IFoS 2010

Sl. No. 2666

B–JGT–K–QIA

PHYSICS

Paper—I

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

The number of marks carried by each part of a question is indicated against each.

Answers must be written in ENGLISH only.

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise indicated, symbolic notations carry usual meaning.

Useful Constants

Electron charge (e)	$= 1\cdot 602\times 10^{-19}$ C
Electron rest mass (m_e)	$= 9\cdot 109\times 10^{-31}$ kg
Proton mass (m_p)	$= 1\cdot 672\times 10^{-27}$ kg
Vacuum permittivity (ε_0)	$= 8\cdot 854\times 10^{-12}$ farad/m
Vacuum permeability (μ_0)	$= 1\cdot 257\times 10^{-6}$ henry/m
Velocity of light in free space (c)	$= 3\times 10^{8}$ m/s
Boltzmann constant (k)	$= 1\cdot 38\times 10^{-23}$ J/K
Electron volt (eV)	$= 1\cdot 602\times 10^{-19}$ J
Planck's constant (h)	$= 6\cdot 62\times 10^{-34}$ J-s
Stefan's constant (σ)	$= 5\cdot 67\times 10^{-8}$ W m^{-2} K^{-4}
Avogadro's number (N)	$= 6\cdot 02\times 10^{26}$ kmol^{-1}
Gas constant (R)	$= 8\cdot 31\times 10^{3}$ J kmol^{-1} K^{-1}
exp (1)	$= 2\cdot 7183$

Section—A

1. Answer any *four* of the following :

   (a) Establish the relation between the angular momentum and torque of a particle. Show that this relation leads one to the principle of conservation of angular momentum. 10

   (b) How can one introduce the constraints of motion through the concept of generalized coordinate systems? Write down the set of transformation equations for a system of N particles relating the generalized coordinates with the real coordinates. 10

   (c) Consider a system of two thin lenses of focal lengths $+15$ cm and -20 cm separated by a distance of 25 cm in air. Determine the system matrix. For an object of height 1 cm placed at a distance of 27·5 cm in front of the convex lens, find the size and position of the image. 10

   (d) A slit 0·25 mm wide is placed in front of a convex lens and illuminated by plane waves of wavelength 500 nm. The Fraunhofer diffraction pattern is formed in the focal plane of the lens. In the pattern, the distance from the third minimum on the left to the third minimum on the right is found to be 3 mm. Find the focal length of the lens. 10

   (e) *(i)* Between multimode and single-mode fibers, explain critically how and why single-mode fiber is chosen for communication. 5

(ii) Why are the two wavelengths 1·30 μm and 1·55 μm important in single-mode fiber-optical communication system? 5

2. (a) (i) For a particle of mass m moving with velocities $\vec{v}_s$ and $\vec{v}_r$ relative to the space and rotating axis, respectively, show that the equation of motion is obtained as

$$\vec{F} - 2m(\vec{\omega} \times \vec{v}_r) - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) = m\vec{a}_r$$

with $\vec{\omega}$ and $\vec{a}_r$ being the angular velocity and acceleration in rotating coordinates. 15

(ii) Which term in the above equation represents the Coriolis force? 5

(iii) Discuss the important role of Coriolis forces in the circulation pattern of winds. 5

(b) Show that the magnitudes of the centripetal and Coriolis accelerations of the earth while rotating counter-clockwise about the north pole are around 0·034 m sec^{-2} and $(1 \cdot 46 \times 10^{-4} v)$ m sec^{-2}, respectively. In obtaining these results, consider the ratio of sidereal days to solar days in a year as (366·5/365·5) and the radius of the earth along the equator as 6400 km. 15

3. (a) (i) An electron of rest mass m_0 moves with a velocity v such that its total energy is double of its rest mass energy. What is the electron velocity? 10

(ii) At what velocity, the electron momentum will be $m_0 c$? 5

(b) (i) What is birefringence? Indicate how it can be used to obtain plane and circularly polarized light. 15

(ii) A left circularly polarized beam of light having $\lambda_0 = 5893$ Å is incident on a calcite crystal with its optic axis cut parallel to the surface. The crystal has thickness $d = 0{\cdot}005141$ mm, $n_0 = 1{\cdot}65836$ and $n_e = 1{\cdot}48641$ at this λ_0. What will be the state of polarization of the incident beam? 10

4. (a) With a suitable diagram, deduce an expression for numerical aperture (NA) for an optical fiber having refractive indices of core and cladding n_1 and n_2, respectively and being placed in a medium of index n_0. 15

(b) What do you mean by underfilled and overfilled conditions with reference to numerical aperture in exciting light in fiber? 5+5

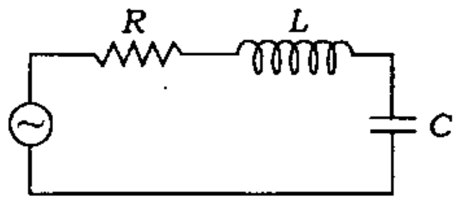
(c) Consider a bare fiber having $n_1 = 1 \cdot 48$ and $n_2 = 1$ (air). Find out the NA. What is the maximum incident angle up to which light can be guided through the fiber? 5+5

(d) Why is the NA of the single-mode optical fiber low as compared to multimode fiber? 5

Section—B

5. Answer any *four* of the following :

(a) Consider the L-C-R circuit shown below :



$R = 0{\cdot}1\ \Omega$
$L = 1$ nH
$C = 1$ nF

(i) Determine its resonance frequency f_0. 4

(ii) Determine its quality factor Q and width of resonance Δf. 6

(b) Consider a long, line charge with charge density $\rho_l = 10^{-6}$ coulomb/m. Find the force acting on a dust particle carrying -10^{-9} coulomb, 1 metre away from the line charge in free space. 10

