SY-227

Reg. No. : .....

Name : .....

SECOND YEAR HIGHER SECONDARY
SECOND TERMINAL EXAMINATION, DECEMBER-2024

Part – III

Time : 2 Hours

MATHEMATICS (SCIENCE)

Cool-off time : 15 Minutes

Maximum : 60 scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമേ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores. **(6 × 3 = 18)**

1. Consider a set A = {1, 2, 3}.

   (i) The relation R = {(2, 3)} on A is **(1)**

   (a) reflexive only (b) symmetric only

   (c) transitive only (d) equivalence relation

   (ii) Consider a relation R = {(1, 2), (2, 3)} on A.

   Add a minimum number of ordered pairs so that the enlarged relation is an equivalence relation. **(2)**

2. (i) The principal value range of $f(x) = \tan^{-1}x$. **(1)**

   (a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $[0, \pi]$

   (c) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

   (ii) For the principal value range, evaluate **(2)**

   $\cot^{-1}(-1) + \operatorname{cosec}^{-1}\left(-\sqrt{2}\right) + \sec^{-1}(2)$

3. (i) If A and B are symmetric matrices of same order, then AB – BA is **(1)**

   (a) Symmetric matrix (b) Skew symmetric matrix

   (c) Identity matrix (d) Zero matrix

   (ii) For what value of x **(2)**

$$[1\ 2\ 1]\begin{bmatrix}1 & 2 & 0\\ 2 & 0 & 1\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 2\\ x\end{bmatrix} = 0\ ?$$

4. Find the intervals in which $f(x) = \sin x - \cos x$, $0 \le x \le \pi$ is **(3)**

   (i) strictly increasing

   (ii) strictly decreasing

5. Find the integrals :

(i) $\int \frac{\cos x - \sin x}{\cos x + \sin x}\, dx$ (1)

(ii) $\int \frac{1}{x^2 - 6x + 13}\, dx$ (2)

6. (i) Write the order and degree of the differential equation : (1)

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0$$

(ii) Solve the differential equation : (2)

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

7. (i) If 'θ' is the angle between any two vectors $\vec{a}$ & $\vec{b}$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then θ is _____ . (1)

(ii) Find $|\vec{a} - \vec{b}|$ if two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$. (2)

8. (i) The Cartesian equation of a line is $\frac{x-5}{4} = \frac{y+4}{7} = \frac{z-6}{2}$.

Write its vector equation. (1)

(ii) Find the angle between the lines

$\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k})$ &

$\vec{r} = 2\hat{i} - \hat{j} - 5\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$ (2)

Answer any 6 questions from 9 to 16. Each carries 4 scores. $(6 \times 4 = 24)$

9. (i) Which of the following is not a continuous function ? (1)

(a) $|x|$ (b) $\lceil x \rceil$

(c) $\log x$ (d) $\frac{1}{x}, x \neq 0$

(ii) If $x = a(\cos\theta + \theta \sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, find $\frac{dy}{dx}$. (3)

10. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ? (4)

11. Evaluate :

(i) $\int_{-\pi/2}^{\pi/2} \sin^5 x \cdot dx$ (1)

(ii) $\int_{-1}^{2} |x^3 - x| \, dx$ (3)

12. (i) Area of the region bounded by $y^2 = 4x$, y-axis and the line $y = 3$ is (1)

(a) 2 (b) $\frac{9}{2}$

(c) $\frac{9}{4}$ (d) 9

(ii) Find the area of the region bounded by $\frac{x^2}{4} + \frac{y^2}{9} = 1$, using integration. (3)

13. Solve the differential equation $x^2ydx - (x^3 + y^3)dy = 0$. (4)

14. Solve the differential equation : (4)

$$(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$$

15. If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of points A, B, C and D respectively, then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. (4)

16. Find the shortest distance between the lines (4)

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Answer any 3 questions from 17 to 20. Each carries 6 scores. (3 × 6 = 18)

17. Check the consistency of system of equations and solve using matrix method : (6)

$x + y + z = 6$

$y + 3z = 11$

$x - 2y + z = 0$

18. (i) The function $f(x) = x + \cos x$ is (1)

(a) always increasing

(b) always decreasing

(c) increasing for certain range of x

(d) neither increasing nor decreasing

(ii) The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm ? (2)

(iii) Find the local maxima and local minima of the function $f(x) = -x^3 + 12x + 5$, $x \in (-3, 3)$ (3)

19. Find the integrals:

(i) $\int \frac{1-\sin x}{\cos^2 x} dx$ (1)

(ii) $\int \frac{e^x(x^2+1)}{(x+1)^2} dx$ (2)

(iii) $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$ (3)

20. (i) Consider the points A(1, 2, –3) and B(–1, –2, 1). Find a unit vector in the direction of $\overrightarrow{AB}$. (2)

(ii) If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular. (2)

(iii) Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$. (2)