SECOND YEAR HIGHER SECONDAY SECOND TERMINAL EXAMINATION, MARCH 2025.

ANSWER KEY

1. a) B) one-one but not onto

   b) R is one-one but not onto (Every element of B has at least one element in A - 7 of B has no pre-image.). $\therefore R$ is not bijective.

2. a) I

   b) $\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} x+y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 1 & 8 \end{bmatrix}$

   $\begin{bmatrix} 2+x+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 1 & 8 \end{bmatrix}$

   $2x + 2 = 8 \Longrightarrow 2x = 8 - 2 = 6 \Longrightarrow x = \frac{6}{2} = 3$

   $2 + x + y = 8 \Longrightarrow 2 + 3 + y = 8 \Longrightarrow y = 8 - 5 = 3$

3. a) A) $y = \sin^{-1} x$

   b) Put $x = tan\theta \Longrightarrow \tan^{-1} x = \theta$

   $$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\frac{sec\theta - 1}{tan\theta}\right) = \tan^{-1}\left(\frac{1-cos\theta}{sin\theta}\right)$$

   $$= \tan^{-1}\left(\frac{1-cos\theta}{sin\theta}\right) = \tan^{-1}\tan\left(\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$$

4. a) C) x^3

   $b)\ f(x) = x^2 - 4x + 6$

   $f'(x) = 2x - 4$

   For increasing or decreasing, $f'(x) = 0 \Longrightarrow 2x - 4 = 0 \Longrightarrow x = 2$

   Therefore, the intervals may be $(-\infty, 2)\ and\ (2, \infty)$

   When $x \in (-\infty, 2): say\ x = 1, f'(1) = 2 - 4 = -2 < 0\ \downarrow$

   When $x \in (2, \infty): say\ x = 3, f'(3) = 6 - 4 = 2 > 0\ \uparrow$

   $\therefore f\ is\ increasing\ in\ (2, \infty)\ and\ decreasing\ in\ (-\infty, 2).$

5. a) $\lim_{x\to\pi^-}(kx+1) = \lim_{x\to\pi^-} cosx$

$\pi k + 1 = cos\pi \Longrightarrow \pi k + 1 = -1 \Longrightarrow \pi k = -1 - 1 = -2$

$k = -\frac{2}{\pi}$

$b)\ f(2) = -\frac{2}{\pi}(2) + 1 = -\frac{4}{\pi} + 1 = \frac{\pi - 4}{\pi}$

6. a) $e^x sinx$

b) put $\tan^{-1} x = t$

$\frac{1}{1+x^2}dx = dt$

When $x = 0, t = \tan^{-1} 0 = 0$

When $x = 1, t = \tan^{-1} 1 = \frac{\pi}{4}$

$\therefore I = \int_0^{\frac{\pi}{4}} t\,dt = \left[\frac{t^2}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[\frac{\pi^2}{16} - 0^2\right] = \frac{\pi^2}{32}$

7. a) C) $\frac{\pi}{4}$

b) $\vec{a} = \hat{\imath} - \hat{k}$

$|\vec{a}| = \sqrt{1+1} = \sqrt{2}$

Unit vector perpendicular to $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$= \frac{\hat{\imath} - \hat{k}}{\sqrt{2}}$

8. a) $P(B/A) = \frac{P(A\cap B)}{P(A)}$

$P(A \cap B) = P(A).P(B/A) = 0.8 \times 0.4 = 0.32$

b) $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.32}{0.5} = 0.64$

9. a) 3

b) a) $For\ any\ a \in A,\ |a - a| = 0, is\ even$

$\therefore R$ is reflexive.

b) Let $(a, b) \in R,\ then\ |a - b| is\ even\ \Longrightarrow |b - a|\ is\ even \Longrightarrow (b, a) \in R$

$\therefore R\ is\ symmetric.$

c) Let $a, b, c\ \in A$.

Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow |a-b|$ is $even, |b-c|$ is $even$

But $a-b+b-c=a-c$

Hence, $|a-c|$ is $also$ $even$.

$\therefore R$ is an equivalence relation.

