FIRST YEAR HIGHER SECONDAY SECOND TERMINAL EXAMINATION, MARCH 2025.

ANSWER KEY

1. a) $A = \{x : n^2, n \in N, n \leq 10\}$

   b) $\{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1\}, \{2\}, \{3\}, \phi$

2. a) 240^0

   b) $sinx = -\frac{4}{5}$

   $tanx = \frac{4}{3}$

3. a) $3x - 3 \leq 2x - 6$

   $3x - 2x \leq -6 + 3$

   $x \leq -3 \Rightarrow x \in (-\infty, -3]$

   b)

4. a) No. of 4 digit numbers $= 9P_4 = 3024$

   b) $\frac{9!}{(9-5)!} = \frac{9!}{4!} = 15120$

5. a) D) $(-2, -2, -2)$

   b) Distance $= \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$

   $= \sqrt{81 + 9 + 36} = \sqrt{126} = 3\sqrt{14}$

6. $2g = -4 \Rightarrow g = -2$

   $2f = -8 \Rightarrow f = -4$

   Centre $= (-g, -f) = (2,4)$

   $r = \sqrt{g^2 + f^2 - c} = \sqrt{4 + 16 + 45} = \sqrt{65}$

7. a) $\lim_{x \to 0} \frac{tan2x}{x} = 2 \times \lim_{2x \to 0} \frac{tan2x}{2x} = 2 \times 1 = 2$

   b) $\lim_{x \to 0} \frac{1 - cosx}{x} = \lim_{x \to 0} \frac{2\sin^2\frac{x}{2}}{x} = \lim_{x \to 0} \frac{\sin^2\frac{x}{2}}{\frac{x}{2}}$

$$= \lim_{x \to 0} \frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}} \times \sin\left(\frac{x}{2}\right)$$

$$= 1 \times \sin\left(\frac{0}{2}\right) = sin0 = 0 \qquad | \, As\ x \to 0, \frac{x}{2}\ is\ also\ tend\ to\ 0$$

8. a) $x = 0$

b) Midpoint of PQ $= \left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$

$Slope = \frac{-2-0}{4-0} = -\frac{2}{4} = -\frac{1}{2}$

9 a) $\{x : x \in R, -1 < x < 2\}$

b) i) $A \cup B = \{1,3,5,6,7,8,9,10\}$

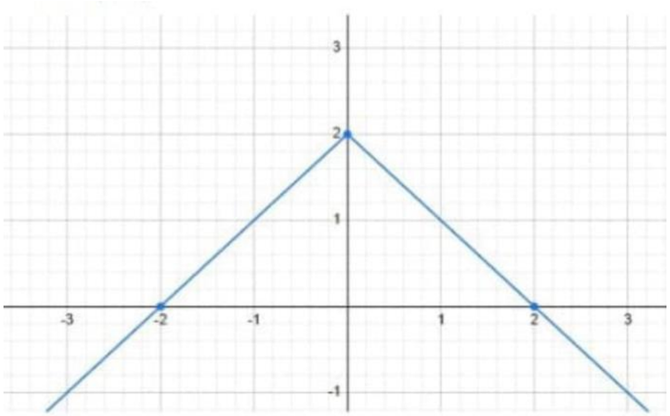
ii) $A' = \{2,4,6,8,10\}, B' = \{1,2,3,4\}$

iii) $(A \cup B)' = \{2,4\}$

$A' \cap B' = \{2,4\}$

Hence $(A \cup B)' = A' \cap B', verified.$

10 a)



b) Domain= $(-\infty, \infty)$

Range= $(-\infty, 2]$

11 a) $z = \left(3 + \frac{i}{3}\right)^3 = 27 + 3(3^2)\left(\frac{i}{3}\right) + 3(3)\left(\frac{i}{3}\right)^2 + \left(\frac{i}{3}\right)^3$

$$= 27 + 9i + 9 \times \frac{i^2}{9} + \frac{i^3}{27} = 27 + 9i + (-1) + \frac{-i}{27}$$

$$= 26 + i\left(9 - \frac{1}{27}\right) = 26 + i\left(\frac{243 - 1}{27}\right) = 26 + \frac{242}{27}i$$

b) $z = 4 - i3$

$$\frac{1}{z} = \frac{1}{4-i3} = \frac{1}{4-i3} \times \frac{4+i3}{4+i3} = \frac{4+i3}{16+9}$$

$$= \frac{4+i3}{25} = \frac{4}{25} + i\frac{3}{25}$$

12. a) No. of selections $= \frac{4C_1 \times 48C_4}{52C_5}$

b) $n = 9 + 8 = 17$

$17C_{17} = 1$

13. a) Equation is $y - y_1 = m(x - x_1)$

$$y - 3 = \frac{1}{4}(x+2) \Rightarrow 4y - 12 = x + 2 \Rightarrow x + 2 - 4y + 12 = 0$$

$x - 4y + 14 = 0$

b) Distance $= \left|\frac{3(3)-4(-5)-26}{\sqrt{3^2+(-4)^2}}\right| = \left|\frac{9+20-26}{5}\right| = \frac{3}{5}$ units.

