0278

IFoS 2010

B-JGT-K-TUB

STATISTICS
Paper II

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

Candidates should attempt questions 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

All questions carry equal marks.

Marks allotted to each part of a question are indicated against each.

Assume suitable data, if considered necessary, and indicate the same clearly.

Answers must be written in ENGLISH only.

Unless otherwise indicated, symbols and notations have their usual meanings.

SECTION A
(Industrial Statistics and Optimization Techniques)

1. Attempt any ***four*** of the following : *4×10=40*

(a) Explain briefly the dominance principle as applied to game theory.

Find the optimal strategies for both players and the value of the game whose pay-off matrix is

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 3 & 3 & 4 & -1 \\ 5 & 4 & 3 & 7 \end{bmatrix} \end{array}$$

(b) Between a single sampling plan with $n = 20$, $c = 2$ and a double sampling plan with $n_1 = 10$, $c_1 = 0$, $n_2 = 10$, $c_2 = 2$, can it be said that the second inspection scheme is more economical than the first ? Give reasons justifying your answer.

(c) A study about a population showed that the mobility of population of a state to a village, town and city is in the following percentages :

	Village	Town	City
Village	60	25	15
Town	10	70	20
City	10	30	60

What will be the proportion of population in village, town and city after one year and two years, given that the present population has proportions of 0·50, 0·40 and 0·10 in the village, town and city respectively ?

(d) An engineer approximates the reliability of a system by

$$R(t) = \begin{cases} \left(1 - \dfrac{t}{t_o}\right)^2, & 0 \le t < t_o \\ 0, & t \ge t_o \end{cases}$$

Determine the failure rate. Does the failure rate increase or decrease with time ?

(e) What is a multi-channel queuing problem ? Deduce differential-difference equations for the (M/M/c) : (∞/FCFS) queuing system.

2. (a) Distinguish between process control and product control. Justify the use of three-sigma limits in the construction of control charts.
Obtain the control limits for p-chart when the sample size is fixed for each sample. *12*

(b) Distinguish between balanced and unbalanced transportation problems. Solve the following transportation problem : *14*

	Destinations						Requirement
Ports	5	3	7	3	8	5	3
	5	6	12	5	7	11	4
	2	8	3	4	8	2	2
	9	6	10	5	10	9	8
Availability	3	3	6	2	1	2	

(c) Stating the underlying assumptions, find the expression of the probability of happening of n events in the time interval (0, t) in Poisson process. Hence or otherwise, obtain the distribution of inter-arrival times. *14*

3. (a) Discuss the role of random numbers in simulation. Explain the basic steps of Monte-Carlo simulation.

Let a six-faced homogeneous die be thrown. Let N be the generated random number with the following rules for determining the outcome of the throw :

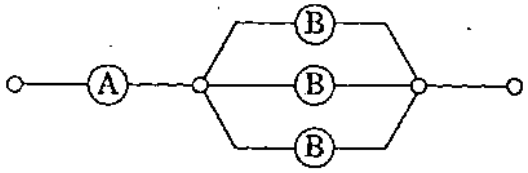
Outcome	Rule
1	$0 \leq N \leq \frac{1}{6}$
2	$\frac{1}{6} < N \leq \frac{1}{3}$
3	$\frac{1}{3} < N \leq \frac{1}{2}$
4	$\frac{1}{2} < N \leq \frac{2}{3}$
5	$\frac{2}{3} < N \leq \frac{5}{6}$
6	$\frac{5}{6} < N \leq 1$

If the 10 random numbers drawn are 058, 673, 479, 948, 613, 593, 934, 178, 347 and 564, then find the sequence of outcomes of ten throws. *13*

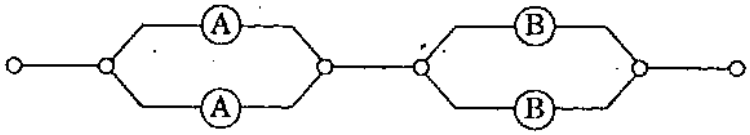
(b) What is the basic difference between deterministic and probabilistic inventory models ? Derive an expression for finding optimum inventory level in a single period stochastic model without set-up cost with continuous demand and discrete replenishment unit. *15*

(c) Explain the need of censoring in life testing giving suitable examples. In case of (i) number of failure censoring and (ii) time censoring, discuss the estimation of parameter of exponential distribution. *12*

