भारतीय वन सेवा परीक्षा 2011

Indian Forest Services Examination ....

Sl. No. 5181 D–VSF–L–ZNB

MATHEMATICS

Paper—II

Time Allowed : Three Hours | Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

Answers must be written in ENGLISH only.

Assume suitable data, if considered necessary, and indicate the same clearly.

Symbols and notations have their usual meanings, unless indicated otherwise.

Section—A

1. Answer any *four* parts from the following :

   (a) Let G be a group, and x and y be any two elements of G. If $y^5 = e$ and $yxy^{-1} = x^2$, then show that $O(x) = 31$, where e is the identity element of G and $x \neq e$. 10

(b) Let Q be the set of all rational numbers. Show that

$$Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$$

is a field under the usual addition and multiplication. 10

(c) Determine whether

$$f(x) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}$$

is Riemann-integrable on $[0, 1]$ and justify your answer. 10

(d) Expand the function

$$f(z) = \frac{2z^2 + 11z}{(z+1)(z+4)}$$

in a Laurent's series valid for $2 < |z| < 3$. 10

(e) Write the dual of the linear programming problem (LPP) :

$$\text{Minimize } Z = 18x_1 + 9x_2 + 10x_3$$

subject to

$$x_1 + x_2 + 2x_3 \geq 30$$
$$2x_1 + x_2 \geq 15$$
$$x_1, x_2, x_3 \geq 0$$

Solve the dual graphically. Hence obtain the minimum objective function value of the above LPP. 10

2. (a) Let G be the group of non-zero complex numbers under multiplication, and let N be the set of complex numbers of absolute value 1. Show that G / N is isomorphic to the group of all positive real numbers under multiplication. 13

(b) Let the function f be defined by

$$f(x) = \frac{1}{2^t}, \quad \text{when} \quad \frac{1}{2^{t+1}} < x \leq \frac{1}{2^t}$$

$$(t = 0, 1, 2, 3, \ldots)$$

$$f(0) = 0$$

Is f integrable on $[0, 1]$? If f is integrable, then evaluate $\int_0^1 f\, dx$. 13

(c) Sketch the image of the infinite strip $1 < y < 2$ under the transformation $w = \frac{1}{z}$. 14

3. (a) Examine the convergence of

$$\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$$

and evaluate, if possible. 13

(b) Let G be a group of order $2p$, p prime. Show that either G is cyclic or G is generated by $\{a, b\}$ with relations $a^p = e = b^2$ and $bab = a^{-1}$. 13

(c) Reduce the feasible solution $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ for the linear programming problem

Maximize $Z = x_1 + 2x_2 + 3x_3$

subject to

$$x_1 - x_2 + 3x_3 = 4$$
$$2x_1 + x_2 + x_3 = 6$$
$$x_1, x_2, x_3 \geq 0$$

to a basic feasible solution. 14

4. *(a)* Evaluate

$$\iint \sqrt{4x^2 - y^2}\, dx\, dy$$

over the triangle formed by the straight lines $y = 0,\ x = 1,\ y = x$. 13

(b) State Cauchy's residue theorem. Using it, evaluate the integral

$$\int_C \frac{e^{z/2}}{(z+2)(z^2-4)}\, dz$$

counterclockwise around the circle $C : |z+1| = 4$. 13

(c) A steel company has three open-hearth furnaces and four rolling mills. Transportation costs (rupees per quintal) for shipping steel from furnaces to rolling mills are given in the following table :

	M_1	M_2	M_3	M_4	*Supply* (quintals)
F_1	29	40	60	20	7
F_2	80	40	50	70	10
F_3	50	18	80	30	18
Demand (quintals)	4	8	8	15	

Find the optimal shipping schedule. 14

Section—B

5. Answer any *four* parts from the following :

(a) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=0$$

to its canonical form and solve. 10

(b) For the data

x	:	0	1	2	5
$f(x)$	:	2	3	12	147

find the cubic function of x. 10

(c) Solve by Gauss-Jacobi method of iteration the equations

$$27x+6y-z=85$$
$$6x+15y+2z=72$$
$$x+y+54z=110$$

(correct to two decimal places) 10

(d) Find the Lagrangian for a simple pendulum and obtain the equation describing its motion. 10

(e) With usual notations, show that ϕ and ψ for a uniform flow past a stationary cylinder are given by

$$\phi = U\cos\theta\left(r+\frac{a^2}{r}\right)$$

$$\psi = U\sin\theta\left(r-\frac{a^2}{r}\right)$$ 10

6. *(a)* A uniform string of length l is held fixed between the points $x = 0$ and $x = l$. The two points of trisection are pulled aside through a distance ε on opposite sides of the equilibrium position and is released from rest at time $t = 0$.

Find the displacement of the string at any latter time $t > 0$.

What is the displacement of the string at the midpoint? 16

(b) Draw a flow chart to declare the results for the following examination system : 12

60 candidates take the examination.

Each candidate writes one major and two minor papers.

A candidate is declared to have passed in the examination if he/she gets a minimum of 40 in all the three papers separately and an average of 50 in all the three papers put together.

Remaining candidates fail in the examination with an exemption in major if they obtain 60 and above and exemption in each minor if they obtain 50 and more in that minor.

(c) Find the smallest positive root of the equation $x^3 - 6x + 4 = 0$ correct to four decimal places using Newton-Raphson method. From this root, determine the positive square root of 3 correct to four decimal places. 12

7. *(a)* For a steady Poiseuille flow through a tube of uniform circular cross-section, show that

$$w(R) = \frac{1}{4}\left(\frac{p}{\mu}\right)(a^2 - R^2)$$ 16

(b) Find the complementary function and particular integral of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$ 12

(c) The velocity of a particle at time t is as follows :

t (seconds)	:	0	2	4	6	8	10	12
v (m/sec)	:	4	6	16	36	60	94	136

Find its displacement at the 12th second and acceleration at the 2nd second. 12

8. *(a)* From a uniform sphere of radius a, a spherical sector of vertical angle 2α is removed. Find the moment of inertia of the remainder mass M about the axis of symmetry. 14

(b) Draw a flow chart to solve a quadratic equation with non-zero coefficients. The roots be classified as real distinct, real repeated and complex. 12

(c) Is

$$\vec{q} = \frac{k^2(x\hat{j} - y\hat{i})}{x^2 + y^2}$$

a possible velocity vector of an incompressible fluid motion? If so, find the stream function and velocity potential of the motion. 14

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