Indian Forest Service Examination-2012

D–GT–M–QIA

PHYSICS

Paper—I

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

*Candidates should attempt Question Nos. **1** and **5** which are compulsory, and any **THREE** of the remaining questions, selecting at least **ONE** question from each Section.*

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

*Answers must be written in **ENGLISH** only.*

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise indicated, symbolic notations carry usual meaning.

Important Note :—

All parts/sub-parts of a question must be attempted contiguously. That is, candidates must complete attempting all parts/sub-parts of a question being answered in the answer-book before moving on to the next question.

Pages left blank, if any, in the answer-book must be clearly struck out. Answers that follow pages left blank may not be given credit.

Useful Constants

Electron charge (e)	$= 1{\cdot}602\times10^{-19}$ C
Electron rest mass (m_e)	$= 9{\cdot}109\times10^{-31}$ kg
Proton mass (m_p)	$= 1{\cdot}672\times10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8{\cdot}854\times10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 1{\cdot}257\times10^{-6}$ henry/m
Velocity of light in free space (c)	$= 3\times10^{8}$ m/s
Boltzmann constant (k)	$= 1{\cdot}38\times10^{-23}$ J/K
Electron volt (eV)	$= 1{\cdot}602\times10^{-19}$ J
Planck's constant (h)	$= 6{\cdot}62\times10^{-34}$ J-s
Stefan's constant (σ)	$= 5{\cdot}67\times10^{-8}$ W m^{-2} K^{-4}
Avogadro's number (N)	$= 6{\cdot}02\times10^{26}$ $kmol^{-1}$
Gas constant (R)	$= 8{\cdot}31\times10^{3}$ J $kmol^{-1}$ K^{-1}
exp (1)	$= 2{\cdot}7183$

SECTION—A

1. Answer the following :— $8\times5=40$

   (a) Express Lagrange's equation of motion for the cyclic coordinate q_j and show that the result leads to the general conservation theorem for the generalized momentum coordinates.

   (b) Using the concept of the operator equation for a given vector quantity of a rotating body, show that

   $$\vec{v}_s = \vec{v}_r + \vec{\omega}\times\vec{r}$$

   where $\vec{v}_s$ and $\vec{v}_r$ are the velocities of the particle relative to space and rotating sets of axes while $\vec{\omega}$ is the angular velocity of the earth relative to the inertial system. Obtain the expression for the Coriolis force for such a moving system of mass m.

(Contd.)

(c) A man has a mass of 100 kg on earth. When he is on the space craft an observer from the earth registers his mass as 102 kg. Determine the speed of the space craft.

(d) Two lenses of focal lengths 8 cm and 4 cm are placed at a certain distance apart. Calculate the distance between the lenses if they form an achromatic combination.

(e) A step-index silica fiber consists of core of refractive index 1·450 and diameter 90 μm. If the numerical aperture of the fiber is 0·16, calculate :

(i) the acceptance angle of the fiber,

(ii) refractive index of the cladding, and

(iii) if the fiber is immersed in water of refractive index 1·33, how does the acceptance angle change ?

2. (a) Starting with Newton's second law of motion, establish D'Alembert's principle and discuss its significance. 8

(b) Using D'Alembert's principle, show that the following relation can be obtained for a system of particles under generalized coordinates

$$\sum_i \vec{F}_i \cdot \delta\vec{r}_i = \sum_j Q_j \, \delta q_j$$

with $Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j}$.

What is the significance of Q_j ? 16

(Contd.)

(c) State Hamilton's principle for the motion of a monogenic system.

