1. A projectile is fired at an angle 60° with some velocity u. If the angle is changed infinitesimally, let the corresponding fractional changes in the range and the time of flight be x and y, respectively, Then y is

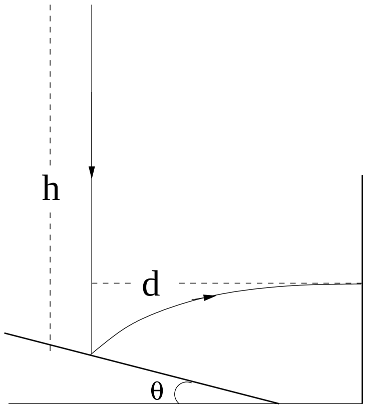
(A) $\frac{2}{3}x$

(B) $-\frac{2}{3}x$

(C) $2x$

(D) $-2x$

2. A ball is dropped down vertically from a tall building. After falling a height h it bounces elastically from a table inclined at an angle θ and hits a wall at a distance d from the point of earlier impact horizontally, then



(A) $\theta = (1/2)\ \sin^{-1}(2d/h)$

(B) $\theta = (1/2)\ \tan^{-1}(d/h)$

(C) $\theta = (1/4)\ \sin^{-1}(d/h)$

(D) $\theta = (1/2)\ \tan^{-1}(2d/h)$

3. A photon with an initial frequency 10^{11}Hz scatters off an electron at rest. Its final frequency is 0.9×10^{11} Hz. The speed of the scattered electron is close to ($h = 6.63 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg)

(A) $4 \times 10^3\,\mathrm{m\,s^{-1}}$

(B) $3 \times 10^2\,\mathrm{m\,s^{-1}}$

(C) $2 \times 10^6\,\mathrm{m\,s^{-1}}$

(D) $30\,\mathrm{m\,s^{-1}}$

Space for rough work

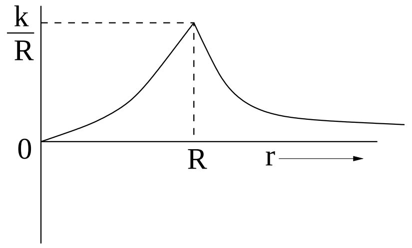
Comprehensive (4 - 5)

An α-particle experiences the following force due to a nucleus $(k > 0)$

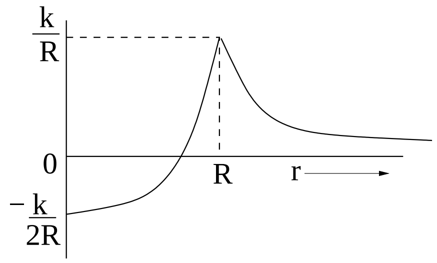
$$\vec{F} = \frac{k}{r^2}\hat{r} \quad \text{if} \quad r > R$$
$$= \frac{-k}{R^3}\vec{r} \quad \text{if} \quad r < R$$

4. The correct potential energy diagram for the above force is

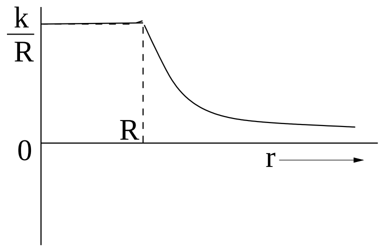
(A)



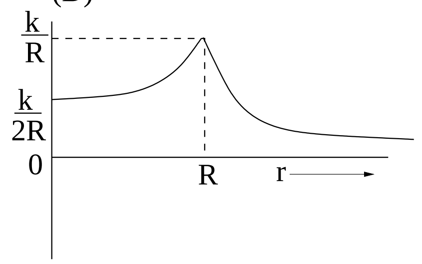
(B)



(C)



(D)



5. Suppose the particle starts from $r = \infty$ with a kinetic energy just enough to reach $r = R$. Its kinetic energy at $r = R/2$ will be

(A) k/R

(B) $(5/8)(k/R)$

(C) $(3/8)(k/R)$

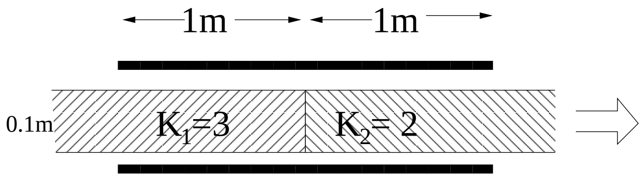
(D) 0

Space for rough work

6. Let a particle have an instantaneous position $\vec{r}(t)$, velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$. Necessary conditions for it to be considered as an instantaneous circular motion about the origin are

   (A) $\vec{r}.\vec{v} = 0$; $\vec{a}.\vec{v} = 0$; $\vec{a}.\vec{r} < 0$
   (B) $\vec{r}.\vec{v} = 0$; $\vec{a}.\vec{v} = 0$; $\vec{a}.\vec{r} > 0$
   (C) $\vec{r}.\vec{v} > 0$; $\vec{a}.\vec{v} = 0$; $\vec{a}.\vec{r} = 0$
   (D) $\vec{r}.\vec{v} = 0$; $\vec{a}.\vec{v} > 0$; $\vec{a}.\vec{r} < 0$

