# HIGHER SECONDARY FIRST TERMINAL EXAMINATION - 2025

Max. Score : 60

**Second Year** Time : 2 Hrs

PART – III Cool-off Time : 15 Mts

**MATHEMATICS (SCIENCE 60)**

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**Answer any 6 questions from 1 to 8. Each carries 3 scores.**

1. i) R is transitive only

   ii) $(d, d)$ and $(d, c)$

   iii) 16

2. i) The expression $= -\frac{\pi}{4} + -\frac{\pi}{4} + \frac{\pi}{3} = -\frac{2\pi}{4} + \frac{\pi}{3} = -\frac{\pi}{2} + \frac{\pi}{3} = \frac{\pi}{6}$

3. i) 16

   ii) $\begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

   $\begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

   $2 + y = 5 \implies y = 5 - 2 = 3$

   $2x + 2 = 8 \implies 2x = 6 \implies x = 3$

4. i) $2x - 15 = 5 \implies 2x = 5 + 15 \implies 2x = 20 \implies x = 10$

   ii) Area$= 4$

   $\left| \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} \right| = 4 \implies \frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4$

   $\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 8 \implies k[0-2] - 0 + 1[8-0] = \pm 8$

   $\implies -2k + 8 = \pm 8$

   $\implies -2k + 8 = 8 \ or - 2k + 8 = -8$

   $\implies -2k = 8 - 8 \ or - 2k = -8 - 8$

   $\implies -2k = 0 \ or - 2k = -16$

   $\implies k = 0 \ ork = 8$

5. i) $|x| + 1$

   ii) Sum of a modulus and polynomial function is continuous everywhere.

   Hence, $|x| + 1$ is a continuous function.

   iii) It is not differentiable at $x = 0$

6. i) D) $f: N \rightarrow N, f(x) = x^3$

ii) $f(x) = cosx$

$g(x) = 3x^2$

$fog = f[g(x)] = f(3x^2) = \cos(3x^2)$

$gof = g[f(x)] = g(cosx) = 3(cosx)^2 = 3\cos^2 x$

$fog \neq gof$

7 i) Put $x = tan\theta$

$\tan^{-1} x = \theta$

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{tan\theta}\right) = \tan^{-1}\left(\frac{sec\theta-1}{tan\theta}\right) = \tan^{-1}\left[\frac{\frac{1-cos\theta}{cos\theta}}{\frac{sin\theta}{cos\theta}}\right]$$

$$= \tan^{-1}\left[\frac{1-cos\theta}{sin\theta}\right] = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}cos\frac{\theta}{2}}\right) = \tan^{-1}\tan\left(\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$$

8. i) $y = \sqrt{tanx}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{tanx}} \times \sec^2 x$$

ii) $x^2 + xy + y^2 = 100$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(x+2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)}$$

**Answer any 6 questions from 9 to 16. Each carries 4 scores.**

9 i) $|a-a| = |0| = 0, even\ \forall\ (a,a) \in R$

$\therefore R$ is reflexive.

$If\ |a-b|\ is\ even, then\ |b-a| = |-(a-b)| = |a-b|\ is\ also\ even.$

$i$.e., if $(a,b) \in R, then\ (b,a) \in R$

$\therefore R$ is symmetric.

$If\ |a-b|\ is\ even\ and\ |b-c|\ is\ even\ then\ |a-b+b-c| = |a-c|\ is\ also\ even.$

$i$.e., if $(a,b) \in R\ and\ (b,c) \in R\ \Longrightarrow (a,c) \in R$

$\therefore$ $R$ is transitive.

$Hence, R\ is\ an\ equivalence\ relation.$

ii) Since 5 is an odd, the other odd numbers are 1 and 3.

$\therefore$ the set elements related to 5 is $\{1,3,5\}$

10. a) iii)

b) iv)

c) i)

d) ii)

11. i) Order of AB is $3 \times 3$

ii) $AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & -12 \\ 4 & 12 & 24 \\ 5 & 15 & 30 \end{bmatrix}$

$(AB)' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & 24 & 30 \end{bmatrix}$ ............. (1)

$B'A' = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ -12 & 24 & 30 \end{bmatrix}$ ............... (2)

From (1) and (2) we have, $(AB)' = B'A'$

12. i) If a square matrix is singular, det = 0

$$\begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & 5 \end{vmatrix} = 0$$

$(2+x)[-5-2] - 3[5-2x] + 4[1--x] = 0$

$(2+x)(-7) - 3(5-2x) + 4(1+x) = 0$

$-14 - 7x - 15 + 6x + 4 + 4x = 0$

$3x - 25 = 0$

$3x = 25$

$\therefore x = \frac{25}{3}$

ii) $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 - -2 = 5$

$\text{adj } A = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

$A(\text{adj } A) = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3+2 & 1-1 \\ 6-6 & 2+3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \ldots\ldots\ldots\ldots.(1)$

$|A|.\, I = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \ldots\ldots\ldots\ldots..(2)$

From (1) and (2) we have

$A(\text{adj } A) = |A|.\, I$

13) i) $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

If A is symmetric, $A' = A$

$\begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

$2b = 3 \implies b = \frac{3}{2}$

$3a = -2 \implies a = -\frac{2}{3}$

ii) $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$7I = 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\text{LHS} = A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

=RHS

Hence proved.