10. a) $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, A' = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

$Let\ P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}$

$P' = \frac{1}{2}\begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix} = P, symmetric$

$Let\ Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$Q' = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q, is\ skew\ symmetric.$

$Now, P + Q = \frac{1}{2}\left(\begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 6 & 2 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = A$, a square matrix.

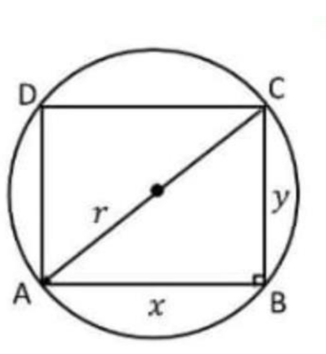
Hence, verified.

b) $A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$\therefore\ A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 0$

11.



$x^2 + y^2 = (2r)^2 \Rightarrow y^2 = 4r^2 - x^2 \dots\dots\dots\dots(1)$

Area, $A = xy = x\sqrt{4r^2 - x^2}$

Let $Z = A^2 = x^2(4r^2 - x^2) = 4r^2x^2 - x^4$

$$\frac{dZ}{dx} = 4r^2(2x) - 4x^3 = 8r^2x - 4x^3$$

$$\frac{d^2Z}{dx^2} = 8r^2 - 12x^2$$

For maximum area,

$$\frac{dZ}{dx} = 0 \Longrightarrow 8r^2x - 4x^3 = 0 \Longrightarrow 8r^2 = 4x^2 \Longrightarrow x^2 = 2r^2 \Longrightarrow x = \sqrt{2}r$$

$$\therefore \frac{d^2Z}{dx^2} = 8r^2 - 12(2r^2) = 8r^2 - 24r^2 = -12r^2 < 0$$

Hence, Area is maximum.

$$\therefore y^2 = 4r^2 - 2r^2 = 2r^2 \Longrightarrow y = \sqrt{2}r = x$$

Hence, the rectangle becomes a square.

12. a) dr's of required line are a, b, c

Since the required line is perpendicular to both the lines, we have

$$a - b - 2c = 0$$

$$3a - 5b - 4c = 0$$

Solving we have

$$\frac{a}{4 - 10} = \frac{b}{-6 + 4} = \frac{c}{-5 + 3}$$

$$\frac{a}{-6} = \frac{b}{-2} = \frac{c}{-2} \Longrightarrow \frac{a}{3} = \frac{b}{1} = \frac{c}{1}$$

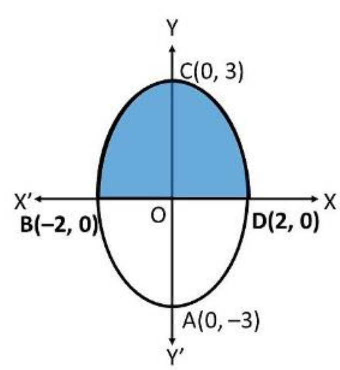
$$\therefore \vec{b} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$$

b) Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Equation is $\vec{r} = \vec{a} + \lambda\vec{b}$

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda\left(3\hat{\imath} + \hat{\jmath} + \hat{k}\right)$$

13. $\frac{y^2}{9} = 1 - \frac{x^2}{4} = \frac{4-x^2}{4}$



$$y^2 = \frac{9}{4}(4 - x^2)$$

$$y = \frac{3}{2}\sqrt{2^2 - x^2}$$

$$A = \int_0^2 y\,dx$$

$$A = \frac{3}{2}\int_0^2 \sqrt{2^2 - x^2}\,dx = \frac{3}{2}\left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= \frac{3}{2}\ [0 + 2\sin^{-1}(1) - (0 + 0)] = \frac{3}{2} \times 2 \times \frac{\pi}{2} = \frac{3}{2}\pi\ sq.units$$

14. a) $(\hat{\imath}.\hat{\imath}) + (\hat{\jmath}.-\hat{\jmath}) + (\hat{k}.\hat{k}) = 1 + (-1) + 1 = 1$

b) $\overrightarrow{OA} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\overrightarrow{OB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overrightarrow{OC} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\overrightarrow{BA} = 0\hat{\imath} - \hat{\jmath} - 2\hat{k}$

$\overrightarrow{BC} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\overrightarrow{BA} \times \overrightarrow{BC} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$

Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{BA} \times \overrightarrow{BC}\right| = \frac{1}{2} \times \sqrt{16 + 4 + 1} = \frac{\sqrt{21}}{2}\ square\ units.$

15. Let E_1, E_2, E_3 be the event of getting the boxes.

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

Let E be the event of getting a gold coin.

