SY-427

SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2025

Part – III

MATHEMATICS (SCIENCE)

Time : 2 Hours

Cool-off time : 15 Minutes

Maximum : 60 Scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Answer any 6 questions from 1 to 8. Each carries 3 scores. **$(6 \times 3 = 18)$**

1. (a) A function f defined on N, the natural numbers as $f(n) = 2n$; $n \in N$. Then f is ____. **(1)**

   (A) one-one and onto (B) one-one but not onto

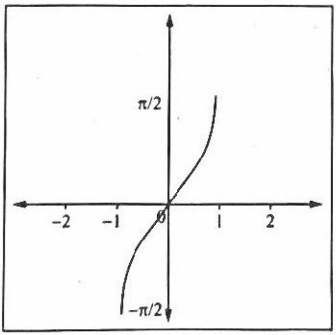
   (C) not one-one and not onto (D) onto but not one-one

   (b) $R = \{(1, 4), (2, 5), (3, 6)\}$ is a relation from $A = \{1, 2, 3\}$ to $B = \{4, 5, 6, 7\}$. Check whether R is bijection or not. Justify your answer. **(2)**

2. (a) If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is ______. **(1)**

   (b) Find x and y if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} x+y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 1 & 8 \end{bmatrix}$ **(2)**

3. (a) The graph below depicts : **(1)**



   (A) $y = \sin^{-1} x$ (B) $y = \cos^{-1} x$

   (C) $y = \operatorname{cosec}^{-1} x$ (D) $y = \cot^{-1} x$

   (b) Write the simplest form of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$; $x \neq 0$. **(2)**

4. (a) Which of the following is an increasing function in $\mathbb{R}$? (1)

(A) $\cos x$ (B) x^3

(C) x^2 (D) $|x|$

(b) Find the interval in which the function $f(x) = x^2 - 4x + 6$ is increasing or decreasing in $\mathbb{R}$. (2)

5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$$

(a) For what value of k, f is continuous ? (1)

(b) Find f(2). (2)

6. (a) The value of the integral $\int e^x (\sin x + \cos x)dx$ is ______ (1)

(b) Evaluate $\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx$ (2)

7. (a) If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$ with $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then θ is : (1)

(A) 0 (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) π

(b) Write a unit vector in the direction of $\vec{a} = \hat{i} - \hat{k}$. (2)

8. If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4, then find

(a) $P(A \cap B)$ (1)

(b) P(A/B) (1)

(c) Check whether A and B are independent events (1)

Answer any 6 questions from 9 to 16. Each carries 4 scores. $(6 \times 4 = 24)$

9. (a) What is the minimum number of ordered pairs to form a reflexive relation on $A = \{1, 2, 3\}$? (1)

(b) Show that the relation $R = \{(a, b) : |a - b| \text{ is even }\}$ defined on $A = \{1, 2, 3, 4, 5\}$ is an equivalence relation. (3)

10. (a) Express $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix. (2)

(b) Prove that A in part(a) satisfies $A^2 - 5A + 7I = 0$. (2)

11. Show that of among all rectangles inscribed in a given circle, the square has the maximum area. (4)

12. Given lines $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$

(a) Obtain a vector perpendicular to both lines. (2)

(b) Find the equation of the line perpendicular to given lines and passing through the point (1, 2, 3). (2)

13. Using integration, find the area of the region bounded by the ellipse

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ in the first quadrant. (4)

14. (a) If $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors in the coordinate system, then the value of $\hat{i}\cdot(\hat{j} \times \hat{k}) + \hat{j}\cdot(\hat{i} \times \hat{k}) + \hat{k}\cdot(\hat{i} \times \hat{j})$ is ______. (1)

(A) 0 (B) −1

(C) 1 (D) 3

(b) Using vectors, find the area of the triangle with vertices (1, 1, 1), (1, 2, 3) and (2, 3, 1). (3)

15. Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold, in box II, both are silver and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ? (4)

16. (a) Prove that $y = \cos x + A$, (A is an arbitrary constant) is a solution of the differential equation $\frac{dy}{dx} + \sin x = 0$. (1)

(b) Consider the differential equation $x\frac{dy}{dx} + 2y = x^2$

(i) Find the integrating factor. (1)

(ii) Write the general solution. (2)

Answer any 3 questions from 17 to 20. Each carries 6 scores. **(3 × 6 = 18)**

17. (a) If $x = ct$, $y = \frac{c}{t}$, then find $\frac{d^2y}{dx^2}$ (2)

(b) Find $\frac{dy}{dx}$ of the following :

(i) $x^{\frac{1}{3}} + y^{\frac{1}{3}} = a^{\frac{1}{3}}$ (2)

(ii) $y = (\sin x)^{\cos x}$ (2)

18. Consider the system of equations,

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

(i) Write its matrix form. (1)

(ii) Prove that the system is consistent. (1)

(iii) Solve the system using matrix method. (4)

19. Evaluate the following :

(i) $\int \frac{x}{(x+1)(x+2)} dx$ (1)

(ii) $\int \frac{1}{x^2 - 6x + 13} dx$ (1)

(iii) $\int_0^4 |x - 1| dx$ (4)

20. Using graphical method, solve the Linear Programming

Maximize, $Z = 3x + 4y$

subject to the constraints

$x + 2y \leq 10$

$3x + y \leq 15; x \geq 0, y \geq 0$

(6)