7. A large parallel plate capacitor is made of two metal plates of size 2m × 1m. It has a dielectric slab made of two dielectrics of permeability K_1 and K_2 as shown in the figure, the distance between them being 0.1m . It is charged by a battery of 1000V after which the battery is disconnected. Now the dielectric slab is pulled out by 10cm. The work done in doing so is (ignore the gap between the plates and the dielectric slab)



   (A) $2.5 \times 10^5 \epsilon_0$ J
   (B) $-2.5 \times 10^5 \epsilon_0$ J
   (C) $5 \times 10^5 \epsilon_0$ J
   (D) $-5 \times 10^5 \epsilon_0$ J

Space for rough work

8. A current I is flowing in a long straight wire along the z-axis. A particle with mass m and charge q has an initial position $x_0\hat{i}$ and velocity $v_0\hat{k}$. The z-component of its velocity after a very short time interval Δt is

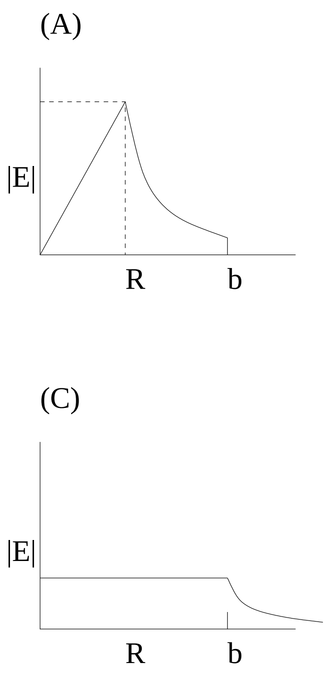
   (A) $v_z(\Delta t) = v_0[1 - \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x_0}\right)^2 \frac{q^2}{m^2}(\Delta t)^2]$

   (B) $v_z(\Delta t) = v_0[1 + \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x_0}\right)^2 \frac{q^2}{m^2}(\Delta t)^2]$

   (C) $v_z(\Delta t) = v_0[1 - \frac{3}{2}\left(\frac{\mu_0 I}{2\pi x_0}\right)^2 \frac{q^2}{m^2}(\Delta t)^2]$

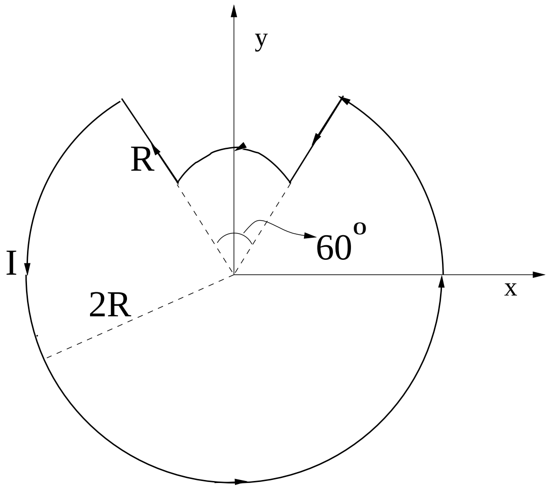
   (D) $v_z(\Delta t) = v_0$

9. A non-conducting sphere of radius R has a charge Q distributed uniformly over its volume. The sphere is surrounded by a thin metal shell of radius b ($b > R$) with a charge $-Q$. The space between the shell and the sphere is filled with air. Which of the following graphs correctly represents the corresponding electric field?



Space for rough work

10. The magnetic field at the center of a loop carrying a current I in the circuit shown is given by



(A) $\frac{\mu_0 I}{3R}\frac{7}{8}\hat{k}$

(B) $\frac{\mu_0 I}{3R}\frac{5}{8}\hat{k}$

(C) $-\frac{\mu_0 I}{3R}\frac{7}{8}\hat{k}$

(D) $-\frac{\mu_0 I}{3R}\frac{5}{8}\hat{k}$

11. A current I is flowing in a wire of length l. The total momentum carried by the charge carrier of mass m and charge q is

(A) $\frac{m}{q}\, I\, l$

(B) $\frac{2m}{q}\, I\, l$

(C) $\frac{q}{m}\, I\, l$

(D) $\frac{2q}{m}\, I\, l$

Space for rough work

12. In an oil drop experiment, charged oil drops of mass m and charge q are released at a height h, one at a time, at intervals $\Delta t > \sqrt{2h/g}$. The drops are collected in a large metal sphere of radius R with a small opening at the top. The total number of drops that are able to enter the sphere will be