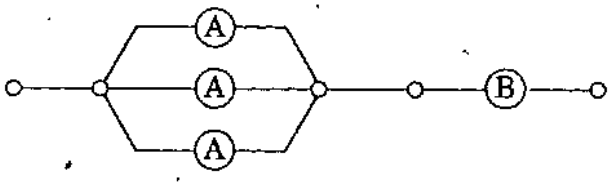
4. (a) A system S consists of two subsystems A and B, each having a reliability p. Subsystems A and B are connected in series in the system. In order to increase reliability of the system, it was decided to use four subsystems, the possible combinations being (1A, 3B), (2A, 2B), (3A, 1B). The possible configurations with these pairs are shown below. Determine which configuration has the highest reliability. *14*



(a)



(b)



(c)

(b) What is replacement problem ? Derive condition for optimal replacement policy of capital equipment that deteriorates gradually.

For a machine, the following data are available :

Year	0	1	2	3	4	5	6
Cost of spares (Rs.)	–	200	400	700	1000	1400	1600
Salary of maintenance staff (Rs.)	–	1200	1200	1400	1600	2000	2600
Losses due to breakdown (Rs.)	–	600	800	700	1000	1200	1600
Resale value (Rs.)	12000	6000	3000	1500	800	400	400

Determine the optimum period for replacement of the above machine. *14*

(c) What do you mean by transition probability matrix (T.P.M.) of a Markov chain ? Given the following T.P.M. of a Markov chain having three states 1, 2 and 3, with initial distribution

$$\pi_0 = [0{\cdot}7,\ 0{\cdot}2,\ 0{\cdot}1]$$

find the value of

$$P[X_3 = 2,\ X_2 = 3,\ X_1 = 3,\ X_0 = 2]$$

$$\begin{bmatrix} 0{\cdot}1 & 0{\cdot}5 & 0{\cdot}4 \\ 0{\cdot}6 & 0{\cdot}2 & 0{\cdot}2 \\ 0{\cdot}3 & 0{\cdot}4 & 0{\cdot}3 \end{bmatrix}$$

12

SECTION B

(Quantitative Economics and Official Statistics)

5. Attempt any ***four*** of the following : *4×10=40*

(a) Enumerate the items on which information was collected in 2001 population census of India.

(b) Describe the link relative method for measuring seasonal variations in a time series.

(c) Prepare the life table upto age 3 starting with radix l_0 = 100000 on the basis of the following values in thousands :

$L_0 = 90$, $L_3 = 50$, $d_0 = 10$, $d_1 = 8$, $d_2 = 6$, $d_3 = 5$.

(d) What is base shifting ? From the index numbers given below, find the index numbers by shifting base year from 1976 to 1979 :

Year	Index Number
1976	100
1977	110
1978	175
1979	250
1980	300

(e) What do you mean by autocorrelation ? Discuss its consequences on OLS estimators in a linear regression model.

6. (a) Show how would you study the cyclical component by harmonic analysis. For the autoregressive series

$$u_{t+2} + au_{t+1} + bu_t = \varepsilon_{t+2}$$

show that if ε_t is a random variable and the series is long then

$$\frac{\text{var } u_t}{\text{var } \varepsilon_t} = \frac{1 + b}{(1 - b)\{(1 + b)^2 - a^2\}}.$$ *14*

(b) What do you mean by stable and quasi-stable populations ? What are the basic assumptions of stable population theory ? Derive three basic equations which provide information about the intrinsic growth rate, birth rate and age distribution of the population. *14*

(c) Write a note on the process of collection, compilation and publication of trade and price statistics in India. *12*

7. (a) Starting from a suitable assumption regarding the relative growth rate of the population, derive the logistic equation. *14*

(b) What do you mean by reliability of a test ? Compare the relative merits and demerits of various methods of finding reliability. Show that when the reliability coefficient is zero, the standard error of an obtained score equals the standard deviation of the test. *14*

(c) What are index numbers ? State their uses and limitations. Show that the harmonic mean of the price relatives using current year's expenditure as weights gives the same result as obtained by Paasche's formula. *12*

8. (a) Explain the meaning of multicollinearity of the regression models with examples. Present a brief account of the methods to overcome the problem of multicollinearity. Describe a test for testing multicollinearity. *14*

(b) Explain crude and standardized death rates. In what sense is standardized death rate superior to crude death rate ? Discuss briefly the direct and indirect methods of finding standardized death rates. *14*

(c) What is the concept of factor analysis ? Mention briefly the steps for performing factor analysis and its application to Psychometry. *12*