Lines and Angles

Fundamental concepts of Geometry:

Point: It is an exact location. It is a fine dot which has neither length nor breadth nor thickness but has position i.e., it has no magnitude.

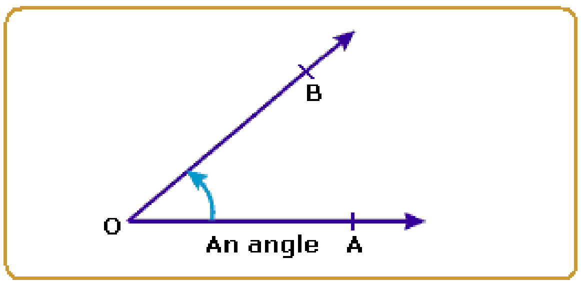
Line segment: The straight path joining two points A and B is called a line segment $\overline{AB}$. It has and points and a definite length.

Ray: A line segment which can be extended in only one direction is called a ray.

Intersecting lines: Two lines having a common point are called intersecting lines. The common point is known as the point of intersection.

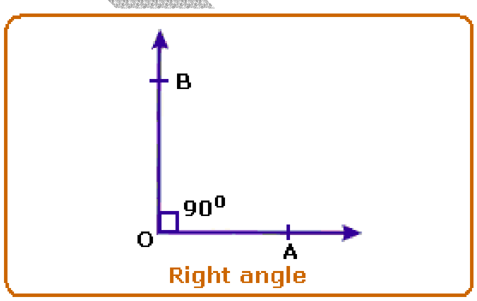
Concurrent lines: If two or more lines intersect at the same point, then they are known as concurrent lines.

Angles: When two straight lines meet at a point they form an angle.

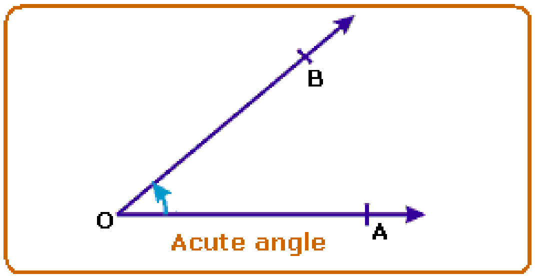


In the figure above, the angle is represented as ∠AOB. OA and OB are the arms of ∠AOB. Point O is the vertex of ∠AOB. The amount of turning from one arm (OA) to other (OB) is called the measure of the angle (ÐAOB).

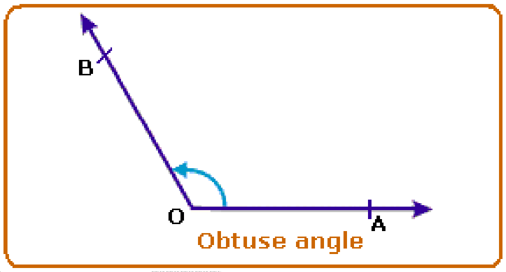
Right angle: An angle whose measure is 90° is called a right angle.



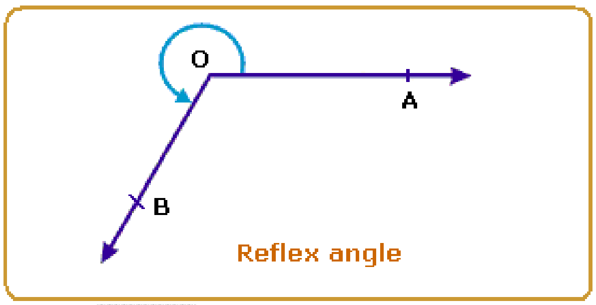
Acute angle: An angle whose measure is less then one right angle (i.e., less than 90°), is called an acute angle.



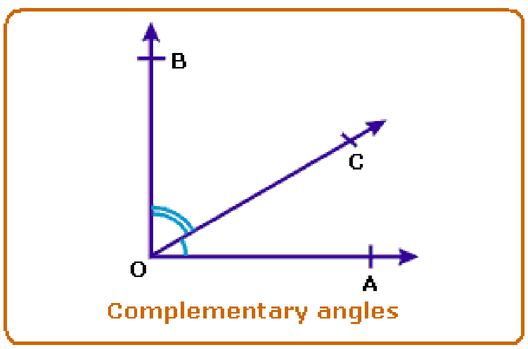
Obtuse angle: An angle whose measure is more than one right angle and less than two right angles (i.e., less than 180° and more than 90°) is called an obtuse angle.



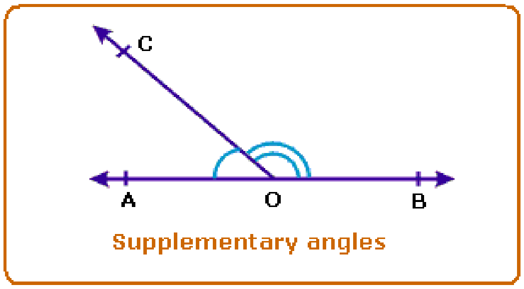
Reflex angle: An angle whose measure is more than 180° and less than 360° is called a reflex angle.



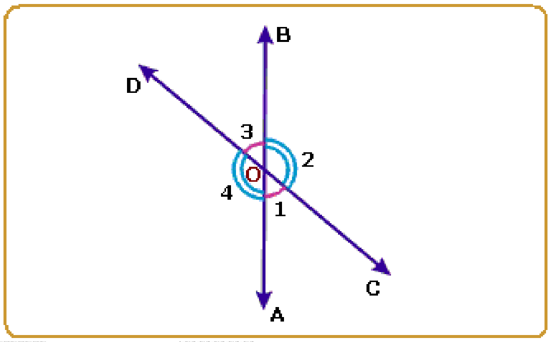
Complementary angles: If the sum of the two angles is one right angle (i.e., 90°), they are called complementary angles. Therefore, the complement of an angle θ is equal to $90° - \theta$.



