945

STATISTICS

Paper II

| Time Allowed : Three Hours | Maximum Marks : 200 |

INSTRUCTIONS

Candidates should attempt questions 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Assume suitable data, if considered necessary, and indicate the same clearly.

Answers must be written in ENGLISH.

Unless otherwise indicated, symbols and notations have their usual meanings.

SECTION A
(Industrial Statistics and Optimization Techniques)

1. Attempt any *four* of the following :   4×10=40

   (a) Define a Markov chain.
   Obtain Chapman – Kolmogorov equation for higher order transition probabilities in a Markov chain.

(b) Define hazard rate in life testing models. When are censored observations needed in experiments ? Describe Type I and Type II censoring.

(c) For a (M/M/1) : (∞/FIFO) queueing system with arrival rate λ and service rate μ, state the expression for the probability that there are n customers in the system. Hence obtain the expressions for average number of customers in the queue and in the system.

(d) If a linear programming problem has an optimal solution, then prove that it attains its optimum at an extreme point of the convex set generated by the set of all feasible solutions to the linear programming problem.

(e) Write briefly on the SPSS software package.

2. (a) How do single and double sampling plans for attributes differ with respect to their ASN and ATI curves ? Explain the situation when a double sampling plan should be preferred to a single sampling plan. *12*

(b) Suppose that three electronic devices have a failure law given by an exponential distribution with parameters α_1, α_2 and α_3 respectively. Suppose also that these three devices function independently and are connected in parallel to form a single system.

(i) Obtain an expression for the reliability of the system.

(ii) Find the mean time to failure of the system. *14*

(c) What do you understand by the term "Sensitivity analysis" ? Discuss the changing cost coefficients in a linear programming problem without affecting the optimal solution of the problem. *14*

3. (a) Define Monte – Carlo simulation technique and mention its steps. Describe how you would generate random numbers for the probability distribution $f(x) = \lambda \exp(-\lambda x);\ x > 0$. *14*

   (b) Explain assignable and chance causes of quality variation. When is a production process said to be under statistical control ? Describe $(\overline{X}, \sigma)$ chart and mention its advantages and disadvantages over $(\overline{X}, R)$ chart. *12*

   (c) What do you mean by probabilistic inventory control models ? Deduce an expression for optimum inventory level when the demand is stochastic and is instantaneous and replenishment unit is discrete. *14*

4. (a) Describe a two-person zero sum game and in this context explain the terms (i) pay-off matrix, (ii) saddle point, (iii) mixed strategies. Let $f(i, j)$ be a real-valued function and be defined whenever $i \in A, j \in B$. Suppose both

   $\max\limits_{i}\min\limits_{j} f(i, j)$ and $\min\limits_{j}\max\limits_{i} f(i, j)$ exist. Then

   prove that a necessary and sufficient condition that $\max\limits_{i}\min\limits_{j} f(i, j) = \min\limits_{j}\max\limits_{i} f(i, j)$ is that

   function f possesses a saddle point. *14*

(b) Explain clearly individual and group replacement policies. The following mortality rates have been observed for certain type of electric bulbs :

Week	Percentage failing
1	10
2	25
3	50
4	80
5	100

The total number of bulbs is 1000. The individual replacement cost is Rs. 10 while cost for replacement of all the bulbs is Rs. 2·50 per bulb. Calculate the optimal group replacement interval. *12*

(c) Find the value of the rectangular game and optimal strategies of both the players whose pay-off matrix below is : *14*

Player B

		B_1	B_2	B_3	B_4	B_5
	A_1	9	3	1	8	0
	A_2	7	4	4	6	7
Player A	A_3	2	4	3	3	8
	A_4	5	6	2	· 2	1

SECTION B
(Quantitative Economics and Official Statistics)

5. Attempt any *four* of the following :   *4×10=40*

   (a) Explain the mathematical tests for a 'good' index number. Define Fisher's price index number and examine whether it satisfies these tests.

   (b) Discuss the consequences of multicollinearity in the data. Also suggest remedial measures to overcome the undesirable consequences.

   (c) Write a note on the present official statistical system in India relating to agriculture.

   (d) What are reproduction rates ? Explain the meaning of Gross Reproduction Rate (G.R.R.) and Net Reproduction Rate (N.R.R.)

   (e) What is the problem of scaling ? Describe z-scaling and t-scaling.

6. (a) Explain what is meant by consumer price index number and mention its uses. Outline any two methods of constructing such an index number.   *12*

   (b) What do you mean by 'seasonal index' in the analysis of a time series ? Discuss the different steps involved in the computation of the seasonal indices in the link relatives method.   *14*

   (c) Explain the life table and the meaning of various columns of a life table. Determine the values of l_{81}, l_{82} and l_{83}, given that $l_{80} = 13987$, $d_{80} = 2018$, $q_{81} = 0.15649$ and $p_{82} = 0.83042$.   *14*

7. (a) State the functions of (i) Central Statistical Organization (C.S.O.), (ii) National Sample Survey Organization (N.S.S.O.) and (iii) Labour Bureau. *12*

   (b) Discuss autocorrelation. Show that the series determined by

   $$u_{t+1} = a\, u_t + v_{t+1}, \quad |a| < 1$$

   where u_t has zero mean, has an autocorrelation given by

   (i) $\rho_k = a^k$, if the successive values of v_t are independent,

   (ii) $\rho_k =$

   $$\frac{1}{(b-a)(1+ab)} \{(b^2 - 1)\, a^{k+1} - (a^2 - 1)\, b^{k+1}\},$$

   if v_t itself obeys a relation of the form

   $v_{t+1} = b\, v_t + w_{t+1}$, $|b| < 1$, where successive values of w_t are independent. *14*

   (c) Discuss the problem of estimation of structural parameters in simultaneous equation system. For the following system of equations :

   $$C_t = \alpha + \beta Z_t + U_t$$
   $$Y_t = C_t + Z_t$$

   obtain indirect least squares estimators of the parameters. *14*

8. (a) Explain how you would fit a Gompertz curve to the data for the purpose of population projection. *14*

(b) Distinguish between T-scores and standard scores. Discuss the method of converting raw scores into T-scores. *12*

(c) Define crude and standardized death rates. How is standardized death rate superior to crude death rate ? Discuss briefly the direct and indirect methods of finding standardized death rates. *14*