \{1, 2, 3, 4\}$
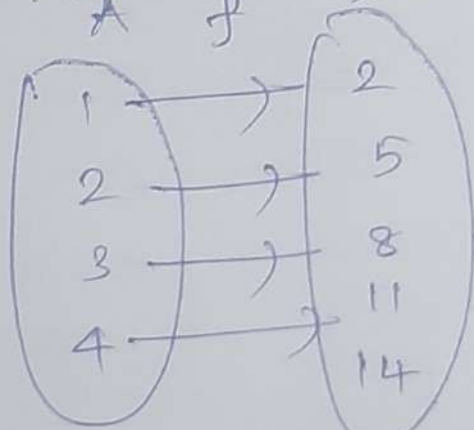
$B = \{2, 5, 8, 11, 14\}$

$f(x) = 3x - 1$

$f(1) = 3(1) - 1 = 2$

$f(2) = 3(2) - 1 = 5$

$f(3) = 3(3) - 1 = 8$

$f(4) = 3(4) - 1 = 11.$

(1) Arrow diagram.

(ii) a table form

x	1	2	3	4
$f(x)$	2	5	8	11

(iii) Orderd pair

$\{(1, 2), (2, 5), (3, 8), (4, 11)\}$

(iv) graph.

31) $f(x) = x - 1$ $\quad g(x) = 3x+1$

$h(x) = x^3$.

LHs $(fog) oh$.

$fog = x - 1 \circ (3x+1)$

$3x + 1 - 1 = 3x$.

$(fog) oh = 3x \circ (x^3)$

$3x^3$ $\qquad$ —①

RHs. $3x^3$.

$fo (goh)$

$goh = 3x+1 \circ (x^3)$

$= 3x^3 + 1$

$fo (goh) = x - 1 \circ (3x^3 + 1)$

$= 3x^3 + 1 - 1$

$= 3x^3$ $\qquad$ ②

From ① and ②

proved.

32). $396, 504, 636$ (Find HCF)

$a = bq + r$.

$396, 504$

$$396 = \underline{108} \times 3 + \underline{72}$$

$$108 = \underline{72} \times 1 + \underline{36}$$

$$\underline{72} = 36 \times 2 + 0$$

$$\therefore \quad HCF = 36.$$

$$36, \ 636$$

$$636 = \underline{36} \times 17 + \underline{24}$$

$$36 = \underline{24}(1) + \underline{12}$$

$$24 = \underline{12}(2) + 0$$

No's : HCF = 12.

between 602 & 902.

33. $603 + 604 + \cdots \cdots + 901$

$$a = 603 \ ; \ d = 1 \ ; \ l = 901$$

$$n = \frac{l - a}{d} + 1 = \frac{901 - 603}{1} + 1$$

$$n = 298 + 1 = 299$$

$$\therefore \quad S_n = \frac{n}{2} [a + l]$$

$$S_{299} = \frac{299}{2} (\underbrace{603 + 901}_{752})$$

$$S_{299} = \frac{299}{2} \times \overset{752}{\cancel{1504}} = \underline{224848}$$

$\div$ by 4, between 602 & 902

$$604 + 608 + \cdots + 900$$

$$a = 604 ; \ d = 4 ; \ l = 900$$

$$n = \frac{l - a}{d} + 1 = \frac{900 - 604}{4} + 1 = \frac{296}{4} + 1$$

n $74 + 1 = 75$

$S_n = \frac{n}{2}[a + \ell] = \frac{75}{2}(604 + 900)$

$= \frac{75}{2} \times 1504 = \underline{56400}$

$\therefore$ Ans $224848 - 56400 = \underline{\underline{168448}}$

34/ $10^2 + 11^2 + \cdots + \cdots + 24$

$$\frac{n(n+1)(2n+1)}{6}$$

$1^2 + 2^2 + \cdots + 24^2 - (1^2 + 2^2 + \cdots + 9^2)$

$\dfrac{\overset{4}{24} \times 25 \times 49}{\cancel{6}} - \dfrac{\overset{3}{9} \times \overset{5}{10} \times 19}{\cancel{6}\,\cancel{2}}$