Supplementary angles: Two angles are said to be supplementary, if the sum of their measures is 180°. **Example:** Angles measuring 130° and 50° are supplementary angles. Two supplementary angles are the supplement of each other. Therefore, the supplement of an angle θ is equal to $180° - \theta$.



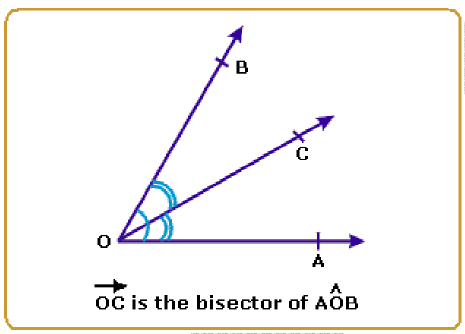
Vertically opposite angles: When two straight lines intersect each other at a point, the pairs of opposite angles so formed are called vertically opposite angles.



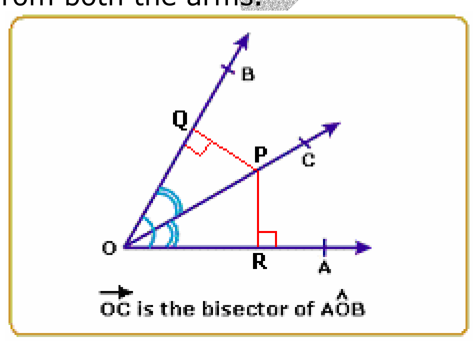
In the above figure, ∠1 and ∠3 and angles ∠2 and ∠4 are vertically opposite angles.

Note: Vertically opposite angles are always equal.

Bisector of an angle: If a ray or a straight line passing through the vertex of that angle, divides the angle into two angles of equal measurement, then that line is known as the Bisector of that angle.



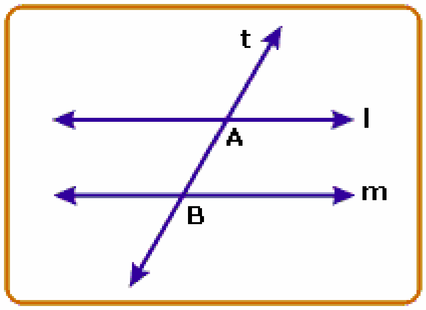
A point on an angle is equidistant from both the arms.



In the figure above, Q and R are the feet of perpendiculars drawn from P to OB and OA. It follows that PQ = PR.

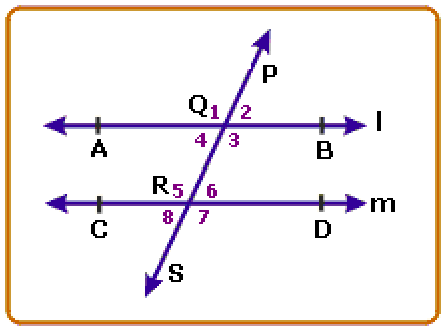
Parallel lines: Two lines are parallel if they are coplanar and they do not intersect each other even if they are extended on either side.

Transversal: A transversal is a line that intersects (or cuts) two or more coplanar lines at distinct points.



In the above figure, a transversal t is intersecting two parallel lines, l and m, at A and B, respectively.

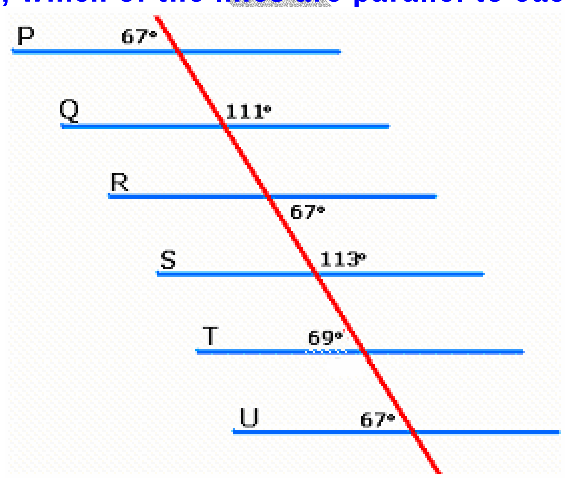
Angles formed by a transversal of two parallel lines:



In the above figure, l and m are two parallel lines intersected by a transversal PS. The following properties of the angles can be observed:

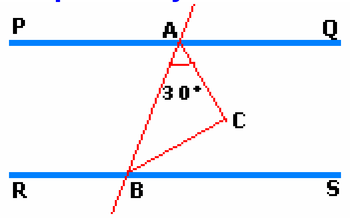
$\angle 3 = \angle 5$ and $\angle 4 = \angle 6$ [**Alternate angles**]
$\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 4 = \angle 8$, $\angle 3 = \angle 7$ [**Corresponding angles**]
$\angle 4 + \angle 5 = \angle 3 + \angle 6 = 180°$ [**Supplementary angles**]

In the figure given below, which of the lines are parallel to each other?



Answer: As 67° + 113° = 180°, lines P and S, R and S, and S and U are parallel. Therefore, lines P, R, S and U are parallel to each other. Similarly, lines Q and T are parallel to each other.

In the figure given below, PQ and RS are two parallel lines and AB is a transversal. AC and BC are angle bisectors of ∠BAQ and ∠ABS, respectively. If ∠BAC = 30°, find ∠ABC and ∠ACB.

