SECOND YEAR HIGHER SECONDAY SECOND TERMINAL EXAMINATION, MARCH 2025.

COMMERCE - ANSWER KEY

1. i) $\begin{bmatrix} x+2 & 2 \\ 6 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ y-2 & 4 \end{bmatrix}$

$x + 2 = 4 \Longrightarrow x = 4 - 2 = 2$

$y - 2 = 6 \Longrightarrow y = 6 + 2 = 8$

ii) $a_{ij} = 2i - j$

$a_{11} = 2(1) - 1 = 1$

$a_{12} = 2(1) - 2 = 0$

$a_{21} = 2(2) - 1 = 3$

$a_{22} = 2(2) - 2 = 2$

$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

2. i) *Symmetric matrix*

ii) $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$A^2 = A.A = A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

LHS= $A^2 - A + 2I = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2+2 & 0+0 \\ 0+0 & -2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = RHS$

3. i) $2x - 36 = 0 \Longrightarrow 2x = 36 \Longrightarrow x = 18$

Ans: (c)

ii) $\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4 \Longrightarrow \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 8$

$\Longrightarrow k(0-2) - 0 + 1(8-0) = \pm 8$

$\Longrightarrow -2k + 8 = \pm 8$

$\Longrightarrow -2k = \pm 8 - 8$

$-2k = 8 - 8 \;\; and - 2k = -8 - 8$

$-2k = 0 \;\; and - 2k = -16$

$k = 0 \;\; and \;\; k = 8$

4. If $f(x)$ is continuous at $x = 2$

$\lim_{x \to 2^-} x^2 = \lim_{x \to 2^-} 2k \Longrightarrow 2^2 = 2k \Longrightarrow k = 2$

5. i) $A = \pi r^2 \Longrightarrow \frac{dA}{dr} = 2\pi r = 2\pi \times 3 = 6\pi \; cm^2/cm$

ii) $f(x) = x^3 + x$

$f'(x) = 3x^2 + 1 > 0, \forall \, x \in R$

Hence, $f(x)$ is increasing on R

6. i) a)

ii) d)

iii) b)

7. i) b)

ii) $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx \quad \dots \dots \dots (1)$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}} dx$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx \; = \int_0^a \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx \dots \dots .. (2)$

$(1) + (2) \Longrightarrow$

$$I + I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx \; + \int_0^a \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$$

$$2I = \int_0^a \left[\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} + \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\right] dx$$

$$2I = \int_0^a \left[\frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}\right] dx = \int_0^a 1 \, dx = [x]_0^a = a - 0 = a$$

$I = \frac{a}{2}$

8. a) $P(A \cap B) = P(A)P(B) = 0.3 \times 0.4 = 0.12$

b) $P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.3 + 0.4 - 0.12 = 0.7 - 0.12 = 0.58$

c) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.4} = 0.3$

9. i) c) (3,3)

ii) $f(x) = x$

iii) a) f is not one-one. Because, the x elements 4 and 5 of A have same image in B.

b) Not onto. Because, the y element 9 has no pre-image in A.

10. i) $0 \leq y \leq \pi$

ii) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

iii) $\tan^{-1}\left[2\cos\left(2\,sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2 \times \frac{1}{2}\right] = \tan^{-1} 1 = \frac{\pi}{4}$

11. $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}, A' = \begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$

i) $A + A' = \begin{bmatrix} 2 & 7 \\ 7 & 10 \end{bmatrix}$

$(A + A')' = \begin{bmatrix} 2 & 7 \\ 7 & 10 \end{bmatrix} = A + A'$, is symmetric.

Now, $A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$, is skew-symmetric.

ii) Let $P = \frac{1}{2}(A + A')$ is symmetric and $Q = \frac{1}{2}(A - A')$ is skew-symmetric.

$P + Q = \frac{1}{2}[(A + A') + (A - A')] = \frac{1}{2}\left(\begin{bmatrix} 2 & 7 \\ 7 & 10 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\right)$

$= \frac{1}{2}\begin{bmatrix} 2 & 6 \\ 8 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} = A, a\ square\ matrix.$

Hence verified.