$P(E/E_1) = \frac{2}{2}, P(E/E_2) = \frac{0}{2}, P(E/E_3) = \frac{1}{2}$

$Required\ probability = P(E_1/E)$

$$= \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2) + P(E_3)\,P(E/E_3)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{2}}{\frac{1}{3} \times \frac{2}{2} + \frac{1}{3} \times \frac{0}{2} + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{2}{2}}{\frac{2}{2} + \frac{0}{2} + \frac{1}{2}} = \frac{1}{1 + \frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

16. a) $\frac{dy}{dx} + sinx = 0$

$\frac{dy}{dx} = -sinx$

$dy = -sinx\ dx\ is\ in\ VS$

$\int dy = -\int sinx\ dx$

$y = -(-cosx) + A$

$y = cosx + A$, proved.

b) $x\frac{dy}{dx} + 2y = x^2$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

$$P = \frac{2}{x}\,, \qquad Q = x$$

$$\int Pdx = 2\int \frac{1}{x}dx = 2logx = logx^2$$

i) $IF = e^{\int Pdx} = e^{logx^2} = x^2$

ii) $\int Qe^{\int Pdx}dx = \int x.x^2dx = \int x^3dx = \frac{x^4}{4} + C$

Solution is $ye^{\int Pdx} = \int Qe^{\int Pdx}dx$

$$y\,x^2 = \frac{x^4}{4} + C \Longrightarrow y = \frac{x^2}{4} + \frac{c}{x^2}$$

17.

$a)$ $x = ct \Longrightarrow \frac{dx}{dt} = c$

$$y = \frac{c}{t} \Longrightarrow \frac{dy}{dt} = -\frac{c}{t^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -\frac{c}{t^2} \times \frac{1}{c} = -\frac{1}{t^2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{1}{t^2}\right) = \frac{d}{dt}\left(-\frac{1}{t^2}\right) \times \frac{dt}{dx} = -1 \times\, -\frac{2}{t^3} \times \frac{1}{c} = \frac{2}{ct^3}$$

b) i) $x^{\frac{1}{3}} + y^{\frac{1}{3}} = a^{\frac{1}{3}}$

$$\frac{1}{3}x^{\frac{1}{3}-1} + \frac{1}{3}y^{\frac{1}{3}-1}\frac{dy}{dx} = 0 \Longrightarrow x^{-\frac{2}{3}} + y^{-\frac{2}{3}}\frac{dy}{dx} = 0$$

$$\Longrightarrow y^{-\frac{2}{3}}\frac{dy}{dx} = -x^{-\frac{2}{3}} \Longrightarrow \frac{dy}{dx} = -\frac{x^{-\frac{2}{3}}}{y^{-\frac{2}{3}}} = -\frac{y^{\frac{2}{3}}}{x^{\frac{2}{3}}}$$

iii) $y = sinx^{cosx}$

$$logy = cosx\,\log(sinx)$$

$$\frac{1}{y}\frac{dy}{dx} = cosx.\frac{1}{sinx}.cosx + \log(sinx)$$

$$\frac{dy}{dx} = y[cosxcotx + \log(sinx)(-sinx)] = (sinx)^{cosx}[cosxcotx - sinx\,\log(sinx)]$$

18. i) Let $AX = B$

$$where,\ A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

ii) $|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 10 + 15 + 15 = 40 \neq 0$

$\therefore$ *the system is consistent.*

Hence, A^{-1} *exists.*

iii) $A_{11} = 5,\ A_{12} = 5,\ A_{13} = 5$

$A_{21} = 3, A_{22} = -13, A_{23} = 11$

$A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$adj\ A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$x = 1, y = 2\ and\ x = -1$