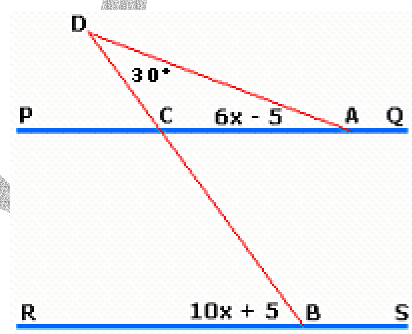


Answer: ∠BAQ + ∠ABS = 180° [Supplementary angles]

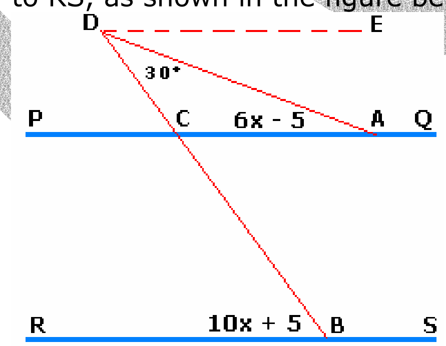
$\Rightarrow \frac{\angle BAQ}{2} + \frac{\angle ABS}{2} = \frac{180^\circ}{2} = 90^\circ \Rightarrow \angle BAC + \angle ABC = 90^\circ$

Therefore, ∠ABC = 60° and ∠ACB = 90°.

For what values of x in the figure given below are the lines P-A-Q and R–B-S parallel, given that AD and BD intersect at D?

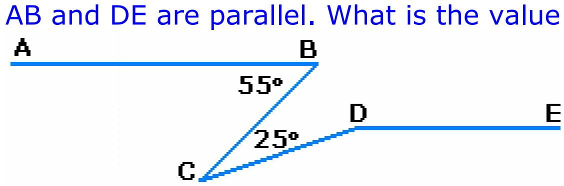


Answer: We draw a line DE, parallel to RS, as shown in the figure below:

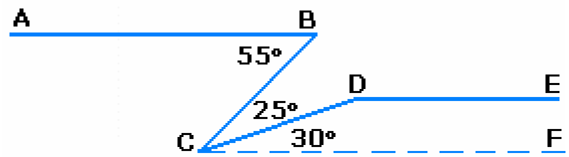


In the above figure, ∠CDE = ∠RBD = 10x + 5 ⇒ ∠CDA = 10x + 5 –30 = 10x – 25.
Let the line PQ and RS be parallel. Therefore, PQ // DE.
⇒ ∠EDA = ∠CAD = 10x – 25 = 6x – 5 ⇒ x = 5.

In the figure given below, lines AB and DE are parallel. What is the value of ∠CDE?



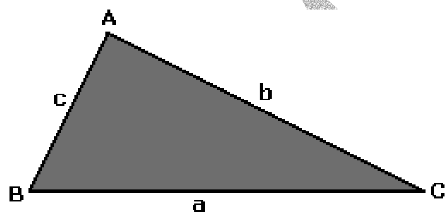
Answer: We draw a line CF // DE at C, as shown in the figure below.



∠BCF = ∠ABC = 55° ⇒ ∠DCF = 30°.
⇒ CDE = 180° – 30° = 150°.

TRIANGLES

Triangles are closed figures containing three angles and three sides.

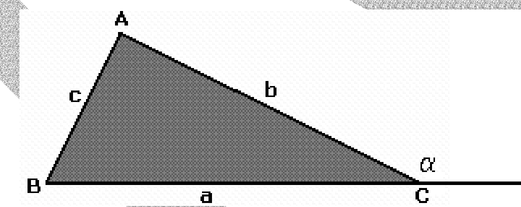


General Properties of Triangles:

1. The sum of the two sides is greater than the third side: a + b > c, a + c > b, b + c > a

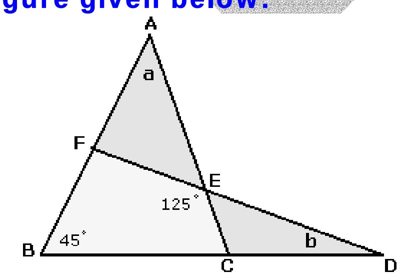
The two sides of a triangle are 12 cm and 7 cm. If the third side is an integer, find the sum of all the values of the third side.
Answer: Let the third side be of x cm. Then, x + 7 > 12 or x > 5. Therefore, minimum value of x is 6. Also, x < 12 + 7 or x < 19. Therefore, the highest value of x is 18. The sum of all the integer values from 6 to 18 is equal to 156.

2. The sum of the three angles of a triangle is equal to 180°: In the triangle below ∠A + ∠B + ∠C = 180°

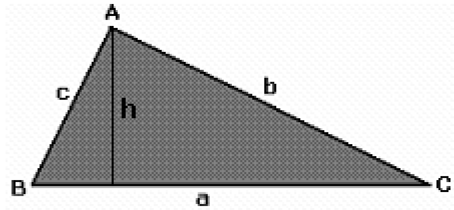


Also, the exterior angle α is equal to sum the two opposite interior angle A and B, i.e. α = ∠A + ∠B.

Find the value of a + b in the figure given below:



Answer: In the above figure, ∠CED = 180° – 125° = 55°. ∠ACD is the exterior angle of ΔABC. Therefore, ∠ACD = a + 45°. In ΔCED, a + 45° + 55° + b = 180° ⇒ a + b = 80°



3. **Area of a Triangle:**

Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times a \times h$

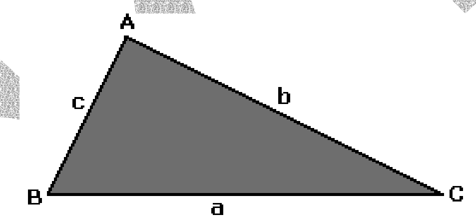
Area of a triangle $= \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B$

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

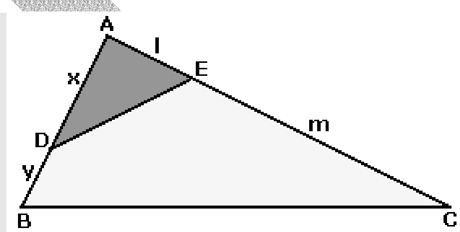
Area of a triangle $= \frac{abc}{4R}$ where R = circumradius