12. $f(x) = x^3 - 3x + 3$

$f'(x) = 3x^2 - 3$

$f''(x) = 6x$

For maxima and minima, $f'(x) = 0 \Longrightarrow 3x^2 - 3 = 0$

$\Longrightarrow 3x^2 = 3 \Longrightarrow x^2 = 1 \Longrightarrow x = \pm 1$

$Now, f''(-1) = 6(-1) = -6 < 0$

$\therefore f(x)\ is\ maximum\ and\ the\ maximum\ value\ is\ f(-1) = (-1)^3 - 3(-1) + 3 = 5$

$And, f''(1) = 6(1) = 6 > 0$

$\therefore f(x)\ is\ minimum\ and\ the\ minimum\ value\ is\ f(1) = (1)^3 - 3(1) + 3 = 1$

13. i) dc's are: $-1 - 3, 1 - 5, 2 - -4 = -4, -4, 6$

ii) Equation of the line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$$= (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$$

14. Let E_1 and E_2 be the events of getting boxes.

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Let E be the event of getting a black ball.

$$P(E/E_1) = \frac{3}{7}\ ,\ P(E/E_2) = \frac{2}{4}$$

$$Required\ probability = P(E_2/E) = \frac{P(E_2)\,P(E/E_2)}{P(E_1)\,P(E/E_1) +\ P(E_2)\,P(E/E_2)}$$

$$= \frac{P(E_2)\,P\left(\frac{E}{E_2}\right)}{P(E_1)\,P\left(\frac{E}{E_1}\right) +\ P(E_2)\,P\left(\frac{E}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{2}{4}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{2}{4}} = \frac{\frac{2}{4}}{\frac{3}{7}+\frac{2}{4}}$$

$$= \frac{\frac{1}{4}}{\frac{12+14}{28}} = \frac{\frac{1}{4}}{\frac{26}{28}} = \frac{1}{4}\times\frac{28}{26} = \frac{7}{26}$$

15. Area of the shaded region $= \int_0^2 \sqrt{2^2 - x^2}\,dx$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= [0 + 2\sin^{-1}(1) - \{0 + 0\}] = 2\times\frac{\pi}{2} = \pi\ sq.units$$

16. i) order = 1, degree =1

ii) $\frac{dy}{dx} - cosx = 0$

$$\frac{dy}{dx} = cosx \Longrightarrow dy = cosx\ dx\ \ is\ in\ VS$$

$$\int dy = \int cosxdx \Longrightarrow y = sinx + C$$

17. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

i) $adjA = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$

ii) $|A| = 3 - 4 = -1$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

iii) $Let\ AX = B$

where, $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$

$$X = A^{-1}B = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \end{bmatrix} = \begin{bmatrix} -15+18 \\ 10-9 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\therefore x = 3, y = 1$

18. i) $y = (3x+1)^3$

$$\frac{dy}{dx} = 3(3x+1)^2 \times 3(1+0)$$

$$= 9(3x+1)^2$$

ii) $x = at^2, y = 2at$

$$\frac{dx}{dt} = 2at, \quad \frac{dy}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2a}{2at} = \frac{1}{t}$$

iii) $y = 5cosx + 3sinx$

$$\frac{dy}{dx} = 5(-sinx) + 3cosx = -5sinx + 3cosx$$

$$\frac{d^2y}{dx^2} = -5cosx + 3(-sinx) = -(5cosx + 3sinx) = -y$$

$$\frac{d^2y}{dx^2} + y = 0, proved.$$

19. i) $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$$\vec{a} + \vec{b} = 2\hat{\imath} + 2\hat{k}$$

$$|\vec{a} + \vec{b}| = \sqrt{4+4} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Unit vector perpendicular to $\vec{a} + \vec{b} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{2\hat{\imath}+2\hat{k}}{2\sqrt{2}}$

$$= \frac{\hat{\imath} + \hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{k})$$

ii) $\vec{a} - \vec{b} = 2\hat{\jmath}$

$$(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 2(0) + 0(2) + 2(0) = 0$$

iii) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 1 \\ 0 & 2 & 0 \end{vmatrix} = \hat{\imath}(0-2) - \hat{\jmath}(0-0) + \hat{k}(4-0)$

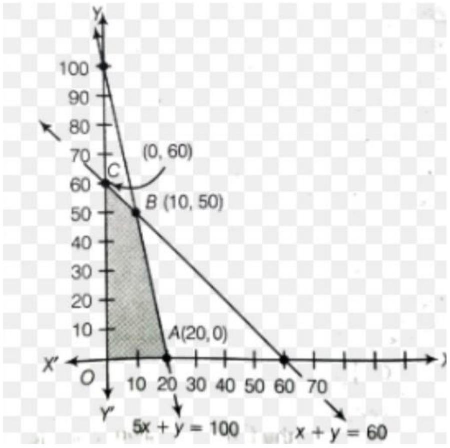
$$= -2\hat{\imath} + 4\hat{k}$$

20. $x + y \leq 60$

x	0	60
y	60	0

$5x + y \leq 100$

x	0	20
y	100	0



$Point$	$Z = 250x + 75y$
$O(0,0)$	$Z = 250(0) + 75(0) = 0$
$A(20,0)$	$Z = 250(20) + 75(0) = 5000$
$B(10,50)$	$Z = 250(10) + 75(50) = 6250 \leftarrow$ Max
$C(0,60)$	$Z = 250(0) + 75(60) = 4500$

$Z_{max} = 6250\ at\ B(10,50)$