$4900 \qquad - 15 \times 19 = 4900 - 285$

$= \underline{4615}$

35/

$$\begin{array}{r}
x^2 - 6x + 3 \\
x^2 \overline{\smash{\big)}\, x^4 - 12x^3 + 42x^2 - 36x + 9} \\
\end{array}$$

$ x^4$
$(-) x^2$

$2x^2 - 6x \quad \begin{array}{l} -12x^3 + 42x^2 \\ -12x^3 + 36x^2 \\ (+) \qquad (-) \end{array}$

$2x^2 - 12x + 3 \quad \begin{array}{l} 6x^2 - 36x + 9 \\ 6x^2 - 36x + 9 \\ (-) \quad (+) \quad (-) \end{array}$
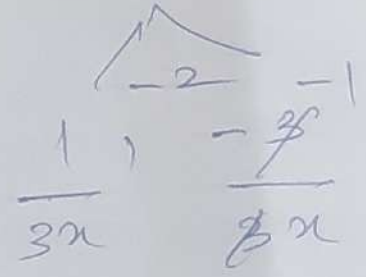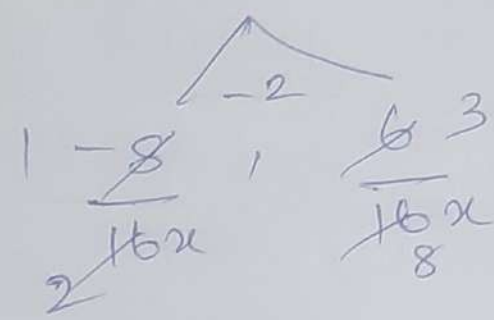
$\underline{0}$

$\therefore |x^2 - 6x + 3|$

36.

$$\frac{16x^2 - 2x - 3}{3x^2 - 2x - 1} \times \frac{3x^2 - 11x - 4}{8x^2 + 11x + 3}$$

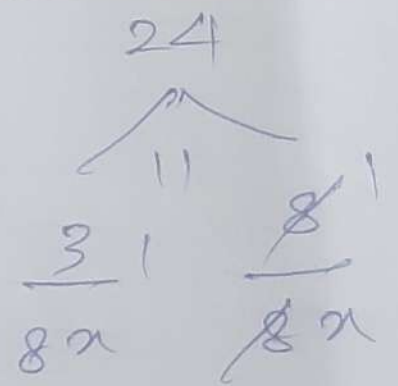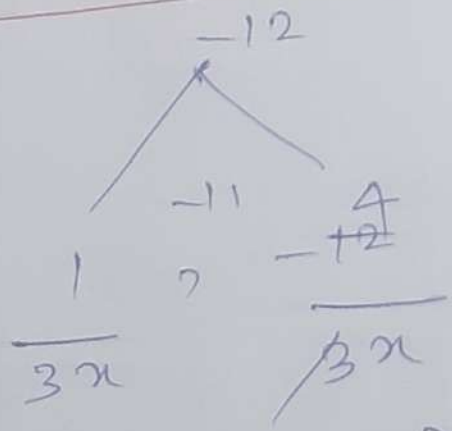$$\boxed{\frac{16x^2 - 2x - 3}{-48}} \qquad \boxed{\frac{3x^2 - 2x - 1}{-3}}$$

$$1 \;-8 \;, \; \frac{6\;3}{\cancel{16}x} \qquad \frac{1}{3x} \;, \; \frac{-\cancel{3}\;-1}{\cancel{8}x}$$

$$(2x - 1)(8x + 3) \qquad \frac{(3x+1)(x-1)}{}$$

$$\boxed{3x^2 - 11x - 4} \qquad \boxed{8x^2 + 11x + 3}$$

$$\frac{1}{3x} \;, \; \frac{-\cancel{12}\;-4}{\cancel{3}x} \qquad \frac{3}{8x} \;, \; \frac{\cancel{8}\;1}{\cancel{8}x}$$

$$(3x+1)(x-4) \qquad (8x+3)(x+1)$$

$$\therefore \frac{(2x-1)(8x+3)}{(3x+1)(x-1)} \times \frac{(3x+1)(x-4)}{(8x+3)(x+1)}$$

$$\frac{(2x-1)(x-4)}{(x-1)(x+1)} = \frac{2x^2 - 9x + 4}{x^2 - 1}$$

34. Let the original speed of the bus $= x$ km/hr. Time $= \dfrac{Distance}{Speed}$.

Time taken by the bus $T_1 = \dfrac{90}{x}$.