Area of a triangle $= r \times s$ where r = inradius and $s = \frac{a+b+c}{2}$

4. **More Rules:**



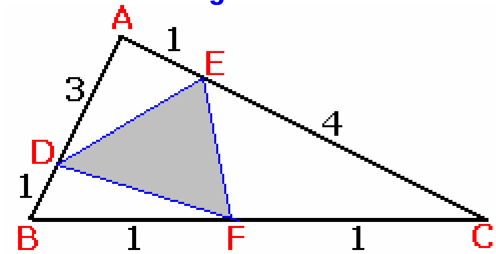
- Sine Rule: $\frac{\text{Sin}A}{a} = \frac{\text{Sin}B}{b} = \frac{\text{Sin}C}{c}$
- Cosine Rule: $\text{Cos}A = \frac{b^2 + c^2 - a^2}{2bc}$, $\text{Cos}B = \frac{a^2 + c^2 - b^2}{2ac}$, $\text{Cos}C = \frac{b^2 + a^2 - c^2}{2ab}$



Let D and E be on sides AB and AC of triangle ABC such that $\frac{AD}{DB} = \frac{x}{y}$ and $\frac{AE}{EC} = \frac{l}{m}$. Then, area triangle ADE = $\frac{1}{2} lx \sin A$ and area triangle ABC = $\frac{1}{2}(l+m)(x+y) \sin A$ .Therefore,

$$\frac{\text{Area } \Delta ADE}{\text{Area } \Delta ABC} = \frac{lx}{(x+y)(l+m)}$$

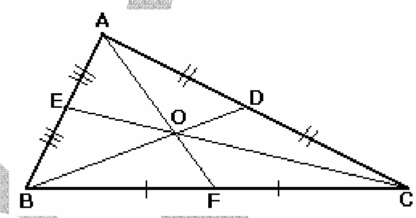
Points D, E and F divide the sides of triangle ABC in the ratio 1: 3, 1: 4, and 1: 1, as shown in the figure. What fraction of the area of triangle ABC is the area of triangle DEF?



Answer: $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta ABC} = \frac{1 \times 3}{4 \times 5} = \frac{3}{20}$, $\frac{\text{Area } \Delta BDF}{\text{Area } \Delta ABC} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$, $\frac{\text{Area } \Delta CFE}{\text{Area } \Delta ABC} = \frac{4 \times 1}{5 \times 2} = \frac{2}{5}$

Therefore, $\frac{\text{Area } \Delta DEF}{\text{Area } \Delta ABC} = 1 - \left(\frac{3}{20} + \frac{1}{8} + \frac{2}{5}\right) = \frac{13}{40}$

5. **Medians of a triangle:**

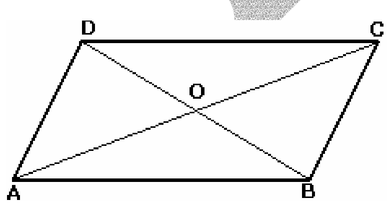


The medians of a triangle are lines joining a vertex to the midpoint of the opposite side. In the figure, AF, BD and CE are medians. The point where the three medians intersect is known as the **centroid**. O is the centroid in the figure.

- The medians divide the triangle into two equal areas. In the figure, area ΔABF = area ΔAFC = area ΔBDC = area ΔBDA = area ΔCBE = area ΔCEA = $\frac{\text{Area } \Delta ABC}{2}$
- The centroid divides a median internally in the ratio 2: 1. In the figure, $\frac{AO}{OF} = \frac{BO}{OD} = \frac{CO}{OE} = 2$
- Apollonius Theorem: $AB^2 + AC^2 = 2(AF^2 + BF^2)$ or $BC^2 + BA^2 = 2(BD^2 + DC^2)$ or $BC^2 + AC^2 = 2(EC^2 + AE^2)$ REMEMBER!

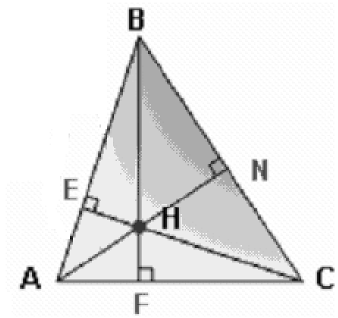
ABCD is a parallelogram with AB = 21 cm, BC = 13 cm and BD= 14 cm. Find the length of AC.

Answer: The figure is shown below. Let AC and BD intersect at O. O bisects AC and BD. Therefore, OD is the median in triangle ADC.



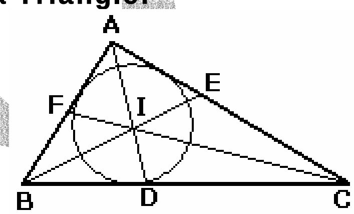
$\Rightarrow AD^2 + CD^2 = 2(AO^2 + DO^2) \Rightarrow AO = 16$. Therefore, AC = 32.

6. **Altitudes of a Triangle:**



The altitudes are the perpendiculars dropped from a vertex to the opposite side. In the figure, AN, BF, and CE are the altitudes, and their point of intersection, H, is known as the orthocenter.
Triangle ACE is a right-angled triangle. Therefore, $\angle ECA = 90° - \angle A$. Similarly in triangle CAN, $\angle CAN = 90° - \angle C$. In triangle AHC, $\angle CHA = 180° - (\angle HAC + \angle HCA) = 180° - (90° - \angle A + 90° - \angle C)$
$= \angle A + \angle C = 180° - \angle B$.
Therefore, ∠AHC and ∠B are supplementary angles.