19.

$$i.\ \int \frac{x}{(x+1)(x+2)}dx$$

$$Let \frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \quad \ldots\ldots\ldots.(1)$$

$x = A(x+1) + B(x+1)$

$Put\ x = -2$

$-2 = B(-2+1) \Longrightarrow -B = -2 \Longrightarrow B = 2$

$Put\ x = -1$

$-1 = A(-1+2) + 0 \ \Longrightarrow A = -1$

(1) :

$$I = \int \left[\frac{A}{x+1} + \frac{B}{x+2}\right] dx = \int \left[\frac{-1}{x+1} + \frac{2}{x+2}\right] dx$$

$$= -\log|x+1| + 2\log|x+2| + C = \log\left[\frac{(x+2)^2}{|x+1|}\right] + C$$

ii) $I = \int \frac{1}{x^2 - 6x + 13} dx$

$$= \int \frac{dx}{x^2 - 6x + 3^2 + 2^2} = \int \frac{dx}{(x-3)^2 + 2^2} = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

iii) $\int_0^4 |x-1| dx$

$x - 1 = 0 \Rightarrow x = 1$

$$I = \int_0^1 -(x-1)dx + \int_1^4 (x-1)dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= \left[1 - \frac{1}{2} - 0\right] + \left[\frac{16}{2} - 4 - \left(\frac{1}{2} - 1\right)\right] = \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 = 5$$

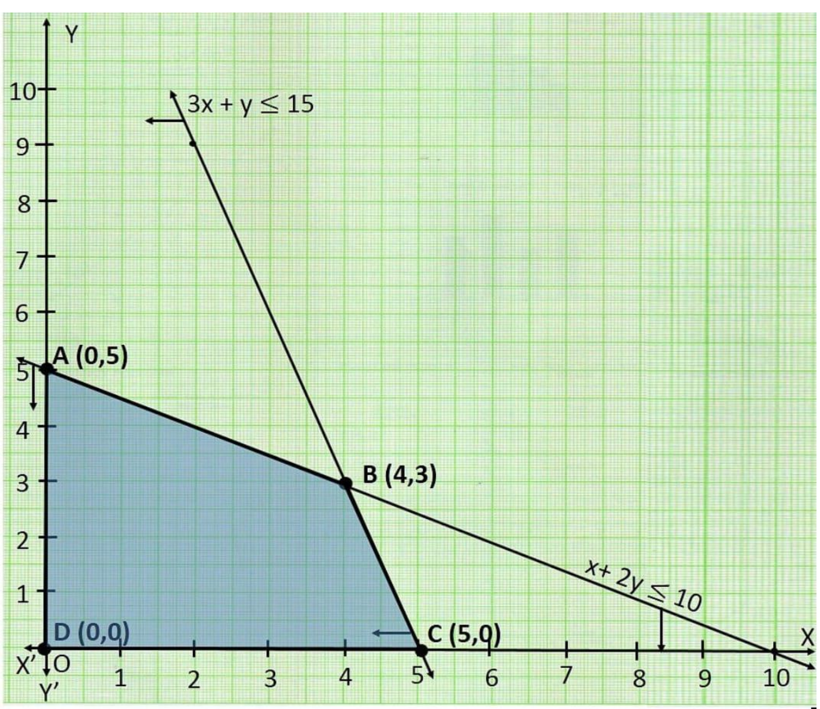
20. $Z = 3x + 4y$

$x + 2y \leq 10$

x	0	10
y	5	0

$3x + y \leq 15$

x	0	5
y	15	0



$Point$	$Z = 3x + 4y$
$A(0,5)$	$Z = 3(5) + 4(0) = 15$
$B(4,3)$	$Z = 3(4) + 4(3) = 24$ ← Max
$C(5,0)$	$Z = 3(5) + 4(0) = 15$
$D(0,0)$	$Z = 0$

$\therefore Maximum\ of\ Z = 24\ at\ B(4,3)$