

Serial No.

1541

D-VSF-L-FGB

STATISTICS

Paper—II

Time Allowed : Three Hours

Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Questions 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

Marks allotted to each part of a question are indicated against each.

Assume suitable data, if considered necessary, and indicate the same clearly.

Answers must be written in ENGLISH only.

Unless otherwise indicated, symbols and notations have their usual meanings.

SECTION—A

**(INDUSTRIAL STATISTICS AND
OPTIMIZATION TECHNIQUES)**

1. Attempt any **FOUR** of the following :— $4 \times 10 = 40$

(a) Describe the nature of a two person zero sum game. Define the terms : (i) strategy of a player (ii) pay off and (iii) a fair game using one example.

(b) Solve the following linear programming problem :

$$\text{Maximize } Z = 2x_1 + 3x_2$$

$$\text{subject to } x_1 + 2x_2 \leq 4,$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 9,$$

$$x_1, x_2 \geq 0.$$

(c) Explain how a control chart helps to control the quality of a manufactured product. Describe the basis of a control chart. Distinguish clearly between the charts for variables and charts for attributes.

(d) What is Average Sample Number (ASN) and Average Total Inspection (ATI) ? Explain the method of their calculation for single sampling plan. Why are ASN and ATI calculated ?

- (e) Let $\{X_n, n \geq 0\}$ be a Markov chain with the state space $S = \{1, 2, 3, 4\}$ and the transition probability matrix :

$$P = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 \\ 1/3 & 0 & 0 & 2/3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (i) Classify the states and determine the period of each state.
- (ii) Find the mean recurrence time of each of the states.
2. (a) Explain the different control charts for attributes used in industrial inspection of manufactured units.

During an examination of equal length of cloth, the following are the number of defects observed :

2, 3, 4, 0, 5, 6, 7, 4, 3, 2

How will you draw a control chart for the number of defects and comment whether the process is under control or not ? 12

- (b) Define the average sample number (ASN) and the average outgoing quality (AOQ) in the case of double sampling inspection and indicate their usefulness in choosing a sampling scheme. Describe the general method of plotting OC curve of double sampling plan.

14

- (c) A firm is contemplating the introduction of three products 1, 2, 3 in its three plants A, B and C. Only a single product is decided to be introduced in each of the plants. The unit cost of producing i -th product in j -th plant is given in the following matrix of Table I :

Table I

Table II

	A	B	C	Product	Quantity (in Units)
1	8	12	—	1	2,000
2	10	6	4	2	2,000
3	7	6	6	3	10,000

- (i) How should the products be assigned so that the total unit cost is minimised ?
- (ii) If the quantity of different products to be produced is given in Table II then what assignment shall minimise the aggregate production cost ? Hence obtain the optimum total cost.

- (iii) It is expected that the selling prices of the products produced by different plants would be different as shown in Table III assuming that the quantities mentioned in Table II would be produced and sold, how should the products be assigned to the plants to obtain maximum profit? Hence obtain the maximum total profit :

	A	B	C	
	1	15	18	—
<u>Table III</u>	2	18	16	10
	3	12	10	8

3. (a) A scientific equipment manufacturing company is engaged in producing equipment for use of science laboratories. The processing time for two different assembly lines is regarded as a random variable and is described by the following distributions :

Processing time (minutes)	Assembly A_1	Assembly A_2
20	0.20	0.10
21	0.40	0.15
22	0.20	0.40
23	0.15	0.25
24	0.05	0.10

Using the following random numbers, generate data on the process times for six units of the item and compute the expected process time for the product :

3441 7674 4349 4383 8311 1519

For the purpose, read the numbers horizontally, taking the first two digits for the processing time on assembly A_1 and the last two digits for processing time on assembly A_2 . 14

(b) What is a statistical software ? Mention a few of them. Given a sample of 20 observations $\{x_1, \dots, x_{20}\}$ describe input, output and processing commands to obtain simple arithmetic average and variance of sample mean. 14

(c) A 24 hour supermarket has the following minimal requirements for cashiers :

Period	1	2	3	4	5	6
Time of day	3-7	7-11	11-15	15-19	19-23	23-3
Minimum No.	7	20	14	20	10	5