(A) $\frac{m\, g\, 4\, \pi\, \epsilon_0\, (h-R)}{q^2}$

(B) $\frac{m\, g\, 4\, \pi\, \epsilon_0\, (h-R)^2}{q^2}$

(C) $\frac{m\, g\, 4\, \pi\, \epsilon_0\, (h-R)}{q}$

(D) $\frac{m\, g\, 4\, \pi\, \epsilon_0}{(h-R)q}$

13. Two lenses, one biconvex of focal length f_1 and another biconcave of focal length f_2 are placed along the same axis. They are separated by a certain distance such that a parallel beam of light incident on the convex lens also emerges parallel from the concave lens subsequently. The magnification of the combination is given by

(A) $M = {f_1}^2/{f_2}^2$

(B) $M = f_2/f_1$

(C) $M = f_1/f_2$

(D) $M = (f_1 f_2)/({f_1}^2 + {f_2}^2)$

14. The central fringe in a Young's double slit experiment with the He-Ne laser ($\lambda = 632.8$ nm) has intensity I_0. If one of the slits is covered by a 5μm thick film of plastic (refractive index = 1.4), the intensity becomes I_1. The ratio I_1/I_0 is close to

(A) 3/16

(B) 1/4

(C) 1/2

(D) 3/4

Space for rough work

15. A polarizer is introduced in the path of a beam of unpolarized light incident on a block of transparent material (refractive index $= \sqrt{3}$). The polarizer can be placed such that its axis is parallel (P) or normal (N) to the plane of incidence. The incident beam makes an angle θ with the surface of the block. The light will be completely transmitted if

 (A) $\theta = 30°$ and the polarizer is placed in P
 (B) $\theta = 30°$ and the polarizer is placed in N
 (C) $\theta = 60°$ and the polarizer is placed in P
 (D) $\theta = 60°$ and the polarizer is placed in N

16. A submarine traveling at 10 ms^{-1} is chasing another one in front of it. It locates its position and speed by sending Sonar (ultrasonic sound) towards it and recording the time of its travel and return frequency. The frequency of the Sonar is 25000 Hz and the frequency of the reflected signal is 24900 Hz. If the speed of sound in water is 1500 ms^{-1}, the speed of the submarine being chased is

 (A) 16 ms^{-1}
 (B) 14 ms^{-1}
 (C) 13 ms^{-1}
 (D) 11 ms^{-1}

17. When light of intensity I reflects from a surface separating two media with refractive index μ_1 and μ_2 ($\mu_2 > \mu_1$), the intensity of the reflected light is $(\mu_2 - \mu_1)^2/(\mu_2 + \mu_1)^2$. To make reflection zero a thin layer of a material of refractive index μ of thickness t is inserted between the two media. The value of μ and t such that wavelength of light λ is not reflected at all is

 (A) $\mu = (\mu_1 + \mu_2)/2; \quad 2\mu t = \lambda(2n+1)/2$
 (B) $\mu = (\mu_1 + \mu_2)/2; \quad 2\mu t = 2n\lambda$
 (C) $\mu = \sqrt{\mu_1 \mu_2}; \quad 2\mu t = \lambda(2n+1)/2$
 (D) $\mu = \sqrt{\mu_1 \mu_2}; \quad 2\mu t = 2n\lambda$

Space for rough work

18. A point object is placed below a wide glass plate of refractive index n. As an observer moves from left to right above the glass plate, the angle subtended by the apparent object is

(A) $2\tan^{-1}\frac{n}{n^2-1}$

(B) $\tan^{-1}\frac{n}{\sqrt{n^2-1}}$

(C) $2\tan^{-1}\frac{n}{\sqrt{n^2-1}}$

(D) $\tan^{-1}\frac{2n}{\sqrt{n^2-1}}$

19. A light sensor is fixed at one corner of the bottom of a rectangular tank of depth 10 m full of a liquid of refractive index $2/\sqrt{3}$. The illuminated area through which the light can exit at the top of the tank is

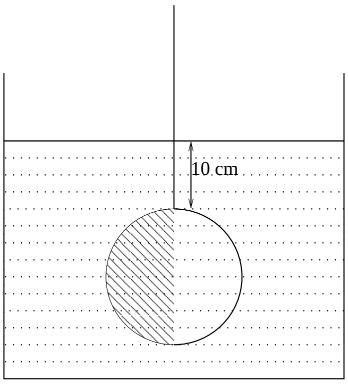
(A) a quarter of a circle of radius > 10 m

(B) a quarter of a circle of radius < 10 m

(C) a quarter of a circle of radius $= 10\sqrt{3}$ m

(D) a quarter of a circle of radius $> 10\sqrt{3}$ m

20. The average pressure on a sphere submerged in water is the pressure at the depth of its center. A sphere of radius 10 cm made of steel is held in water as shown in the figure. The force that water applies on the surface of the shaded hemisphere is (g $= 10$ms^{-2}; $\rho_{\text{water}} = 10^3$kgm^{-3})



(A) 0 N

(B) 63 N

(C) 126 N

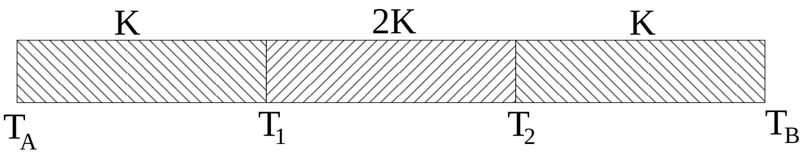
(D) 252 N

Space for rough work

21. Laplace correction to the speed of sound is made only for gases and not for solids and liquids. This is because, in comparison to gases, liquids and solids have

(A) larger thermal conductivity
(B) much smaller compressibility
(C) much smaller coefficient of thermal expansion
(D) much smaller relative pressure change when the wave is passing through them.