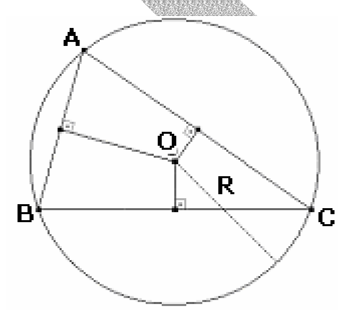
7. **Internal Angle Bisectors of a Triangle:**



In the figure above, AD, BE and CF are the internal angle bisectors of triangle ABC. The point of intersection of these angle bisectors, I, is known as the incentre of the triangle ABC, i.e. centre of the circle touching all the sides of a triangle.

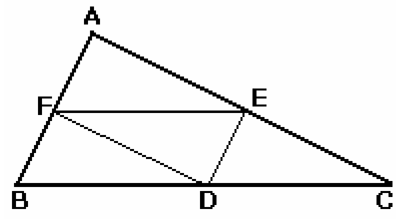
- $\angle BIC = 180° - (\angle IBC + \angle ICB) = 180 - \left(\frac{B}{2} + \frac{C}{2}\right) = 180 - \left(\frac{B+C}{2}\right) = 180 - \left(\frac{180 - A}{2}\right) = 90 + \frac{A}{2}$
- $\frac{AB}{AC} = \frac{BD}{CD}$ (internal bisector theorem) REMEMBER!

8. **Perpendicular Side Bisectors of a Triangle:**



In the figure above, the perpendicular bisectors of the sides AB, BC and CA of triangle ABC meet at O, the circumcentre (centre of the circle passing through the three vertices) of triangle ABC. In figure above, O is the centre of the circle and BC is a chord. Therefore, the angle subtended at the centre by BC will be twice the angle subtended anywhere else in the same segment.
Therefore, $\angle BOC = 2\angle BAC$.

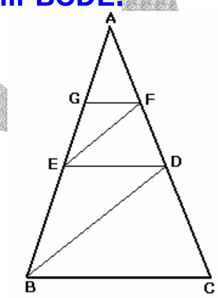
9. **Line Joining the Midpoints:**



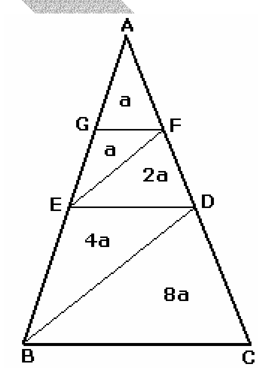
In the figure above, D, E and F are midpoints of the sides of triangle ABC. It can be proved that:

- FE // BC, DE // AB and DF // AC.
- $FE = \frac{BC}{2}$, $DE = \frac{AB}{2}$, $FD = \frac{AC}{2}$
- Area ΔDEF = Area ΔAFE = Area ΔBDF = Area ΔDEC = $\frac{\text{Area } \Delta ABC}{4}$
- **Corollary:** If a line is parallel to the base and passes through midpoint of one side, it will pass through the midpoint of the other side also.

In the figure given below: AG = GE and GF / / ED, EF / /BD and ED / / BC. Find the ratio of the area of triangle EFG to trapezium BCDE.

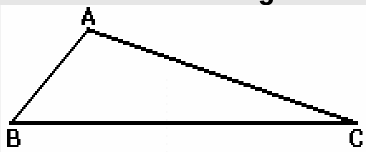
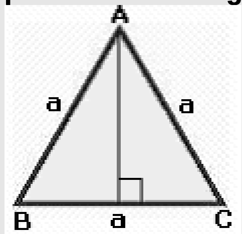
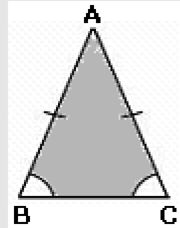
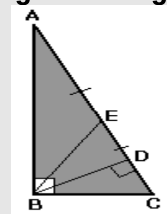


Answer: We know that line parallel to the base and passing through one midpoint passes through another midpoint also. Using this principle, we can see that G, F, E and D are midpoints of AE, AD, AB, and AC respectively. **Therefore, GF, EF, ED, and BD are medians in triangles AFE, AED, ADB and ABC.**

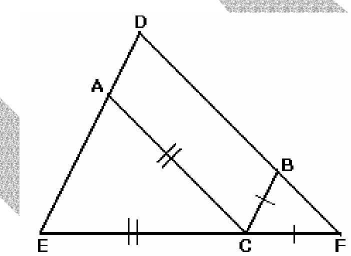


We know that medians divide the triangle into two equal areas. Let the area of triangle AGF = a. Therefore, the areas of the rest of the figures are as shown above.
The required ratio = a/12a = 1/12.

Types of triangles:

Scalene Triangle	Equilateral Triangle	Isosceles Triangle	Right Triangle
 No side equal. All the general properties of triangle apply	 Each side equal Each angle = 60° Length of altitude $= \frac{\sqrt{3}}{2}a$ Area $= \frac{\sqrt{3}}{4}a^2$ Inradius $= \frac{a}{2\sqrt{3}}$ Circumradius $= \frac{a}{\sqrt{3}}$	 Two sides equal. The angles opposite to the opposite sides are equal.	 One of the angles is a right angle, i.e. 90°. Area $= \frac{1}{2} AB \times BC$ $AC^2 = AB^2 + BC^2$ Altitude BD $= \frac{AB \times BC}{AC}$ The midpoint of the hypotenuse is equidistant from all the three vertices, i.e. EA = EB = EC

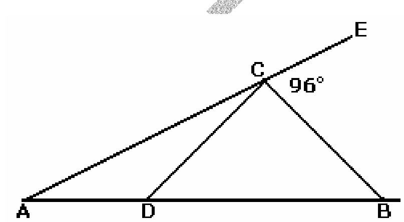
In triangle DEF shown below, points A, B, and C are taken on DE, DF and EF respectively such that EC = AC and CF = BC. If angle D = 40° then what is angle ACB in degrees? (CAT 2001)



1. 140 2. 70 3. 100 4. None of these

Answer: Let ∠AEC = ∠EAC = α and ∠CBF = ∠CFB = β. We know that α + β = 180° – ∠D = 140°. ∠ACB = 180° – (∠ECA + ∠BCF) = 180° – (180° – 2α + 180° – 2β) = 100°.

