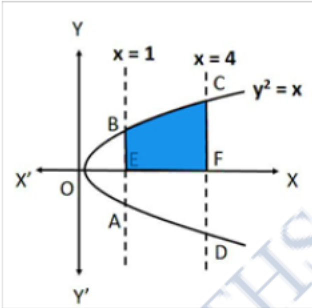
SECOND TERMINAL EXAMINATION – 2022
MATHAMATICS (SCIENCE) HSE II - ANSWERS

	QUESTIONS FROM 1 TO 8. 3 MARK	MARK
1 (a)	$(i) y$	1
(b)	$f(x) = x^2 f(x_1) = {x_1}^2 \quad f(x_2) = {x_2}^2$ $f(x_1) = f(x_2)$ ${x_1}^2 = {x_2}^2 \Rightarrow x_1 = x_2$ or $x_1 = -x_2$ *it is not none – one* $let f(x) = y$ $x^2 = y$ $x = \pm\sqrt{y}$ ***y is real number so can be negatice too hence*** x^2 is ***not onto***	2
2 (a)	$(iii)\ \frac{\pi}{4}$	1
(b)	$cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6}$ $= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$	2
3 (a)	$a_{ij} = 2i - j$ $A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$	1
(b)	$AB = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 7 & 6 \end{bmatrix}$	2
4	*Write equation as* $AX = B$ $\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ *calculate* $\lvert A\rvert = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix} = -11$ $AX = B$ $X = A^{-1}B$ *Here* $A^{-1} = \frac{1}{\lvert A\rvert} adj(A) adj(A) = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$ *Now* $A^{-1} = \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$ $X = A^{-1}B \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ $x = 3, y = -1$	3

5	$Let\ x\ be\ the\ length\ of\ an\ edge\ of\ cube\ and\ V\ be\ the\ volume\ of\ the\ cube. Then,\ V = x^3$ $\therefore\ Rate\ of\ change\ of\ volume\ w.r.t\ time$ $\frac{dV}{dt} = \frac{d(x^3)}{dt} = 3x^2\frac{dx}{dt}$ $It\ is\ given\ that\ edge\ of\ the\ cube\ is\ increasing\ at\ the$ $rate\ of\ 3\ cm/s, so\ \frac{dx}{dt} = 3cm/s \quad \therefore \frac{dV}{dt} = 3x^2(3) = 9x^2cm^3/s$ $thus\ , when x = 10cm\ , \frac{dV}{dt} = 9(10)^2 = \mathbf{900cm^3/s}$ **the volume of the cube is increasing at the rate of 900 cm^3/s when the edge is 10 cm long**	3
6	$\int 2xsin(x^2+1).dx$ $let\ u = x^2 + 1$ $\frac{du}{dx} = 2xdu = 2x.dx$ $subtituting\ \int 2xsin(x^2+1)dx = \int sinu\ du$ $= (-cosu) + c$ $putting\ value\ of\ u = x^2 + 1$ $= -\cos(x^2+1) + c$	3
7(a)	$order = 2\ degree = 1$	2
(b)	0	1
8(a)	$x = 2, y = 3$	1
(b)	$drs = (1,2,2)$ $dcs = (\frac{1}{3},\frac{2}{3},\frac{2}{3})$	2
	QUESTIONS FROM 9 TO 16. 4 MARK	
9(a)	$2x + 3y = sinx$ $Differentiating\ both\ sides\ w.r.t.\ x, we\ obtain$ $\frac{d}{dx}(2x+3y) = \frac{d}{dx}(sinx) \quad \Rightarrow \frac{d}{dx}(2x) + \frac{d}{dx}(3y) = cosx$ $\Rightarrow 2 + 3\frac{dy}{dx} = cosx$ $\Rightarrow 3\frac{dy}{dx} = cosx - 2$	2

	$\therefore \frac{dy}{dx} = \frac{cosx - 2}{3}$	
(b)	$let\ y = e^{sin-1}x$ $Differentiating\ both\ sides\ w.r.t.\ x,$ $\frac{d(y)}{dx} = \frac{d(e^{sin-1}x)}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{e^{sin-1}x}{\sqrt{1-x^2}}$	2
10(a)	$(i) f\ is\ increasing\ in [a, b]\ if\ f'(x) > 0$	1
(b)	$f'(x) = -\sin x$ (i)for each $x \in (0, \pi)$, $\sin x > 0$, $f'(x) < 0$ and so f is decreasing in $(0, \pi)$. (ii)for each $x \in (\pi, 2\pi)$, $\sin x < 0$, $f'(x) > 0$ and so f is increasing in $(\pi, 2\pi)$.	3
11	$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ $f'(x) = 12x^3 + 12x^2 - 24x = 12x(x-1)(x+2)$ $f'(x) = 0\ at\ x = 0, x = 1\ and\ x = -2.$ $Now\ f''(x) = 36x^2 + 24x - 24 = 12(3x^2 + 2x - 2)$ $f''(0) = -24 < 0$ $\quad 0 is\ the\ local\ maxima$ $f''(1) = 36 > 0$ $f''(-2) = 72 > 0$ $\quad -2\ is\ the\ loacal\ minima$ Therefore, by second derivative test, $x = 0$ is a point of local maxima and local maximum value of f at $x = 0$ is $f(0) = 12$ while $x = 1$ and $x = -2$ are the points of local minima and local minimum values of f at $x = -1$ and -2 are $f(1) = 7$ and $f(-2) = -20$, respectively	4
12	$I = \int e^x sinx dx$ $By\ using\ integration\ by\ parts$ $I = e^x(-cosx) - \int e^x(-cosx)dx$ $= -e^x cosx + \int e^x(cosx)dx$ $I = -e^x cosx + e^x sinx - \int e^x sinx dx$ $I = -e^x cosx + e^x sinx - I$	4