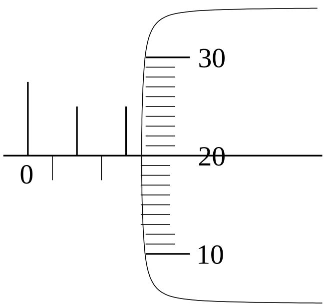
22. Three rods of equal lengths and cross sectional areas are joined as shown in the figure, with respective thermal conductivities K, $2K$and K. The left end is at a temperature T_A and the right end at a temperature T_B. In steady state, the temperatures T_1 and T_2 at the junctions are given by



(A) $T_1 = \frac{3}{5}T_A + \frac{2}{5}T_B$; $T_2 = \frac{2}{5}T_A + \frac{3}{5}T_B$
(B) $T_1 = \frac{4}{5}T_A + \frac{1}{5}T_B$; $T_2 = \frac{1}{5}T_A + \frac{4}{5}T_B$
(C) $T_1 = \frac{3}{5}T_A + \frac{2}{5}T_B$; $T_2 = \frac{1}{5}T_A + \frac{4}{5}T_B$
(D) $T_1 = \frac{4}{5}T_A + \frac{1}{5}T_B$; $T_2 = \frac{2}{5}T_A + \frac{3}{5}T_B$

Space for rough work

23. The diameter of a metal wire is measured using a screw gauge, whose circular scale has 50 divisions. Two full rotations of the circular scale move two main scale divisions of 0.5 mm each. When it is used to measure the diameter of a wire of length 3.14 m, and resistance 10 Ω, its reading is as shown in the figure. The resistivity of the wire is



(A) $4.84 \times 10^{-5} \Omega$ m

(B) $2.42 \times 10^{-5} \Omega$ m

(C) $1.21 \times 10^{-5} \Omega$ m

(D) $2.42 \times 10^{-6} \Omega$ m

24. Which of the following quantities has the least number of significant digits?

(A) 0.80760

(B) 0.08765

(C) 5.7423×10^2

(D) 80.760

25. In an experiment designed to determine the universal gravitational constant, G, the percentage errors in measuring the appropriate mass, length and time variables are given by a, b, c respectively. The total error in determining G is then

(A) $(a + 3b + 2c)$

(B) $(-a + 3b - 2c)$

(C) $(2a + 3b + 2c)$

(D) $(a + 9b + 4c)$

Space for rough work

26. The relative stability of the octahedral complexes of Fe(III) over Fe(II) with the bidentate ligands, (i) HO-CH_2-CH_2-OH, (ii) HO-CH_2-CH_2-NH_2, (iii) H_2N-CH_2-CH_2-NH_2, (iv) H_2N-CH_2-CH_2-SH, follows the order

(A) (i) > (ii) > (iii) > (iv)
(B) (ii) > (i) > (iv) > (iii)
(C) (iii) > (ii) > (iv) > (i)
(D) (iv) > (i) > (iii) > (ii)

27. Number of isomers that [Pt(Cl)(Br)(NO)(NH_3)] exhibit,

(A) 3
(B) 6
(C) 4
(D) 8

28. When a metal is in its low oxidation state, the metal-carbon bond in M-CO is stronger than the metal-chloride bond in M-Cl, because,

(A) chloride is a σ donor and carbon monoxide a π acceptor
(B) chloride is a σ acceptor and carbon monoxide a π donor
(C) chloride is a σ donor and the carbon monoxide is both a σ donor as well as a π acceptor
(D) chloride is both the σ donor as well as π acceptor and the carbon monoxide is both σ and π accepter

Space for rough work

29. Freshly prepared, bright blue coloured, dilute solution of sodium in liquid ammonia can be used to reduce the organic functional moieties. In this, the actual reducing species is,

(A) $[Na(NH_3)_n]^+$
(B) $[H_2(NH_3)_n]$
(C) $[NaNH_2(NH_3)_n]$
(D) $[e(NH_3)_n]^-$ ('e' is an electron)

30. The statement that is NOT correct in case of silicates is,

(A) cement is a silicate
(B) the Si-O bond is 50% covalent and 50% ionic
(C) silicate structures could have holes to fit cations in tetrahedral and octahedral geometries
(D) silicates are mainly built through 'SiO_2' units

31. The $(SiO_3{}^{2-})_n$ are

(A) pyrosilicates
(B) orthosilicates
(C) cyclic silicates
(D) sheet silicates

32. The oxo-acid of sulphur that DOES NOT contain S-S bond is,

(A) pyrosulphurous acid ($H_2S_2O_5$)
(B) dithionus acid ($H_2S_2O_4$)
(C) dithionic acid ($H_2S_2O_6$)
(D) pyrosulphuric acid ($H_2S_2O_7$)