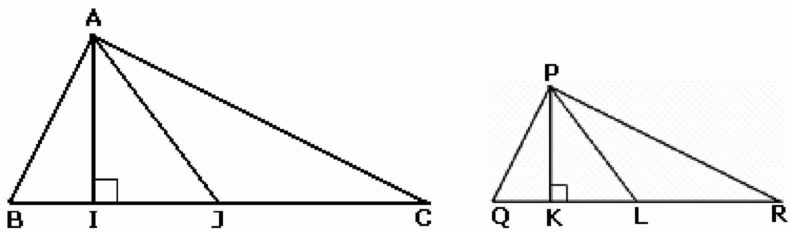
In the figure (not drawn to scale) given below, if AD = CD = BC, and ∠BCE = 96°, how much is ∠DBC? (CAT 2003)



1. 32° 2. 84° 3. 64° 4. Cannot be determined.

Answer: Let ∠DAC = ∠ACD = α and ∠CDB = ∠CBD = β. As ∠CDB is the exterior angle of triangle ACD, β = 2α. Now ∠ACD + ∠DCB + 96° = 180° ⇒ α + 180° – 2β + 96° = 180° ⇒ 3α = 96° ⇒ α = 32° ⇒ β = 64°

Similarity of triangles:

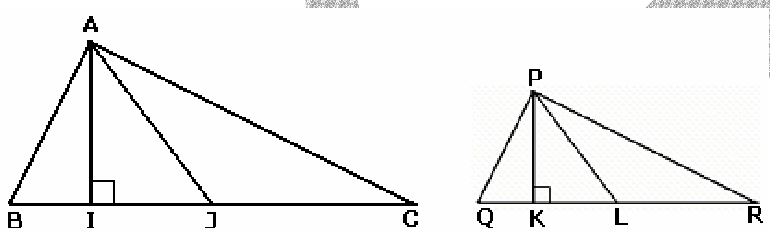


Two triangles are similar if their corresponding angles are equal or corresponding sides are in proportion. In the figure given above, triangle ABC is similar to triangle PQR. Then ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R and

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AI}{PK}\text{(altitudes)} = \frac{AJ}{PL}\text{(medians)}$$

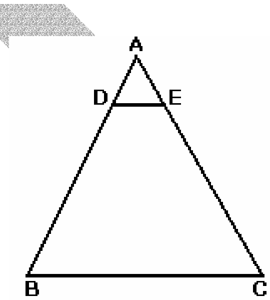
Therefore, if you need to prove two triangles similar, prove their corresponding angles to be equal or their corresponding sides to be in proportion.

Ratio of Areas:



If two triangles are similar, the ratio of their areas is the ratio of the squares of the length of their corresponding sides. Therefore,

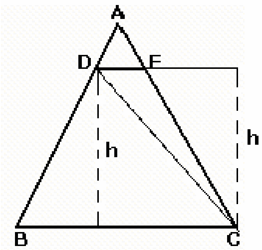
$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle PQR}} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$



In triangle AC, shown above, DE / / BC and $\frac{DE}{BC} = \frac{1}{4}$. If area of triangle ADE is 10, find the area of the trapezium BCED and the area of the triangle CED.

Answer: ΔADE and ΔABC are similar. Therefore,

$\frac{\text{Area of triangle ABC}}{\text{Area of triangle ADE}} = \frac{BC^2}{DE^2} = 16 \Rightarrow \text{Area of triangle ABC} = 160 \Rightarrow$ Area of trapezium BCDE = Area ΔABC – Area ΔADE = 160 – 10 = 150.



To find the area of triangle CDE, we draw altitudes of triangle BDC and CDE, as shown above. Let the length of the altitudes be h.

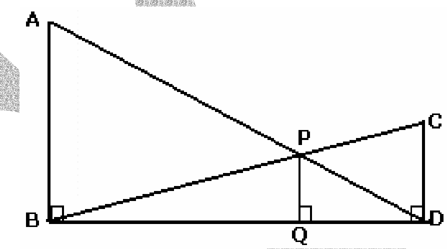
Area of triangle BCD $= \frac{1}{2} \times BC \times h$ and area of triangle CDE $= \frac{1}{2} \times DE \times h$.

$\Rightarrow \frac{\text{Area of triangle BCD}}{\text{Area of triangle CDE}} = \frac{BC}{DE} = 4$

Therefore, we divide the area of the trapezium BCED in the ratio 1: 4 to find the area of triangle CDE.

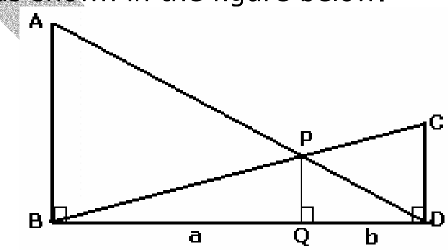
The required area = $= \frac{1}{5} \times 150 = 30$.

In the diagram given below, ∠ABD = ∠CDB = ∠PQD = 90°. If AB: CD = 3: 1, the ratio of CD: PQ is (CAT 2003- Leaked)



1. 1: 0.69 2. 1: 0.75 3. 1: 0.72 4. None of these

Answer: Let BQ = a and DQ = b, as shown in the figure below.

