



**SECOND YEAR HIGHER SECONDARY
SECOND TERMINAL EXAMINATION, DECEMBER-2023**

Part - III

Time : 2 Hours

MATHEMATICS (SCIENCE) Cool-off time : 15 Minutes

Maximum : 60 scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

പ്രസ്താവിക്കപ്പെട്ട പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരവെഴുതിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമായ സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനോ കൺക്കൂലേറ്ററുകൾ വഴിയോ മറ്റ് ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാസാഹചര്യം ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores.

(6 × 3 = 18)

1. (a) A function $f: x \rightarrow y$ is onto if and only if Range of $f =$ _____.

(i) y

(ii) a proper subset of y

(iii) ϕ

(iv) x

(1)

- (b) Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2$ is neither one-one nor onto. (2)

2. (a) The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is _____.

(i) π

(ii) $3\frac{\pi}{4}$

(iii) $\frac{\pi}{4}$

(iv) $\frac{\pi}{3}$

(1)

- (b) Find the value of

$$\cos^{-1}\left(\frac{1}{2}\right) + 2 \sin^{-1}\left(\frac{1}{2}\right) \quad (2)$$

3. (a) Construct a 2×2 matrix $A = [a_{ij}]$ where $a_{ij} = 2i - j$. (1)

- (b) If $B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$, find AB . (2)

4. Solve the following system of equations using matrix method :

$$2x + 5y = 1$$

$$3x + 2y = 7$$

(3)

5. An edge of a cube is increasing at the rate of 3 cm/sec. How fast is the volume of the cube increasing when the edge is 10 cm long ? (3)

6. Find $\int 2x \sin(x^2 + 1) dx$. (3)

7. (a) Write the order and degree of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0 \quad (2)$$

- (b) The number of arbitrary constants in the particular solution of a differential equation of third order are _____. (1)

8. (a) Find the values of x and y so that the vectors $2\vec{i} + 3\vec{j}$ and $x\vec{i} + y\vec{j}$ are equal. (1)

- (b) Find the direction cosines of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. (2)

Answer any 6 questions from 9 to 16. Each carries 4 scores.

(6 × 4 = 24)

9. Find $\frac{dy}{dx}$, if (a) $2x + 3y = \sin x$ (2)

(b) $y = e^{\sin^{-1}x}$ (2)

10. (a) Let the function f be continuous in $[a, b]$ and differentiable in (a, b) , then which among the following is true ?

- (i) f is increasing in $[a, b]$ if $f'(x) > 0$
- (ii) f is increasing in $[a, b]$ if $f'(x) < 0$
- (iii) f is decreasing in $[a, b]$ if $f'(x) > 0$
- (iv) f is decreasing in $[a, b]$ if $f'(x) = 0$

(1)

- (b) Prove that the function $f(x) = \cos x$ is (i) decreasing in $(0, \pi)$ and (ii) increasing in $(\pi, 2\pi)$. (3)

11. Find the local maximum and local minimum values of the function f given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12 \quad (4)$$

12. Find $\int e^x \cdot \sin x \, dx$. (4)

13. (a) The area bounded by the curve $y = f(x)$, the lines $x = a$, $x = b$ and the x -axis is

$$(i) \int_0^a f(x) \, dx$$

$$(ii) \int_0^b f(x) \, dx$$

$$(iii) \int_a^b f(x) \, dx$$

$$(iv) \int_a^b f(x) \, dx \quad (1)$$

- (b) Find the area of the region bounded by the curves $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x -axis in the first quadrant. (3)

14. (a) Write the integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$. (1)

- (b) Find the integrating factor of $x \cdot \frac{dy}{dx} + 2y = x^2$ ($x \neq 0$). (1)

- (c) Find the general solution of the above differential equation. (2)

15. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} - 2\vec{j} + \vec{k}$, find

(a) $\vec{a} \cdot \vec{b}$

(b) the angle between \vec{a} and \vec{b}

(c) the projection of \vec{a} on \vec{b}

16. (a) The direction cosines of x -axis are _____.

(b) Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

Answer any 3 questions from 17 to 20. Each carries 6 scores.

$(3 \times 6 = 18)$

17. (a) Find $\int \frac{1}{\sqrt{7-6x-x^2}} dx$.

(b) Evaluate $\int_1^2 \frac{x dx}{(x+1)(x+2)}$

18. (a) Show that the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is homogeneous.

(b) Solve the differential equation in part (a).

19. (a) Which among the following is correct ?

(i) $\vec{i} \times \vec{i} = 1$

(ii) $\vec{i} \times \vec{j} = \vec{k}$

(iii) $\vec{j} \times \vec{i} = \vec{k}$

(iv) $\vec{i} \times \vec{k} = \vec{j}$

(b) If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$, find

(i) $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ and;

(2)

(ii) a unit vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

(3)

20. (a) Find the vector equation for the line passing through the points $(-1, 0, 2)$ and $(3, 4, 6)$.

(2)

(b) Find the shortest distance between the line in part (a) and the line

$$\vec{r} = \vec{i} + \vec{j} + \mu(2\vec{i} - \vec{j} + \vec{k}).$$

(4)