Space for rough work

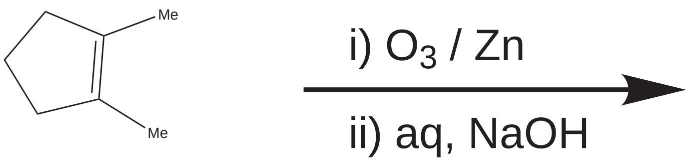
33. The reason for the formation of H^+ when $B(OH)_3$ is dissolved in water is,

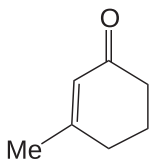
(A) acidic nature of $B(OH)_3$
(B) high polarizing power of B^{3+}
(C) hydrogen bonding between $B(OH)_3$ and water
(D) high electronegativity of oxygen

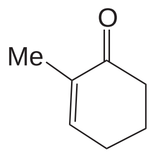
34. An optically active alcohol (X) on catalytic hydrogenation gives an optically inactive alcohol. The alcohol (X) is

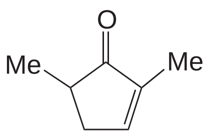
(A) 3-ethyl-3-buten-2-ol
(B) 3-methyl-3-penten-2-ol
(C) 2-ethyl-3-buten-1-ol
(D) 4-methyl-4-penten-2-ol

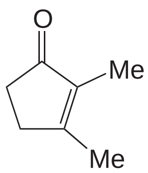
35. The major product formed in the following reaction is



(A) 

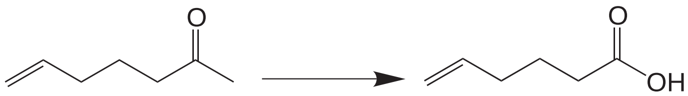
(B) 

(C) 

(D) 

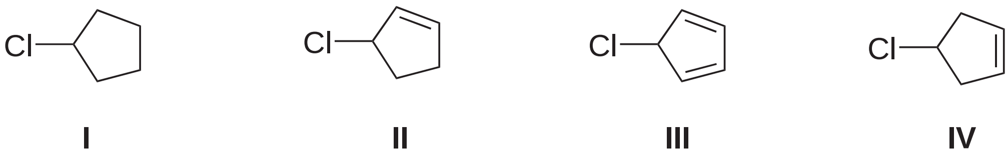
Space for rough work

36. The following transformation is effected by



(A) alkaline $KMnO_4$
(B) $NaOH/CHCl_3$
(C) $NaOH/I_2$
(D) peracetic acid

37. Among the following halides, the one that is least reactive towards solvolysis



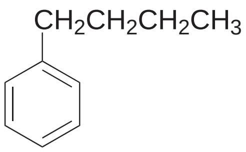
(A) I
(B) II
(C) III
(D) IV

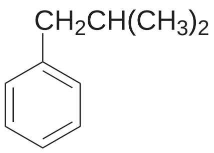
38. Isopropanol can be converted to methylacetate using

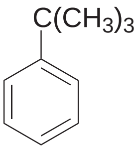
(A) pyridinium chlorochromate followed by peracetic acid
(B) peracetic acid followed by pyridinium chlorochromate
(C) pyridinium chlorochromate followed by $NaOH/I_2$
(D) $NaOH/I_2$ followed by pyridinium chlorochromate

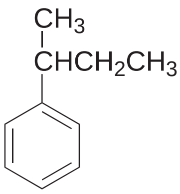
Space for rough work

39. Among the isomeric butylbenzenes, the one that is NOT oxidized by alkaline $KMnO_4$ to benzoic acid is

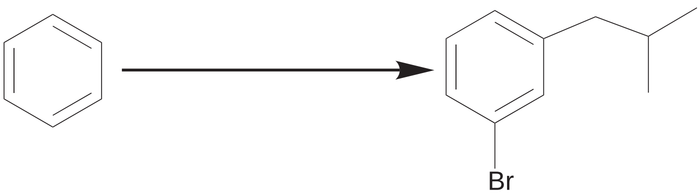
(A) 

(B) 

(C) 

(D) 

40. The following reaction is effected by



(A) i. $(CH_3)_2CHCOCl/AlCl_3$; ii. $Br_2/FeBr_3$; iii. $NH_2.NH_2/KOH$

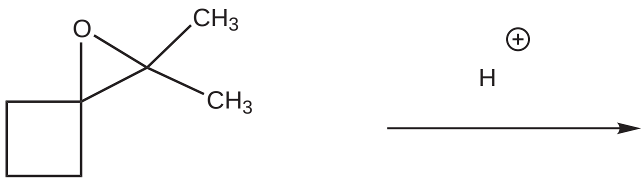
(B) i. $(CH_3)_2CH\text{-}CH_2Cl/AlCl_3$; ii. $Br_2/FeBr_3$

