SECOND TERM EXAMINATION, ANSWER KEY, DEC - 2024

1. (i) (c) Transitive only
(ii) (1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (1, 3), (3, 1)

2. (c) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (ii) $\left(\pi-\frac{\pi}{4}\right)+\left(-\frac{\pi}{4}\right)+\frac{\pi}{3}=\frac{5\pi}{6}$

3. (i) (b) Skew symmetric matrix or
(d) Zero matrix
(ii) $[1\quad 2\quad 1].\begin{bmatrix}1&2&0\\2&0&1\\1&0&2\end{bmatrix}.\begin{bmatrix}0\\2\\x\end{bmatrix}=[6\quad 2\quad 4].\begin{bmatrix}0\\2\\x\end{bmatrix}$
$=[4+4x]\therefore 4+4x=0 \Rightarrow x=-1$

4. $f(x)=\sin x-\cos x \quad f'(x)=\cos x+\sin x$
$f'(x)=0 \Rightarrow \cos x+\sin x=0$
$\Rightarrow \sin x=-\cos x \therefore x=\frac{3\pi}{4}$
There are two intervals $\left(0,\frac{3\pi}{4}\right)$ $\left(\frac{3\pi}{4},\pi\right)$
$f'(x)>0$ on $\left(0,\frac{3\pi}{4}\right)$ & $f'(x)>0$ on $\left(\frac{3\pi}{4},\pi\right)$
f(x) is increasing on $\left(0,\frac{3\pi}{4}\right)$
f(x) is decreasing on $\left(\frac{3\pi}{4},\pi\right)$

5.(i) $\int\frac{\cos x-\sin x}{\cos x+\sin x}dx=\log|\cos x+\sin x|+C$
(ii) $\int\frac{1}{x^2-6x+13}dx=\int\frac{1}{x^2-6x+9+4}dx=$
$\int\frac{1}{(x-3)^2-(2)^2}dx=\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C$

6. (i) Order 2, Degree 1
(ii) $\frac{dy}{1+y^2}=\frac{dx}{1+x^2} \Rightarrow \int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}$
$\therefore \tan^{-1}y=\tan^{-1}x+C$

7. (i) $\theta=\frac{\pi}{4}$
(ii) $\left(|\vec{a}-\vec{b}|\right)^2=(\vec{a}-\vec{b}).(\vec{a}-\vec{b})$
$=\vec{a}.\vec{a}+\vec{a}.\vec{b}-\vec{b}.\vec{a}+\vec{b}.\vec{b}=\vec{a}.\vec{a}-\vec{a}.\vec{b}-\vec{b}.\vec{a}+\vec{b}.\vec{b}$
$=|\vec{a}|^2-2(\overrightarrow{a.b})+|\vec{b}|^2=2^2-2(4)+3^2=5$
$\therefore \vec{a}.\vec{b}=\sqrt{5}$

8. (i) $\vec{r}=5\hat{\imath}-4\hat{\jmath}+6\hat{k}+\lambda(4\hat{\imath}+7\hat{\jmath}+2\hat{k})$
$\vec{b}_1=\hat{\imath}-\hat{\jmath}-2\hat{k} \quad \vec{b}_2=3\hat{\imath}-5\hat{\jmath}-4\hat{k}$
$\vec{b}_1.\vec{b}_2=14 \quad |\vec{b}_1|=\sqrt{6} \quad |\vec{b}_2|=\sqrt{50}$
$\cos\theta=\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1|.|\vec{b}_2|}=\frac{14}{\sqrt{6}\sqrt{50}}=\frac{7}{5\sqrt{3}}$
$\therefore \theta=\cos^{-1}\left(\frac{7}{5\sqrt{3}}\right)$

9. (i) (b) [x]
(ii) $\frac{dx}{d\theta}=a(-\sin\theta+(\theta.\cos\theta+\sin\theta))=a\theta\cos\theta$
$\frac{dy}{d\theta}=a(\cos\theta-(-\theta.\sin\theta+\cos\theta))=a\theta\sin\theta$
$\frac{dy}{dx}=\tan\theta$

10. Let x be the length of the square to be cut off and V be the volume of the box
$V=(45-2x)(24-2x)x$
$=4x^3-138x^2+1080x$
$\frac{dV}{dx}=12x^2-276x+1080 \qquad \frac{d^2V}{dx^2}=24x-276$
$\frac{dV}{dx}=0 \Rightarrow 12(x^2-23x+90)=0$
$\Rightarrow 12(x-18)(x-5)=0$
$\therefore x=18, x=5$
$\frac{d^2V}{dx^2}(x=5)<0 \qquad \frac{d^2V}{dx^2}(x=18)>0$
$\therefore$ Volume is maximum when $x=5$

11. (i) 0
(ii) $\int_{-1}^{0}|x^3-x|dx+\int_0^1|x^3-x|dx+\int_1^2|x^3-x|dx$
$=\int_{-1}^{0}(x^3-x)dx+\int_0^1-(x^3-x)dx+\int_1^2(x^3-x)dx$
$=\int_{-1}^{0}(x^3-x)dx+\int_1^0(x^3-x)dx+\int_1^2(x^3-x)dx$
$=\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_{-1}^{0}+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_1^0+\left[\frac{x^4}{4}-\frac{x^2}{2}\right]_1^2$
$=0-\left(\frac{1}{4}-\frac{1}{2}\right)+0-\left(\frac{1}{4}-\frac{1}{2}\right)+(4-2)-\left(\frac{1}{4}-\frac{1}{2}\right)$
$=\frac{11}{4}$