Time of a day assumes 24 hour clock. Period 1 follows immediately after period 6. A cashier works eight consecutive hours; starting at the beginning of the one of the six periods. Determine a daily employee worksheet which satisfies the requirements with the least number of personnel. Formulate the problem as a linear programming problem (L.P.P.). 12

4. (a) The arrivals at a counter in a bank occur in accordance with a Poisson process at an average rate of 8 per hour. The duration of service of a customer has exponential distribution with a mean of 6 minutes. Find the probability that an arriving customer : (i) has to wait on arrival (ii) finds 4 customers in the system and (iii) has to spend less than 15 minutes in the bank. Estimate also the fraction of total time that the counter is busy. 14

- (b) Describe the concepts of (i) a reliability of a component (ii) series system (iii) parallel system. Consider a system consisting of n components such that the failure of the i^{th} component occurs in accordance with a Poisson process of intensity λ_i .

Find the reliability of the system in series system and parallel system. 14

- (c) Show that the expected number of busy servers in an $M | M | c$ queue in steady state is $c\rho$ and that the expected number of idle servers is $c(1 - \rho)$. 12

SECTION—B
(QUANTITATIVE ECONOMICS AND
OFFICIAL STATISTICS)

5. Attempt any **FOUR** of the following :— $4 \times 10 = 40$
- (a) Define the time reversal test for index numbers. Give one example of index number which satisfies this test and one example which does not.
- (b) Define a time series. Mention its important components with illustrations and describe a method of smoothing of time series by semi averages.
- (c) What do you mean by fertility of a population ? Define (i) Crude Birth Rate (ii) General Fertility Rate and (iii) Total Fertility Rate. Discuss their relative merits and demerits as measures of fertility.

- (d) In studying a pair of related variables, a statistician summarised the data as follows :

$$n = 100; \Sigma x^2 = 1,585,000; \Sigma x = 12,500;$$
$$\Sigma xy = 1,007,425; \Sigma y^2 = 648,100; \Sigma y = 8000.$$

Find \bar{x} , \bar{y} , S_x , S_y , r and the line of regression of y on x . Verify that this line passes through the point of means.

- (e) Describe the population projection by (i) vital statistical method and by (ii) fitting mathematical curves using time series data of populations in different years.
6. (a) Explain the nature of cyclical variations in a time series. How do seasonal variations differ from them ? Give an outline of the moving average method of measuring seasonal variations. 12

- (b) Give the genesis of Logistic curve :

$$y_t = a / [1 + b e^{-ct}].$$

Explain any one method of fitting this curve to time series data regarding production in various years. 14

- (c) Write a note on the present official statistical system in India relating to population. What

do you know about the census collected in 2011 with respect to (i) collection of data by Official Statistical System (ii) processing of data in different organizations (iii) final publications about the results of Population Statistics ?

14

7. (a) Given the following table for l_x , the number of rabbits living at age x , complete the life table for rabbits :

x	0	1	2	3	4	5	6
l_x	100	90	80	75	60	30	0

Let R_1, R_2, R_3 denote the three rabbits of age 1, 2 and 3 years respectively. Find the probability that (i) at least one of them will be alive for one year more (ii) R_1, R_2, R_3 will be alive for two years time (iii) one of the three is alive in two years and (iv) all will be dead in two years time.

14

- (b) Explain briefly the concepts of reliability and validity of scores in educational and psychological experiments. Describe a method of obtaining the reliability coefficient and the validity coefficient.

12

- (c) Describe the indirect method of standardising death rates. Discuss how you would compare mortality between (i) two different communities (ii) two different periods of time (iii) two different causes of death (iv) two different occupations. 14
8. (a) Discuss briefly (i) the problems that are involved in the construction of an index number of prices. (ii) the importance and limitations of index numbers (iii) construction of the wholesale price index numbers. 12
- (b) Define (i) the autocorrelation r_k of order k and (ii) second order auto regressive series. For the series determined by $U_{t+1} = a U_t + \epsilon_{t+1}$, $|a| < 1$, where ϵ has zero mean, find the correlogram if successive values of ϵ are independent. 14
- (c) Describe the problem of multicollinearity in linear regression model with $(k > 1)$ k regressors. What are the consequences on estimation? How will you remove multi-collinearity and obtain the best linear regression equation for response variable? 14