	$2I = e^x(sinx - cosx) + c$ $I = \frac{e^x}{2}(sinx - cosx) + c$	
13(a)	(iv) $\int_a^b f(x)\,dx$	1
(b)	$Area = \int_1^4 ydx = \int_1^4 \sqrt{x}dx$ $= \frac{2}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$ $=\frac{2}{3}[8-1]=\frac{14}{3}$ $units$  	3
14(a)	$integrating factor = e^{\int pdx}$	1
(b)	$x.\frac{dy}{dx} + 2y = x^2$ $p = \frac{2}{x}$, $Q = x$ $\int pdx = \int \frac{2}{x}.dx = 2logx$ $IF = e^{\int pdx} = e^{2logx} = e^{loxx^2} = x^2$	1
(c)	$y.If = \int Q.Ifdx + c$ $y.x^2 = \int x.x^2dx + c$ $x^2y = \int x^3dx + c$ $x^2y = \frac{x^4}{4} + c.$	2
15(a)	$\vec{a}.\vec{b}=10$	1
(b)	$cos\theta = \frac{\vec{a}.\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert} = \frac{10}{\sqrt{14\sqrt{14}}}=\frac{5}{7}$ $\theta = cos^{-1}\left(\frac{5}{7}\right)$	2

(c)	$projection = \frac{10}{\sqrt{14}}$	1
16(a)	$(1,0,0)$	1
(b)	$dc's = \left(\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right)$	3
	QUESTIONS FROM 17 TO 20 . 6 MARK	
17(a)	$\int \frac{1}{\sqrt{7-6x-x^2}} = \int \frac{1}{\sqrt{-x^2-6x+7}} dx$ $= \int \frac{1}{\sqrt{-(x^2+6x-7)}} dx$ $= \int \frac{1}{\sqrt{-((x+3)^2-4^2)}} dx$ $= \int \frac{1}{\sqrt{4^2-(x+3)^2}} dx = \sin^{-1}\left(\frac{x+3}{4}\right) + c$	3
(b)	$\int_1^2 \frac{x.dx}{(x+1)(x+2)}$ $\frac{x.dx}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ $(A = -1, B = 2)$ $\frac{x.dx}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$ $\int \frac{x.dx}{(x+1)(x+2)} = -log\|x+1\| + 2log\|x+2\|$ $\int_1^2 \frac{x.dx}{(x+1)(x+2)} = -3log3 + log2 + 2log4$	3
18(a)	$\frac{dy}{dx} = \frac{x+y}{x}$ $Let\ x = \lambda x, y = \lambda y$ $\therefore \frac{dy}{dx} = \frac{\lambda x + \lambda y}{\lambda x} = \frac{\lambda(x+y)}{\lambda x} = \frac{x+y}{x}$ $\therefore$ *Homogeneous differential eq.*	2

(b)	$Noe\ let\ y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\therefore v + x\frac{dv}{dx} = \frac{x+vx}{x} 1 + v$ $\Rightarrow v + x\frac{dv}{dx} = 1 + v$ $\Rightarrow dv = \frac{dx}{x}$ $\Rightarrow \int dv = \int \frac{dx}{x}$ $\Rightarrow v = logx + c$ $\Rightarrow \frac{y}{x} = logx + c$ $\Rightarrow \boldsymbol{y = xlogx + c.}$	4
19(a)	$\hat{\imath} \times \hat{\jmath} = \hat{k}$	1
(b)(i)	$\vec{P} = \vec{a} + \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ $\vec{Q} = \vec{a} - \vec{b} = 0\hat{\imath} - \hat{\jmath} - 2\hat{k}$	2
(ii)	$unit\ vector = \frac{\vec{P}\times\vec{Q}}{\lvert\vec{P}\times\vec{Q}\rvert} = \frac{-2\hat{\imath}+4\hat{\jmath}-2\hat{k}}{\sqrt{24}}$	3
	20(a) Deleted portion questions **(passing through two points) any related attempt full score**	
20(a)	$\hat{r} = -1i + 0j + 2k + \lambda(4j + 4j + 4k)$	2
(b)	$S.D = \left\lvert\frac{(\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1})}{\lvert\vec{b_1}\times\vec{b_2}\rvert}\right\rvert$ $\vec{b_1}\times\vec{b_2} = 8\hat{\imath} + 4\hat{\jmath} - 12\hat{k}$ $\quad (\vec{b_1}\times\vec{b_2}).(\vec{a_2}-\vec{a_1}) = 44$ $\lvert\vec{b_1}\times\vec{b_2}\rvert = \sqrt{224}$ $S.D = \left\lvert\frac{44}{\sqrt{224}}\right\rvert$	4