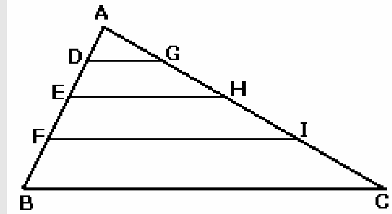


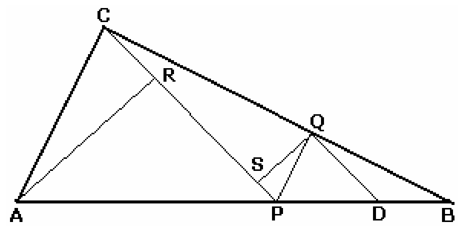
Triangle ABD and triangle PQD are similar. Therefore, $\frac{PQ}{AB} = \frac{b}{a+b}$. Also triangle CBD and triangle PBQ are similar. Therefore, $\frac{PQ}{CD} = \frac{a}{a+b}$

Dividing the second equality by the first, we get. $\frac{AB}{CD} = \frac{a}{b} = 3$. Therefore, $\frac{CD}{PQ} = \frac{a+b}{a} = \frac{4}{3} = 1 : 0.75$

In triangle ABC, lines DG, EH, and FI are parallel to the base BC. Then it can be proved that

$$\frac{AD}{DE} = \frac{AG}{GH}, \frac{DE}{EF} = \frac{GH}{HI}, \frac{EF}{FB} = \frac{HI}{IC}$$





In the figure (not drawn to scale) given below, P is a point on AB such that AP: PB = 4: 3. PQ is parallel to AC and QD is parallel to CP. In ΔARC, ∠ARC = 90°, and in ΔPQS, D PSQ = 90°. The length of QS is 6 cm. What is ratio AP: PD? (CAT 2003)

1. 10: 3 2. 2: 1 3. 7: 3 4. 8: 3

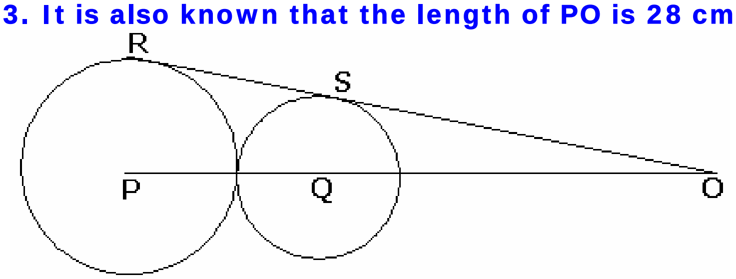
Answer: PQ is parallel to AC $\Rightarrow \frac{AP}{PB} = \frac{CQ}{QB} = \frac{4}{3}$.

Let AP = 4x and PB = 3x.

QD is parallel CP $\Rightarrow \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3} \Rightarrow PD = \frac{4PB}{7} = \frac{12x}{7}$

$\Rightarrow$ AP: PD = 4x: $\frac{12x}{7}$ = 7: 3

In the adjoining figure, I and II are circles with centres P and Q respectively. The two circles touch each other and have a common tangent that touches them at points Rand S respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio 4: 3. It is also known that the length of PO is 28 cm. (CAT 2004)



What is the ratio of the length of PQ to that of QO?

1. 1 : 4 2. 1 : 3 3. 3 : 8 4. 3 : 4

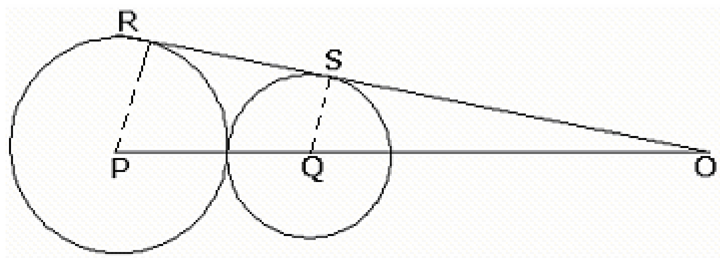
What is the radius of the circle II?

1. 2 cm 2. 3 cm 3. 4 cm 4. 5cm

The length of SO is

1. $8\sqrt{3}$ cm 2. $10\sqrt{3}$ cm 3. $12\sqrt{3}$ cm 4. $14\sqrt{3}$ cm

Answer:

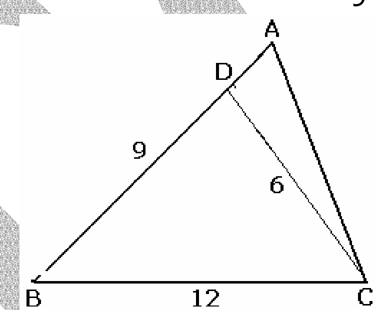


Join R and P, and S and Q. ∠PRO = ∠QSO = 90°. Therefore, ΔPRO and ΔQSO are similar. Therefore, $\frac{PR}{SQ} = \frac{PO}{QO} \Rightarrow QO = \frac{3}{4} \times PO = 21 \Rightarrow PQ = \frac{1}{4} \times PO = 7 \Rightarrow PQ : QO = 1 : 3$. Also, PQ = 7 and the radii are in the ratio 4: 3. Therefore, radius of circle II = 3. Now $SO = \sqrt{QO^2 - SQ^2} = \sqrt{441 - 9} = \sqrt{432} = 12\sqrt{3}$

NOTE: In similar triangles, sides opposite to equal angles are in proportion.

Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and ∠BCD = ∠BAC. What is the ratio of the perimeter of the triangle ADC to that of the triangle BDC? (CAT 2005)

1. $\frac{7}{9}$ 2. $\frac{8}{9}$ 3. $\frac{6}{9}$ 4. $\frac{5}{9}$



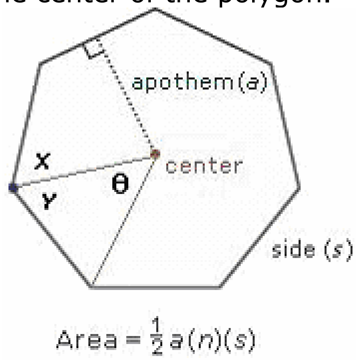
Answer: In ΔBAC and ΔBCD, ∠BCD = ∠BAC, ∠B is common ⇒ ∠BDC = ∠BCA. Therefore, the two triangles are similar.

