SY-27

Reg. No. : .....

Name : .....

SECOND YEAR HIGHER SECONDARY
SECOND TERMINAL EXAMINATION, DECEMBER-2023

Part – III

Time : 2 Hours

MATHEMATICS (SCIENCE)

Cool-off time : 15 Minutes

Maximum : 60 scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores. **(6 × 3 = 18)**

1. (a) A function $f : x \to y$ is onto if and only if Range of f = _____.

   (i) y (ii) a proper subset of y

   (iii) ϕ (iv) x (1)

   (b) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2$ is neither one-one nor onto. (2)

2. (a) The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is _____.

   (i) π (ii) $3\frac{\pi}{4}$

   (iii) $\frac{\pi}{4}$ (iv) $\frac{\pi}{3}$ (1)

   (b) Find the value of

   $$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$ (2)

3. (a) Construct a 2 × 2 matrix A = [aij] where aij = 2i − j. (1)

   (b) If $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$, find AB. (2)

4. Solve the following system of equations using matrix method :

   $2x + 5y = 1$

   $3x + 2y = 7$ (3)

5. An edge of a cube is increasing at the rate of 3 cm/sec. How fast is the volume of the cube increasing when the edge is 10 cm long ? (3)

6. Find $\int 2x \sin(x^2 + 1)\, dx$. (3)

7. (a) Write the order and degree of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$ (2)

(b) The number of arbitrary constants in the particular solution of a differential equation of third order are _____. (1)

8. (a) Find the values of x and y so that the vectors $2\bar{i} + 3\bar{j}$ and $x\bar{i} + y\bar{j}$ are equal. (1)

(b) Find the direction cosines of the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. (2)

Answer any 6 questions from 9 to 16. Each carries 4 scores. **(6 × 4 = 24)**

9. Find $\frac{dy}{dx}$, if (a) $2x + 3y = \sin x$ (2)

(b) $y = e^{\sin^{-1}x}$ (2)

10. (a) Let the function f be continuous in [a, b] and differentiable in (a, b), then which among the following is true ?

(i) f is increasing in [a, b] if $f'(x) > 0$

(ii) f is increasing in [a, b] if $f'(x) < 0$

(iii) f is decreasing in [a, b] if $f'(x) > 0$

(iv) f is decreasing in [a, b] if $f'(x) = 0$ (1)

(b) Prove that the function $f(x) = \cos x$ is (i) decreasing in $(0, \pi)$ and (ii) increasing in $(\pi, 2\pi)$. (3)

11. Find the local maximum and local minimum values of the function f given by

$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ (4)

12. Find $\int e^x \cdot \sin x \, dx$. (4)

13. (a) The area bounded by the curve $y = f(x)$, the lines $x = a$, $x = b$ and the x-axis is _____.

(i) $\int_0^a f(x) \, dx$ (ii) $\int_0^b f(x) \, dx$

(iii) $\int_a^a f(x) \, dx$ (iv) $\int_a^b f(x) \, dx$ (1)

(b) Find the area of the region bounded by the curves $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis in the first quadrant. (3)

14. (a) Write the integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$. (1)

(b) Find the integrating factor of $x \cdot \frac{dy}{dx} + 2y = x^2 \ (x \neq 0)$. (1)

(c) Find the general solution of the above differential equation. (2)

15. If $\bar{a} = \bar{i} - 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} - 2\bar{j} + \bar{k}$, find

(a) $\bar{a} \cdot \bar{b}$ (1)

(b) the angle between $\bar{a}$ and $\bar{b}$ (2)

(c) the projection of $\bar{a}$ on $\bar{b}$ (1)

16. (a) The direction cosines of x-axis are _____. (1)

(b) Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$. (3)

Answer any 3 questions from 17 to 20. Each carries 6 scores. **$(3 \times 6 = 18)$**

17. (a) Find $\int \frac{1}{\sqrt{7 - 6x - x^2}} dx$. (3)

(b) Evaluate $\int_1^2 \frac{x\,dx}{(x+1)(x+2)}$ (3)

18. (a) Show that the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is homogeneous. (2)

(b) Solve the differential equation in part (a). (4)

19. (a) Which among the following is correct ?

(i) $\bar{i} \times \bar{i} = 1$ (ii) $\bar{i} \times \bar{j} = \bar{k}$

(iii) $\bar{j} \times \bar{i} = \bar{k}$ (iv) $\bar{i} \times \bar{k} = \bar{j}$ (1)

(b) If $\bar{a} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{b} = \bar{i} + 2\bar{j} + 3\bar{k}$, find

(i) $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ and; **(2)**

(ii) a unit vector perpendicular to both $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$. **(3)**

20. (a) Find the vector equation for the line passing through the points (–1, 0, 2) and (3, 4, 6). **(2)**

(b) Find the shortest distance between the line in part (a) and the line

$$\bar{r} = \bar{i} + \bar{j} + \mu(2\bar{i} - \bar{j} + \bar{k}).$$ **(4)**