14) i) $y = \sin^{-1} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}.\frac{dy}{dx} = 1$$

Diff. w.r.t. x

$$\sqrt{1-x^2}.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{-x}{\sqrt{1-x^2}} = 0$$

Xing by $\sqrt{1-x^2}$

$$(1-x^2).\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

$$(1-x^2)\frac{d^{2y}}{dx^2} = x\frac{dy}{dx}$$

Hence proved.

ii) $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$ | Put $x = \tan\theta$

$= \cos^{-1}\cos 2\theta$ | $\tan^{-1} x = \theta$

$= 2\theta$

$= 2\tan^{-1} x$

$$\therefore \frac{dy}{dx} = 2 \times \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

15) i) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\sin\left(\pi - \frac{\pi}{3}\right)$

$$= \sin^{-1}\sin\frac{\pi}{3}$$

$$= \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

ii) $\sin^{-1}\left(\frac{8}{17}\right) = \tan^{-1}\left(\frac{8}{15}\right)$

$\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

$\text{LHS} = \tan^{-1}\left(\frac{8}{15}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$$= \tan^{-1}\left[\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{5}\times\frac{3}{4}}\right]$$

$$= \tan^{-1}\left[\frac{32+45}{60-24}\right]$$

$$= \tan^{-1}\left(\frac{77}{36}\right) = \text{RHS}$$

16) i) $f(x) = \cos(x^2)$

Let $h(x) = \cos x$

And $g(x) = x^2$

$\therefore f(x) = (hog)(x) = h[g(x)]$

$= h(x^2)$

$= \cos(x^2)$

The composite function of two continuous functions is also continuous everywhere.

ii) Since f is continuous at $x = 5$

$$\lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x)$$

$$\lim_{x\to5^-}(kx+1) = \lim_{x\to5^+}(3x-5)$$

$5k + 1 = 3(5) - 5$

$5k + 1 = 3(5) - 5$

$5k = 10 - 1$

$5k = 9$

$5k = 9$

$\therefore k = \frac{9}{5}$

17. i) A) = 0

ii) The two functions $f(x) = (x-1)^2 + 1$can be made bijective by redefining its domain as $[1, \infty)$ and its codomain as $[1, \infty)$ . Thus fun be written as f: $[1, \infty) \to [1, \infty)$ by

$f(x) = (x-1)^2 + 1$.

18. i) $aij = 2i - j$

$a_{11} = 2(1) - 1 = 1$

$a_{12} = 2(1) - 2 = 0$

$a_{13} = 2(1) - 3 = -1$

$a_{21} = 2(2) - 1 = 3$

$a_{22} = 2(2) - 2 = 2$

 

$a_{23} = 2(2) - 3 = 1$

$a_{31} = 2(3) - 1 = 5$

$a_{32} = 2(3) - 2 = 4$

$a_{33} = 2(3) - 3 = 3$

$$\therefore A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$$

ii) $$A + A' = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

$$P' = \frac{1}{2}\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

$= P$, is symmetric

Let $Q = \frac{1}{2}(A - A')$

$$= \frac{1}{2}\begin{bmatrix} 1-1 & 0-3 & -1-5 \\ 3-0 & 2-2 & 1-4 \\ 5+1 & 4-1 & 3-3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix}$$

$$Q' = \frac{1}{2}\begin{bmatrix} 0 & 3 & 6 \\ -3 & 0 & 3 \\ -6 & -3 & 0 \end{bmatrix}$$

$= -\frac{1}{2}\begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix} = -Q$, skew-symmetric.

Now, $P + Q = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix}\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix}\right)$$

$= \frac{1}{2}\begin{bmatrix} 2 & 0 & -2 \\ 6 & 4 & 2 \\ 10 & 8 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix} = A$, a square matri

Hence, verified.

**Answer any 3 questions from 17 to 20. Each carries 6 scores.**

19. The system equations can be written in the form $AX = B$

Where, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$$

$A_{11} = 4 \qquad A_{21} = 2 \qquad A_{31} = 2$
$A_{12} = -5 \qquad A_{22} = 0 \qquad A_{32} = 5$
$A_{13} = 1 \qquad A_{23} = -2 \qquad A_{33} = 3$

$$adjA = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$\Longrightarrow x = 2, y = -1\ \ and\ z = 1$

20. i) $a^x loga$

ii) a) $x = acos\theta$

$$\frac{dx}{d\theta} = -asin\theta = -a\ 2sin\frac{\theta}{2}\ cos\frac{\theta}{2} = -2asin\frac{\theta}{2}\ cos\frac{\theta}{2}$$

$y = a(\theta + sin\theta)$

$$\frac{dy}{d\theta} = a(1 + cos\theta) = a \times 2\cos^2\frac{\theta}{2} = 2\,\text{a}\cos^2\frac{\theta}{2}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\Big/\frac{dx}{d\theta}$$

$$= 2\,\text{a}\cos^2\frac{\theta}{2} \times \frac{1}{-2asin\frac{\theta}{2}\ cos\frac{\theta}{2}} = -cot\frac{\theta}{2}$$

b) $y = x^x + x^{sinx}$

$Let\ u = x^x\ \ and\ v = x^{sinx}$

Then $y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \;\ldots\ldots\ldots.(1)$$

Let $u = x^x$

$$logu = xlogx$$

Diff u w.r.t. $x$

$$\frac{1}{u}\frac{du}{dx} = x\,.\frac{1}{x} + logx.1$$

$$\frac{du}{dx} = u\left[x\,.\frac{1}{x} + logx.1\right] = x^x(1 + logx)$$

Let $v = x^{sinx}$

$$logv = sinx.logx$$

Diff w.r.t. $x$

$$\frac{1}{v}\frac{dv}{dx} = sinx \times \frac{1}{x} + logx\;cosx$$

$$\frac{dv}{dx} = v\left[\frac{sinx}{x} + cosxlogx\right] = x^{sinx}\left[\frac{sinx}{x} + cosxlogx\right]$$

$$\frac{dy}{dx} = x^x(1 + logx) + x^{sinx}\left[\frac{sinx}{x} + cosxlogx\right]$$

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