Time taken by the bus when the speed is increased T_2 $\Bigg\} = \dfrac{90}{x+15}$

Gn $T_1 - T_2 = 30$ mins $= \dfrac{1}{2}$ hour.

$$\dfrac{90}{x} - \dfrac{90}{x+15} = \dfrac{1}{2}$$

$$\dfrac{90(x+15) - 90x}{x(x+15)} = \dfrac{1}{2}$$

$$\dfrac{90x + 1350 - 90x}{x^2 + 15x} = \dfrac{1}{2}$$

$$x^2 + 15x = 2 \times 1350 = 2700$$

$$x^2 + 15x - 2700 = 0$$

$$(x + 60)(x - 45) = 0$$

$$x + 60 = 0 \quad \text{or} \quad x - 45 = 0$$

$$x = -60 \quad \text{or} \quad x = 45$$

-2700

```
      15
     /\
   60 , -45
```

Original speed of the bus $\Bigg\} = 45$ km/hr

Thales th^m

A straight line drawn parallel to the side of a triangle, intersecting the other two sides, divides the sides in the same ratio. ABC is a Δ,

Gi: D is a point on AB. E is a point on AC.
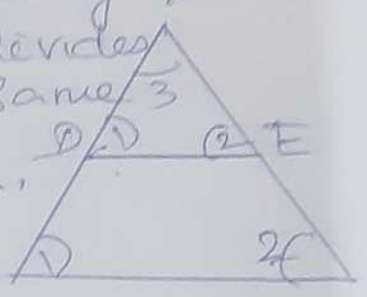
To prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$,

Construction: Draw DE ∥ BC.

$\lfloor ABC = \lfloor ADE = \lfloor 1$	Corr. angles are equal.
$\lfloor ACB = \lfloor AED = \lfloor 2$	" "
$\lfloor BAC = \lfloor DAE = \lfloor 3$	Common angle
$\triangle ABC \sim \triangle ADE$	By AAA rule.
$\dfrac{AB}{AD} = \dfrac{AC}{AE}$	↑
$\dfrac{AD+DB}{AD} = \dfrac{AE+EC}{AE}$	Corr. sides are proportional (From the fig)
$1 + \dfrac{DB}{AD} = 1 + \dfrac{EC}{AE}$	Cancelling 1 on both sides.

$$\frac{DB}{AD} = \frac{EC}{AE}$$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Taking on reciprocals.

Hence the th⁻ᵐ proved.

39.

Area of quadrilateral

$$\frac{1}{2} \left\{ \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{matrix} \right\}$$

$$\frac{1}{2} \left\{ \begin{matrix} -9 & -6 & -1 & -8 & -9 \\ 0 & -3 & -2 & 6 & 0 \end{matrix} \right\} \text{ sq. units}$$

$$\frac{1}{2} \left\{ (27 + 12 - 6 - 0) - (0 + 3 + 16 - 54) \right\}$$

$$\frac{1}{2} \left\{ 33 - (-35) \right\}$$

$$\frac{1}{2} \left\{ 33 + 35 \right\} = \frac{1}{2} \left\{ 68 \right\} = 34 \text{ sq. units.}$$

Ans 34 sq. units.

40. $A(-4, 2)$ $B(6, -4)$

Slope $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{6 + 4} = \frac{-6}{10} = \frac{-3}{5}$

Par slope $m_2 = \frac{5}{3}$. $\left[m_1 \times m_2 = -1 \right]$