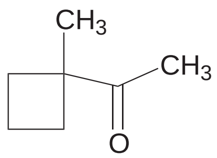
(C) i. $(CH_3)_2CHCOCl/AlCl_3$; ii. $NH_2.NH_2/KOH$; iii. $Br_2/FeBr_3$

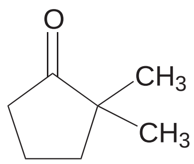
(D) i. $Br_2/FeBr_3$; ii. $(CH_3)_2CH\text{-}CH_2Cl/AlCl_3$

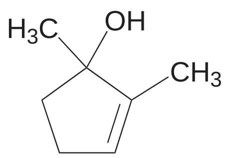
Space for rough work

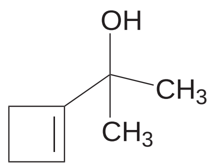
41. The major product of the following reaction is



(A) 

(B) 

(C) 

(D) 

42. Conversion of benzene into 1,3-dibromobenzene is accomplished through

(A) i. $Br_2/FeBr_3$; ii. HNO_3/conc. H_2SO_4; iii. Sn/HCl; iv. $NaNO_2$/HCl, 0-5°C ; v. CuBr
(B) i. $Br_2/FeBr_3$; ii. HNO_3/conc. H_2SO_4; iii. Sn/HCl; iv. $NaNO_2$/HCl, 0-5°C ; v. $CuBr_2$
(C) i. HNO_3/conc. H_2SO_4; ii. $Br_2/FeBr_3$; iii. Sn/HCl; iv. $NaNO_2$/HCl, 0-5°C ; v. CuBr
(D) i. HNO_3/conc. H_2SO_4; ii. $Br_2/FeBr_3$; iii. Sn/HCl; iv. $NaNO_2$/HCl, 0-5°C ; v. $CuBr_2$

Space for rough work

43. Liquid oxygen and liquid nitrogen are allowed to flow between the poles of an electromagnet. Choose the correct observation

(A) Both will be attracted but to opposing pole pieces
(B) Both will be attracted to the same pole
(C) Liquid oxygen will be attracted and liquid nitrogen will be repelled to the same degree
(D) Liquid oxygen will be attracted but liquid nitrogen unaffected

44. The highest transition energy in the Balmer series in the emission spectra of hydrogen is (R_H =109737 cm^{-1})

(A) 4389.48 cm^{-1}
(B) 2194.74 cm^{-1}
(C) 5486.85 cm^{-1}
(D) 27434.25 cm^{-1}

45. A one litre glass bulb is evacuated and weighed. The weight is 500 g. It is then filled with an ideal gas at 1 atm. pressure at 312.5 K. The weight of the filled bulb is 501.2 g. The molar weight of the gas is ($R = 8 \times 10^{-2} L.atm.K^{-1}.mol^{-1}$)

(A) 28
(B) 32
(C) 30
(D) 24

Space for rough work

46. The van der Waals coefficient of the inert gases He, Ar and Xe are given below

Inert gas	a (atm.dm^6 .mol^{-2})	b (10^{-2} dm^3 mol^{-1}
He	0.34	2.38
Ar	1.337	3.20
Xe	4.137	5.16

Choose the appropriate pair to complete the following statement.
"The increase in the value of a signifies the increasing importance of ——- interaction while increase in value in b is due to ———————-"

(A) Ion-ion; increased atomic volume
(B) Induced dipole-induced dipole; increased atomic volume
(C) Induced dipole-dipole; dipole-dipole interaction
(D) Dipole-dipole; decreasing ionization energies

47. Assuming ΔH^0 and S^0 do not change with temperature, calculate the boiling point of liquid A using the thermodynamic data given below

Thermodynamic data	$A_{(liq)}$	$A_{(gas)}$
ΔH^0 (kJ/mol)	-130	-100
S^0 (J/K/mol)	100	200

(A) 300 K
(B) 130 K
(C) 150 K
(D) 50 K

Space for rough work

48. A solution of $CaCl_2$ was prepared by dissolving 0.0112 g in 1 kg of distilled water (molar mass of Ca = 41 g mol^{-1} and Cl = 35.5 g mol^{-1}). The freezing point constant of water is 2 K.kg.mol^{-1}. The depression in the freezing point of the solution is

(A) 0.0002
(B) 0.002
(C) 0.003
(D) 0.0006

49. Of the four values of pH given below which is the closest to the pH of 0.004 M carbonic acid ($K_{a1} = 4 \times 10^{-7}$; $K_{a2} = 2 \times 10^{-12}$)

(A) 4.4
(B) 5.0
(C) 5.4
(D) 4.0

50. The Haber's process for the production of ammonia involves the equilibrium

$$N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$$

Assuming that ΔH^0 and ΔS^0 for the reaction does not change with temperature, which of the statements is true (ΔH^0 = -95 kJ and ΔS^0 = -190 J/K)

(A) Ammonia dissociates spontaneously below 500K
(B) Ammonia dissociates spontaneously above 500K
(C) Ammonia dissociates at all temperatures
(D) Ammonia does not dissociate at any temperature

