I.F.S-2009

№ 007846

B-JGT-J-QIA

PHYSICS

Paper I

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt questions no. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Answers must be written in ENGLISH only.

Assume suitable data, if necessary, and indicate the same clearly.

Unless otherwise indicated, symbolic notations carry usual meaning.

Useful Constants

Electron charge (e) = $1{\cdot}602 \times 10^{-19}$ C

Electron rest mass (m_e) = $9{\cdot}109 \times 10^{-31}$ kg

Proton mass (m_p) = $1{\cdot}672 \times 10^{-27}$ kg

Vacuum permittivity (ε_0) = $8{\cdot}854 \times 10^{-12}$ farad/m

Vacuum permeability (μ_0) = $1{\cdot}257 \times 10^{-6}$ henry/m

Velocity of light in free space (c) = 3×10^{8} m/s

Boltzmann constant (k) = $1{\cdot}38 \times 10^{-23}$ J/K

Electron volt (eV) = $1{\cdot}602 \times 10^{-19}$ J

Planck's constant (h) = $6{\cdot}62 \times 10^{-34}$ Js

Stefan's constant (σ) = $5{\cdot}67 \times 10^{-8}$ Wm^{-2} K^{-4}

Avogadro's constant (N) = $6{\cdot}02 \times 10^{26}$ $kmol^{-1}$

Gas constant (R) = $8{\cdot}31 \times 10^{3}$ J $kmol^{-1}$ K^{-1}

SECTION A

1. Answer any ***four*** questions from the following :
(Each answer must not be more than 150 words long)

4×10=40

(a) Using Euler's equations of motion for a force-free rigid body, show that the kinetic energy remains constant all along the motion of the rigid body.

(b) The vibration of a string fixed at both ends is represented by the equation

$$y = 2 \sin \frac{\pi x}{3} \cos 50\pi t \text{ metre.}$$

If the above stationary wave is produced due to the superposition of two waves of same frequency, velocity and amplitude travelling in opposite directions

$$y_1 = A \sin \frac{2\pi}{\lambda} (x - vt) \text{ and } y_2 = A \sin \frac{2\pi}{\lambda} (x + vt)$$

(i) find the equation of the component waves and

(ii) find the distance between two consecutive nodes of the stationary wave.

(c) Light rays of wavelength 5890 Å and 5896 Å fall on a grating having 5000 lines/cm. If a lens of focal length 100 cm is used to form spectra on a screen, find the distance between the lines in the third order.

(d) For a damped harmonic oscillator with equation

$$m\ddot{x} + \beta\dot{x} + kx = 0,$$

show that the work done against the damping force in an infinitesimal time 'dt' is equal to the loss of energy of the mass 'm' during the same time interval 'dt'.

(e) Explain the mechanisms of pulse dispersion in a step index fibre.

2. (a) A moving particle of charge Ze hits a fixed charge Z′e. Show that the Rutherford scattering cross-section $\sigma(\theta)$ for this phenomenon can be given by

$$\sigma(\theta) = \frac{1}{4}\left(\frac{ZZ'e^2}{2E}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

where θ is the scattering angle and E is the energy of the incident particle. *30*

(b) Discuss the significance of the impact parameter and the scattering angle in the analysis of Rutherford scattering phenomena. *10*

3. (a) Show how the momentum components of a moving particle transform under Lorentz transformation. A particle of momentum $\vec{p}_1$, rest mass m_1 is incident upon a stationary particle of rest mass m_2. Show that the velocity of the centre of mass system is given by

$$\vec{v} = \frac{\vec{p}_1 c^2}{\left(p_1^2 c^2 + m_1^2 c^4\right)^{1/2} + m_2 c^2}$$ 25

(b) When an external sinusoidal force is applied to a vibrating system, we have a situation like forced vibration. Show that in the steady state, the frequency of the forced vibration is the same as that of the external force. 15

4. (a) Describe the construction of a Michelson's interferometer. If the movable mirror is moved through a small distance d and the number of fringes that cross the field of view is n, then show that the wavelength of light is given by $\lambda = 2d/n$. 15

(b) Show that the dispersion D and the resolving power of a grating are given respectively by

(i) $D = \frac{d\theta}{d\lambda} = \frac{m}{d \cos \theta}$ and

(ii) $R = \frac{\lambda}{\Delta\lambda} = N.m,$

where d is the grating element, m is the order number and N is the total number of rulings in the grating. 10

(c) Draw the figures depicting the variation of the electric field vector with time for linearly polarized, circularly polarized and elliptically polarized light beams respectively. Describe the experimental method for detecting the state of polarization of the light beam. *15*

SECTION B

5. Answer any ***four*** questions from the following :
(Each answer must not be more than 150 words long)

4×10=40

(a) In an RC circuit a fully charged capacitor of 1 μF discharges through a resistor of 100 kΩ. After how much time will the energy stored in the capacitor become $\frac{1}{16}^{\text{th}}$ of its initial value ?

(b) Suppose a cavity of volume V contains black-body radiation in equilibrium with the walls of the cavity at a temperature T. For a reversible adiabatic change of volume show that

$$VT^3 = \text{constant}.$$

If the initial temperature is 2000 K and the volume is increased from 10 cm^3 to 1250 cm^3, reversibly and adiabatically, what would be the final temperature of the radiation ?

(c) Consider a system of two particles and three quantum states. Discuss how the particles will distribute themselves among the three states if they obey

(i) Maxwell – Boltzmann statistics

(ii) Bose – Einstein statistics

(iii) Fermi – Dirac statistics.

Ignore the spin degree of freedom.

(d) Write a short note on Saha's ionization formula and discuss its applications.

(e) Calculate the radius of an oxygen molecule, if its coefficient of viscosity at 15° C is $19{\cdot}6 \times 10^{-6}$ Pl (Pl = Poiseuille) and the mean speed of the oxygen molecules is 436 ms^{-1}.

6. (a) What is the Method of Images ? What are the necessary conditions which must be satisfied in order to apply the image method ? Show that a point charge placed at a distance above a perfect conducting plane of infinite extent can be replaced by itself, its image and an equipotential surface in place of the conducting plane. *20*

(b) State Poynting's theorem for a combined system of charges and electromagnetic fields in the form of continuity equation. Explain the physical significance of each term in the equation. *10*

(c) Determine the mutual inductance of two coplanar concentric circular loops of radii 1 m and 2 m. (Usual assumptions may be used, where required, to solve the problem). *10*

7. (a) Derive Planck's law of black-body radiation. Obtain its limiting forms for (i) very low frequencies (ii) very high frequencies. *20*

(b) Consider a system of N non-interacting bosons, occupying a volume V, at a temperature T. Derive an expression for n(E), the average number of bosons occupying the energy state E. *16*

(c) What do you understand by Bose – Einstein condensation ? *4*

8. (a) Derive an expression for the specific heat of a solid based on Einstein's theory. Obtain the limiting form of the specific heat at very low temperatures. *16*

(b) For diamond, the Einstein temperature, $\theta_E = 1450$ K. Calculate the specific heat of diamond at T = 290 K. *4*

(c) For a system of N non-interacting fermions, enclosed in a volume V, at T = 0, find an expression for (i) the internal energy U, (ii) the pressure P. *15*

(d) Metallic silver has a density of $10 \cdot 5 \times 10^3$ kg m^{-3} and its atomic weight is 107. Taking one free electron per silver atom, calculate the pressure of the electron gas in silver at T = 0. *5*