12. (i) (c) $\frac{9}{4}$
(ii) Area of the ellipse $=4.\int_0^2\frac{3}{2}\sqrt{2^2-x^2}dx$
$=6\left[\frac{x}{2}\sqrt{2^2-x^2}+\frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2=6\left[0+2\frac{\pi}{2}-0\right]$
$=6\pi$ Sq. Unit

13. $\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$ Homogeneous DE
Put $y=vx$, $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\quad\frac{dy}{dx}=\frac{x^2vx}{x^3+(vx)^3}=\frac{v}{1+v^3}$

$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$ ∴ $x\frac{dv}{dx} = \frac{v-v(1+v^3)}{1+v^3} = \frac{-v^4}{1+v^3}$

$\frac{1+v^3}{v^4}\,dv = -\frac{dx}{x}$ - Variable separable form

$\int (v^{-4} + v^{-1})dv = -\int \frac{dx}{x}$

$\frac{v^{-3}}{-3} + \log v = -\log x + C$, where $v = \frac{y}{x}$

14. $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\sqrt{x^2+4}}{1+x^2}$ $P = \frac{2x}{1+x^2}$ $Q = \frac{\sqrt{x^2+4}}{1+x^2}$

$IF = e^{\int Pdx} = e^{\int \frac{2xdx}{1+x^2}} = e^{\log(1+x^2)} = 1 + x^2$

$y(1+x^2) = \int \frac{\sqrt{x^2+4}}{1+x^2}.(1+x^2)\,dx$

$y(1+x^2) = \int (\sqrt{x^2+4}\,dx$

$y(1+x^2) = \frac{x}{2}\sqrt{x^2+4} + \frac{4}{2}\log\left|x + \sqrt{x^2+4}\right| + C$

15. $\overrightarrow{AB} = \hat{\imath} + 4\hat{\jmath} - \hat{k}$ $\overrightarrow{CD} = -2\hat{\imath} - 8\hat{\jmath} + 2\hat{k}$

$\overrightarrow{AB}.\overrightarrow{CD} = -36$ $|\overrightarrow{AB}| = \sqrt{18}$ $|\overrightarrow{CD}| = \sqrt{72}$

$\cos\theta = \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{AB}|.|\overrightarrow{CD}|} = \frac{-36}{\sqrt{18}\sqrt{72}} = -1$ ∴ $\theta = \pi$

∴ $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

16. $\vec{a}_1 = -\hat{\imath} - \hat{\jmath} - \hat{k}$ $\vec{b}_1 = 7\hat{\imath} - 6\hat{\jmath} + \hat{k}$

. $\vec{a}_2 = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$ $\vec{b}_2 = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = -4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$ $|\vec{b}_1 \times \vec{b}_2| = \sqrt{116}$

$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = -116$

Shortest Distance $= \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \sqrt{116}$

17. $A = \begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}, B = \begin{bmatrix}6\\11\\0\end{bmatrix}, X = \begin{bmatrix}x\\y\\z\end{bmatrix}$ $|A| = 9$

Cofactor matrix $= \begin{bmatrix}7 & 3 & -1\\-3 & 0 & 3\\2 & -3 & 1\end{bmatrix}$,

Adj A $= \begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}$, $X = A^{-1}B$

$= \frac{1}{9}\begin{bmatrix}7 & -3 & 2\\3 & 0 & -3\\-1 & 3 & 1\end{bmatrix}.\begin{bmatrix}6\\11\\0\end{bmatrix} = \frac{1}{9}\begin{bmatrix}9\\18\\27\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$

x = 1, y = 2, z = 3

18. (i) (a) always increasing

(ii) $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dt} = \frac{1}{2}$ cm/sec r = 1

$\frac{dV}{dt} = \frac{4\pi}{3}3r^2\frac{dr}{dt} = \frac{4\pi}{3}3 \times 1^2 \times \frac{1}{2} = 2\pi$ cm³ / sec

(iii) $f'(x) = -3x^2 + 12$

$f'(x) = 0 \Rightarrow -3x^2 + 12 = 0 \Rightarrow x = \pm 2$

$f(2) = 21$; $f(-2) - 11$

Local maximum = 21 & Local minimum = -11

19. (i) $\int \frac{1-\sin x}{\cos^2 x}dx = \int(\sec^2 x - \tan x \sec x)\,dx$

= tan x - sec x + C

(ii) $\int \frac{e^x(x^2+1)}{(x+1)^2}dx = \int \frac{e^x(x^2-1+2)}{(x+1)^2}dx$

$= \int e^x\left[\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right]dx = \int e^x\left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right]dx$

$= e^x\left[\frac{x-1}{x+1}\right] + C$

(iii) $Nr = A\frac{d}{dx}(Dr) + B$

$5x + 3 = A(2x+4) + B$

Solving $A = \frac{5}{2}$ $B = -7$

$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$

$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{x^2+4x+10}}dx$

$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{x^2+4x+4+6}}dx$

$= 5\int \frac{2x+4}{2\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{(x+2)^2+(\sqrt{6})^2}}dx$

$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C$

20. $\overrightarrow{AB} = -2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ $|\overrightarrow{AB}| = \sqrt{36} = 6$

Required vector $= \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{-2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}}{6}$

(ii) $\vec{a} + \vec{b} = 6\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$

$\vec{a} - \vec{b} = 4\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

$(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 24 - 8 - 16 = 0$

∴ $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular

(iii) $\vec{a} \times \vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\1 & -1 & 3\\2 & -7 & 1\end{vmatrix} = 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{400+25+25} = \sqrt{450} = 15\sqrt{2}$

Area = $15\sqrt{2}$ Sq. Unit

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