Space for rough work

51. Martin throws two dice simultaneously. If the sum of the outcomes is 12, he offers lunch at a five star hotel with probability $\frac{2}{3}$. If the sum is 7, he offers lunch with probability $\frac{1}{2}$. In all the other cases, he offers lunch with probability $\frac{1}{3}$. Given that the lunch was offered, the probability that the sum of the outcomes equals 12 is

(A) $\frac{1}{18}$
(B) $\frac{1}{20}$
(C) $\frac{1}{24}$
(D) $\frac{1}{36}$

52. A species has an initial population 4^{10}. At the end of the first day, the population increases by 50%. At the end of the second day, it decreases by the same percentage. If the process continues in the same pattern, the number of days for the population to reach 3^{10} is

(A) 10
(B) 20
(C) 50
(D) 100

53. If 4 squares are chosen at random on a chessboard (there are 64 squares arranged in 8 rows and 8 columns in a chessboard), then the probability that all the four squares are in the same main diagonal is

(A) $\frac{{}^{8}C_4}{{}^{64}C_4}$
(B) $2\,\frac{{}^{8}C_4}{{}^{64}C_4}$
(C) $3\,\frac{{}^{8}C_4}{{}^{64}C_4}$
(D) $4\,\frac{{}^{8}C_4}{{}^{64}C_4}$

Space for rough work

54. A student was calculating the variance of a data that consists of ten observations. By mistake, he used one of the observations as 1 instead of 10 and found the variance as $\frac{744}{100}$ and mean as $\frac{46}{10}$. The actual variance of the data is

(A) $\frac{825}{100}$
(B) $\frac{725}{100}$
(C) $\frac{625}{100}$
(D) $\frac{525}{100}$

55. A fair coin is tossed 6 times. The probability that the head appears in the sixth trial for the third time is

(A) $\frac{5}{16}$
(B) $\frac{5}{32}$
(C) $\frac{5}{36}$
(D) $\frac{3}{64}$

56. The sum of the roots of the equation $x + 1 - 2\log_2(2^x + 3) + 2\log_4(10 - 2^{-x}) = 0$ is

(A) $\log_2 11$
(B) $\log_2 12$
(C) $\log_2 13$
(D) $\log_2 14$

Space for rough work

57. Let $z = a(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5})$, $a \in \mathbb{R}$, $|a| < 1$. Then $z^{2010} + z^{2011} + \cdots$ equals

(A) $\dfrac{z^{2010}}{1-a}$

(B) $\dfrac{a^{2010}}{1-a}$

(C) $\dfrac{za^{2010}}{1-z}$

(D) $\dfrac{a^{2010}}{1-z}$

58. The locus of the point z satisfying $\arg(z+1) = \alpha$ and $\arg(z-1) = \beta$, when $\alpha, \beta \in (0, \pi)$ vary subject to the condition $\dfrac{1}{\tan\alpha} - \dfrac{1}{\tan\beta} = 2$, is

(A) two parallel lines

(B) a single point

(C) a line parallel to the x-axis

(D) two intersecting lines

59. For the equation, $\sin x + \cos x = \dfrac{1}{\sqrt{2}}\left(a + \dfrac{1}{a}\right)$, $a > 0$,

(A) there is no solution, for any $a > 0$

(B) there is a solution, for infinitely many $a > 0$

(C) there is a solution, for two or more, but finitely many $a > 0$

(D) there is a solution, for exactly one $a > 0$

Space for rough work

60. The number of solutions of the equation $\sin x + \cos x = 1 - \sin 2x$ in the interval $[-2\pi, 3\pi]$ is

(A) 2

(B) 4

(C) 6

(D) 8

61. Consider the circles $C_1 : x^2 + y^2 = 64$ and C_2 with radius 10. If the center of C_2 lies on the line $y = x$ and C_2 intersects C_1 such that the length of the common chord is 16, then the center of the circle C_2 is

(A) $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

(B) $\left(\frac{4}{\sqrt{2}}, \frac{4}{\sqrt{2}}\right)$

(C) $\left(\frac{6}{\sqrt{2}}, \frac{6}{\sqrt{2}}\right)$

(D) $\left(\frac{8}{\sqrt{2}}, \frac{8}{\sqrt{2}}\right)$

62. A line segment joining $(1, 0, 1)$ and the origin $(0, 0, 0)$ is revolved about the x-axis to form a right circular cone. If (x, y, z) is any point on the cone, other than the origin, then it satisfies the equation

(A) $x^2 - 2y^2 - z^2 = 0$

(B) $x^2 - y^2 - z^2 = 0$

(C) $2x^2 - y^2 - 2z^2 = 0$

(D) $x^2 - 2y^2 - 2z^2 = 0$

Space for rough work

63. Let (x, y, z) be any point on the line passing through $(x_0, y_0, 0)$ and parallel to the vector $\vec{i} + \vec{j} + \vec{k}$. If $x_0 + y_0 = 2$, then (x, y, z) lies on the plane normal to the vector