Use the calculus of variation to deduce Lagrange's equation that follows from Hamilton's principle. Explain carefully all the terms used in the derivation. 16

3. (a) Explain mathematically the phenomenon of beats. How does it differ from that of interference ? 15

(b) When two tuning forks were sounded together, 24 beats were produced in 6 seconds. After loading one tuning fork with wax, they produced 30 beats in 6 seconds. If the unloaded fork frequency is 512, what is the frequency of the other ? 15

(c) A body describing simple harmonic motion (SHM) has a maximum acceleration of 8 πm/sec^2 and a maximum speed of 1·6 m/sec. Find the time period (T) and the amplitude (A) of SHM. 10

4. (a) Explain the Fraunhofer diffraction at a single slit and obtain the condition for minima. Derive the expression for resolving power of a grating. 15

(b) An optical waveguide has core and cladding of refractive indices 1·46 and 1·45 respectively.

(i) Calculate the maximum number of modes with wavelength λ = 900 nm that can travel if it is a planar waveguide of core thickness 10 μm.

(Contd.)

(ii) What is the number of modes if it is a cylindrical fiber of diameter 10 μm ? 10

(c) What are Einstein's A and B coefficients ? Establish a relation between them. 15

SECTION—B

5. Answer the following :— 8×5=40

(a) Show that the complex propagation constant k of an electromagnetic wave propagating in an isotropic dielectric medium with conductivity σ can be given by $k = k_r + ik_i$ with

$$k_r \approx \frac{2\pi}{\lambda_o} n\left[1+\frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]$$

and

$$k_i \approx \frac{2\pi}{\lambda_o} n\left[\frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)\right]$$

where $n = \sqrt{\varepsilon/\varepsilon_o}$ and $\lambda_o = \omega/2\pi c$; ε and c being the permittivity of the dielectric and velocity of light in free space, respectively.

(b) Explain the physical significance of the Poynting vector $\vec{S}$. What is represented by the closed integral $\oint \vec{S}\cdot d\vec{a}$ for a closed surface of area $\vec{a}$?

(Contd.)

(c) Explain the thermodynamic behaviour of an ideal Fermi gas. What are fermions ?

(d) Show that if an ideal gas is compressed isothermally its compressibility is $\frac{1}{p}$, whereas if it is compressed adiabatically its compressibility is $\frac{1}{\gamma p}$ where $\gamma = C_p/C_v$.

(e) The dielectric constant of a medium is 3. The Electric field in the dielectric is 10^6 Vm^{-1}. What are the electric displacement and polarization ?

6. (a) Explain the term 'mutual inductance' between two coils carrying current. Describe a method of determining mutual inductance between two coils of wire with relevant theory. 20

(b) A long straight solenoid has 100 turns in the secondary and 3000 turns per centimeter in the primary. The area of cross-section of the solenoid is 3 square centimeters. Calculate the mutual inductance. 15

(c) How do you justify the statement : "A current carrying conductor, although has no net charge experiences a force when placed in a magnetic field." 5

7. (a) If considered a blackbody as a radiation-filled cavity at a uniform temperature T, demonstrate with an appropriate figure the electromagnetic wave patterns inside the cavity of length L for wavelengths $\lambda = L, \frac{3}{2}L$ and 2L. 10

(b) For the above system, show that the energy per unit cavity volume in the frequency range of ν and $\nu + d\nu$ can be given by

$$u(\nu)d\nu = \frac{8\pi\nu^2 k_B T}{c^3} d\nu$$

where k_B is the Boltzmann's constant. Discuss the limitations of this formula and how did Planck put forward the correct analysis. 20

(c) Find out the number of photons in a cavity of volume 1·00 cm^3 under thermal equilibrium at temperature T = 1000 °K.

Assume that

$$\int_0^\infty \frac{x^2 dx}{e^x - 1} = 2{\cdot}405.$$

10

8. (a) Derive the gas equation when it undergoes adiabatic process. 10

(Contd.)

(b) A motor car tyre has a pressure of 2 atmospheres at the room temperature of 27°C. If the tyre suddenly bursts, find the resulting temperature. 10

(c) What are bosons ? Based on Bose-Einstein statistics, derive the expression for Bose-Einstein condensation. 15

(d) Find the efficiency of a Carnot's engine working between 127°C and 27°C. 5