(c) The electric field for a uniform plane wave in free space is given by $\vec{E} = (\hat{x} + \hat{y})10e^{j10z}$. Determine the corresponding magnetic field vector. 10

(d) Show that when the temperature T of a radiating object is not too different from the surrounding temperature T_0, the object obeys Newton's law of cooling. 10

(e) The following inputs are given :

Surface temperature of the sun, T_0 = 5500 K
Radius of the sun, $R = 7 \times 10^{10}$ cm
Radius of the earth, $r = 6 \cdot 4 \times 10^{8}$ cm
Distance between the sun and the earth,
$D = 1 \cdot 5 \times 10^{13}$ cm

Assume that the earth and the sun both absorb all electromagnetic radiations incident on them, and that the earth is at a constant temperature T over the day-night cycle. Calculate T. 10

6. *(a)* A spherical capacitor is made of concentric conductors of radii a and b $(b > a)$. The total charge on the inner sphere of radius a is Q.

(i) Derive an expression for the capacitance C. 8

(ii) The earth may be modeled as a spherical capacitor with $a = 6 \cdot 5 \times 10^6$ m and $b \to \infty$. Determine C, if the medium surrounding the earth is free space. 2

(b) For two isotropic media with $\mu_1 \neq \mu_2$ and $\varepsilon_1 \neq \varepsilon_2$, find an expression for the Brewster angle θ_b for parallel polarization. 10

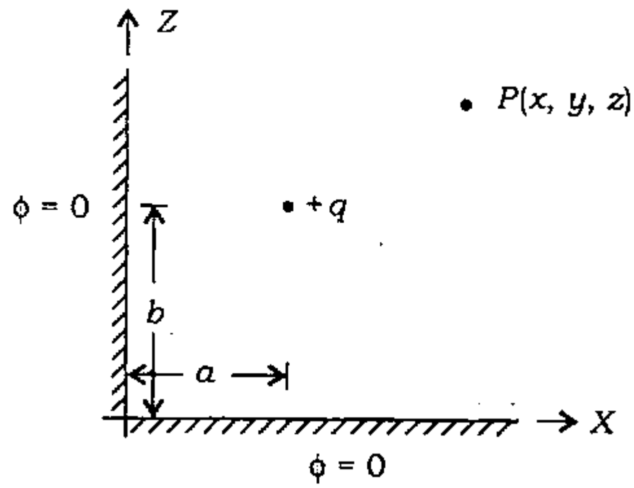
(c) The electromagnetic field inside a device is given by

$$\vec{E} = \hat{y} E_0 \sin(k_x x) e^{-jk_z z}, \quad \text{where } k_x = \frac{n\pi}{a}$$

$$\vec{H} = E_0 [\hat{x} \frac{-k_z}{\omega\mu} \sin(k_x x) + \hat{z} \frac{jk_x}{\omega\mu} \cos(k_x x)] e^{-jk_z z}$$

Obtain an expression for the z-component of time-averaged Poynting vector $\vec{S}$. 10

(d) A point charge $+q$ is located near the corner of a horizontal and a vertical plate as shown below :

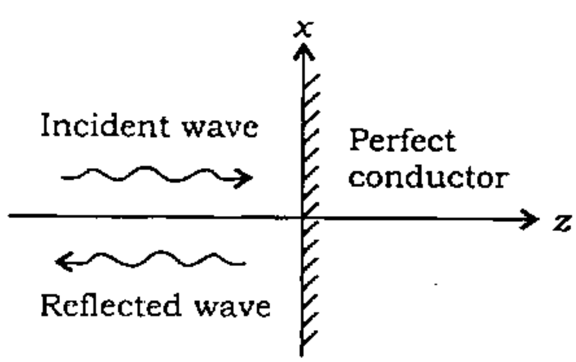


Obtain an expression for the electrostatic potential ϕ_P using the image method. 10

7. (a) A long wire of radius a carries I amperes of current. The magnetic field surrounding it is given by $H_\phi = \dfrac{I}{2\pi\rho}$ for $\rho > a$.

Obtain expressions for (i) the magnetic energy stored per unit length in the region $b \geq \rho \geq a$ and (ii) the equivalent inductance L per unit length. 8+2

(b) Consider a perfectly conducting half-space as shown below :



A uniform plane wave given by

$$\vec{E}^i = \hat{x} E_0 e^{-jkz}$$

$$\vec{H}^i = \hat{y} \frac{E_0}{\eta_0} e^{-jkz} \qquad \eta_0 = 120\pi \text{ ohm}$$

is incident normally on the boundary. Write down the expressions for the reflected electric and magnetic fields. 10

(c) N particles are distributed among three states having energies $E = 0$, $E = kT$ and $E = 2kT$. If the total equilibrium energy of the system is $1000kT$, what is the value of N? 15

(d) Explain why the distribution of speeds of molecules emerging through a small hole in an effusive molecular beam source is not a Maxwellian distribution. 5

8. *(a)* Let f be the Fermi-Dirac distribution function, then show that—

(i) $-\left(\frac{\partial f}{\partial E}\right)$ is a maximum at the Fermi level; 20

(ii) $-\left(\frac{\partial f}{\partial E}\right)$ is symmetric about the Fermi level. 5

(b) A radiation gas of temperature T fills a cavity of volume V. The system expands adiabatically and reversibly to a volume equal to $8V$. By what factor does the temperature change? 15

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