14. $2a = 26 \Rightarrow a = 13$

$c = 5$

$c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 169 - 24 = 144$

$$Equation\ is\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{169} + \frac{y^2}{144} = 1$$

15. a) $(a+b)^4 = a^4 + 4C_1a^3b + 4C_2a^2b^2 + 4C_3ab^3 + b^4$

$(a-b)^4 = a^4 - 4C_1a^3b + 4C_2a^2b^2 - 4C_3ab^3 + b^4$

$(-) \quad (-) \quad (+) \quad (-) \quad (+) \quad (-)$

Subtracting we have,

$(a+b)^4 - (a-b)^4 = 4C_1a^3b + 4C_3ab^3 = 4a^3b + 4ab^3 = 4ab(a^2 + b^2)$

b) $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 4\left(\sqrt{3}\right)\left(\sqrt{2}\right)[3+2] = 20\sqrt{6}$

16. i) $P(not\ A) = 1 - P(A) = 1 - 0.42 = 0.58$

ii) $P(not\ B) = 1 - P(B) = 1 - 0.48 = 0.52$

iii) $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$= 0.42 + 0.48 - 0.16 = 0.74$

17. a) $sin765 = \sin(765 - 2 \times 360) = sin45 = \frac{1}{\sqrt{2}}$

b) $\cos 75 = \cos(45 + 30) = cos45cos30 - sin45sin30$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

c) $tan3x = \tan(2x + x)$

$tan3x = \dfrac{tan2x + tanx}{1 - tan2x\, tanx}$

$tan3x(1 - tan2xtanx) = tan2x + tanx$

$tan3x - tan3xtan2xtanx = tan2x + tanx$

$tan3x - tan2x - tanx = tan3xtan2xtanx$

or

$tan3xtan2xtanx = tan3x - tan2x - tanx$

Hence proved.

18 a) $a_n = 8192$

$ar^{n-1} = 8192$

$2 \times 4^{n-1} = 2 \times 4^6 \Rightarrow n - 1 = 6 \Rightarrow n = 6 + 1 = 7$

b) $S_n = 8 + 88 + 888 + \cdots to\ n\ terms$

$= 8(1 + 11 + 111 + \cdots to\ n\ terms)$

$= \frac{8}{9}[9 + 99 + 999 + \cdots + to\ n\ terms]$

$= \frac{8}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \cdots to\ n\ terms]$

$= \frac{8}{9}[10 + 100 + 1000 + \cdots to\ n\ terms - n(1)]$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] = \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

c) Let the GP is $1, a_2, a_3, a_4, 256$

$a_5 = 256 \Rightarrow ar^4 = 256 \Rightarrow r^4 = 4^4 \Rightarrow r = 4$

$a_2 = 4, a_3 = 16, a_4 = 64$

$\therefore 1, 4, 16, 64, 256\ is\ the\ required\ GP.$

19. a) $f(x) = tanx$

$f(x+h) = \tan(x+h)$

$$f(x+h) - f(x) = \tan(x+h) - tanx = \frac{\sin(x+h)}{\cos(x+h)} - \frac{sinx}{cosx}$$

$$= \frac{\sin(x+h)\,cosx - \cos(x+h)\,sinx}{\cos(x+h)\,cosx}$$

$$= \frac{\sin(x+h-x)}{\cos(x+h)\,cosx} = \frac{sinh}{\cos(x+h)\,cosx}$$

$$f'(x) = \lim_{h\to 0}\frac{1}{h}\,\frac{sinh}{\cos(x+h)\,cosx} = \lim_{h\to 0}\left[\frac{sinh}{h} \times \frac{1}{\cos(x+h)\,cosx}\right]$$

$$= 1 \times \frac{1}{\cos(x+0)\,cosx} = \frac{1}{\cos^2 x} = \sec^2 x$$

b) $y = sinxcosx$

$$\frac{dy}{dx} = sinx(-sinx) + cosx(cosx) = -\sin^2 x + \cos^2 x = \cos^2 x - \sin^2 x = cos2x$$

20.

Class	Mid value x_i	Frequency f_i	u_i	$f_i u_i$	u_i^2	$f_i u_i^2$
30 – 40	35	3	-3	-9	9	27
40 – 50	45	7	-2	-14	4	28
50 – 60	55	12	-1	-12	1	12
60 – 70	65	15	0	0	0	0
70 – 80	75	8	1	8	1	8
80 – 90	85	3	2	6	4	12
90 – 100	95	2	3	6	9	18
		$N = 50$		$\Sigma f_i u_i = -15$		$\Sigma f_i u_i^2 = 105$

i) $Mean = a + \frac{\Sigma f_i u_i}{N} \times h = 65 + \frac{-15}{50} \times 10 = 65 - 3 = 62$

ii) $Variance = \left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \times h^2 = \left[\frac{105}{50} - \left(\frac{-15}{50}\right)^2\right] \times 10^2$

$$= [2.1 - 0.09] \times 100 = 201$$

$$iii)\ Standard\ Deviation = \sqrt{Variance} = \sqrt{201} = 14.18$$

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