(A) $\vec{i} + \vec{j} - 2\vec{k}$

(B) $\vec{i} + \vec{j} + 2\vec{k}$

(C) $\vec{i} + \vec{j} - \vec{k}$

(D) $\vec{i} + \vec{j} + \vec{k}$

64. A tangent to the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ meets the coordinate axes at A and B. If the points A, B and the origin are vertices of an isosceles triangle, then the length of AB is

(A) $\sqrt{41/2}$

(B) $\sqrt{41}$

(C) $\sqrt{82}$

(D) $\sqrt{164}$

65. Let $a_n = \dfrac{1}{n}[(2n+1)(2n+2)\cdots(2n+n)]^{1/n}$, for $n \in \mathbb{N}$ and $\lim_{n\to\infty} a_n = e^L$, then L is

(A) $\displaystyle\int_2^3 \log x \, dx$

(B) $\displaystyle\int_2^3 \log 2x \, dx$

(C) $\displaystyle\int_2^3 \log(1+x) \, dx$

(D) $\displaystyle\int_2^3 \log(2+x) \, dx$

Space for rough work

66. The value of $\lim_{n\to\infty} \dfrac{1^3 + 2^3 + \cdots + (3n)^3}{3n^4}$ is

(A) $1/2$
(B) $9/4$
(C) $27/4$
(D) $81/4$

67. The value of $\int_0^{\pi/2} \dfrac{2 + \sin x}{1 + \cos x} e^{x/2}\, dx$ is

(A) $e^{\pi/4}$
(B) $e^{\pi/2}$
(C) $2e^{\pi/2}$
(D) $2e^{\pi/4}$

68. The differential equation satisfied by the family of curves which cut the curves $y = \alpha x^3$, $\alpha \in \mathbb{R}$ at right angles is

(A) $xy' - 3y = 0$
(B) $x + 3yy' = 0$
(C) $y' - 3\alpha x^2 = 0$
(D) $3\alpha x^2 y' + 1 = 0$

Space for rough work

69. Let $f(x) = x(|x - \pi|)(2 + \cos^2 x),\ x \in \mathbb{R}$. Then the function $f : \mathbb{R} \to \mathbb{R}$ is

(A) one-one but NOT onto

(B) onto but NOT one-one

(C) both one-one and onto

(D) neither one-one nor onto

70. The equation $2x^3 - 3x^2 + p = 0$ has three distinct real roots for all p belonging to

(A) $(0, 1)$

(B) $(2, \infty)$

(C) $(-\infty, 1/2)$

(D) $(-\infty, 0) \cup (1, \infty)$

71. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let $f : [1/2, \infty) \to \mathbb{R}$ be defined by $f(x) = (x - [x])^{[x]} \cos \dfrac{\pi x}{2}$. Then f is

(A) continuous at $x = 1$ but NOT continuous at $x = 2$

(B) continuous at $x = 2$ but NOT continuous at $x = 1$

(C) continuous at both $x = 1$ and $x = 2$

(D) discontinuous at $x = 1$ and $x = 2$

Space for rough work

72. Let $f : (0, \infty) \to \mathbb{R}$ be defined by $f(x) = 2x^{\sin 2x} \cos 2x$. Then $\lim_{x \to 0} f(x)$ is

(A) 1
(B) 2
(C) e
(D) $2e$

73. The distance of the point $(1, 2, 3)$ from the plane $\vec{r} \cdot (2\vec{i} + \vec{j} + 2\vec{k}) = -5$ measured parallel to the line $\vec{r} = (-3\vec{i} + 2\vec{j}) + \lambda(\vec{i} + \vec{j} + \vec{k})$ is

(A) $3\sqrt{3}$
(B) $5\sqrt{2}$
(C) $3\sqrt{5}$
(D) $5\sqrt{6}$

74. If the vector $3\vec{i} + 4\vec{j} + 7\vec{k} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{i} - \vec{j} + \vec{k}$ and $\vec{v}_2$ is perpendicular to $2\vec{i} - \vec{k}$, then $|\vec{v}_1|^2 + |\vec{v}_2|^2$ is

(A) 68
(B) 70
(C) 88
(D) 92

Space for rough work

75. A plane H passes through the intersection of the planes $\vec{r}\cdot(\vec{i}+\vec{j}+\vec{k})=-3$ and $\vec{r}\cdot(\vec{i}-\vec{j}+\vec{k})=2$. If H divides the line segment joining $(3,0,2)$ and $(0,3,-1)$ in the ratio $2:1$ internally, then the equation of H is

(A) $\vec{r}\cdot(3\vec{i}-\vec{j}+3\vec{k})=1$

(B) $\vec{r}\cdot(\vec{i}+\vec{j}+3\vec{k})=3$

(C) $\vec{r}\cdot(5\vec{i}-3\vec{j}+\vec{k})=-1$

(D) $\vec{r}\cdot(2\vec{i}+\vec{j}+3\vec{k})=4$

Space for rough work

Space for rough work

Space for rough work