$\frac{AB}{BC} = \frac{AC}{CD} = \frac{BC}{BD} \Rightarrow AB = \frac{BC^2}{BD} = 16 \Rightarrow AD = 7$, Similarly, $AC = \frac{BC \times CD}{BD} = 8$

Perimeter ΔADC = 7 + 6 + 8 = 21, perimeter ΔBDC = 27. Therefore, ratio = $\frac{7}{9}$

REGULAR POLYGONS

A regular polygon is a polygon with all its sides equal and all its interior angles equal. All vertices of a regular lie on a circle whose center is the center of the polygon.



Each side of a regular polygon subtends an angle $\theta = \frac{360}{n}$ at the centre, as shown in the figure.

Also $X = Y = \frac{180 - \frac{360}{n}}{2} = \frac{180(n-2)}{2n}$. Therefore, interior angle of a regular polygon = $X + Y = \frac{180(n-2)}{n}$.

Sum of all the angles of a regular polygon = $n \times \frac{180(n-2)}{n} = 180(n-2)$.

What is the interior angle of a regular octagon?

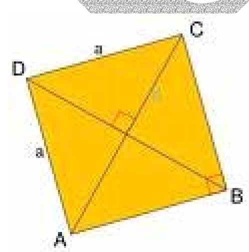
Answer: The interior angle of a regular octagon = $\frac{180(8-2)}{8} = 135^{\circ}$

NOTE! The formula for sum of all the angles of a regular polygon, i.e. $180(n-2)$, is true for all n-sided convex simple polygons.

Let's look at some polygons, especially quadrilaterals:

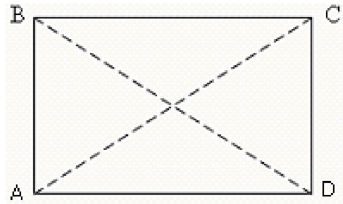
Quadrilateral: A *quadrilateral* is any closed shape that has four sides. The sum of the measures of the angles is *360°*. Some of the known quadrilaterals are square, rectangle, trapezium, parallelogram and rhombus.

Square: A square is regular quadrilateral that has four right angles and parallel sides. The sides of a square meet at right angles. The diagonals also bisect each other perpendicularly.



If the side of the square is a, then its perimeter = 4a, area = a^2 and the length of the diagonal = $\sqrt{2}a$

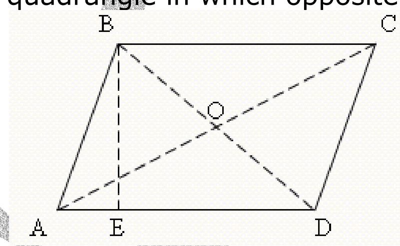
Rectangle: A rectangle is a parallelogram with all its angles equal to right angles.



Properties of a rectangle:

Sides of rectangle are its heights simultaneously.
Diagonals of a rectangle are equal: AC = BD.
A square of a diagonal length is equal to a sum of squares of its sides' lengths, i.e. $AC^2 = AD^2 + DC^2$.
Area of a rectangle = length × breadth

Parallelogram: A parallelogram is a quadrangle in which opposite sides are equal and parallel.

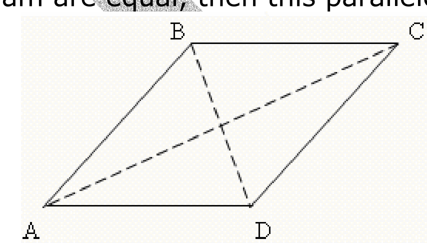


Any two opposite sides of a parallelogram are called *bases*, a distance between them is called a *height*.
Area of a parallelogram = base × height
Perimeter = 2(sum of two consecutive sides)

Properties of a parallelogram:

1. *Opposite sides of a parallelogram are equa l*(AB = CD, AD = BC).
2. *Opposite angles of a parallelogram are equal* (∠A = ∠C, ∠B = ∠D).
3. *Diagonals of a parallelogram are divided in their intersection point into two* (AO = OC, BO = OD).
4. *A sum of squares of diagonals is equal to a sum of squares of four sides*:
   $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$.

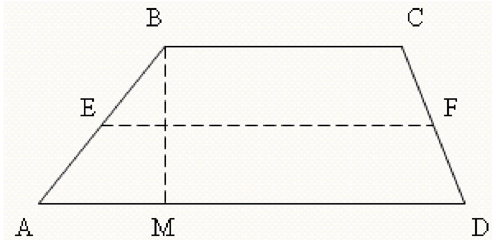
Rhombus: If all sides of parallelogram are equal, then this parallelogram is called a *rhombus*.



Diagonals of a rhombus are mutually perpendicular (AC ⊥BD) and divide its angles into two (∠DCA = ∠BCA, ∠ABD = ∠CBD etc.).

Area of a rhombus = $\frac{1}{2} \times$ product of diagonals $= \frac{1}{2} \times$ AC × BD

Trapezoid: Trapezoid is a quadrangle two opposite sides of which are parallel.



Here AD || BC. Parallel sides are called *bases* of a trapezoid, the two others (AB and CD) are called *lateral sides.* A distance between bases (BM) is a *height.* The segment EF, joining midpoints E and F of the lateral sides, is called a *midline* of a trapezoid. *A midline of a trapezoid is equal to a half-sum of bases:*

$$EF = \frac{AD + BC}{2}$$

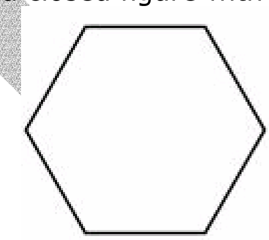
and parallel to them: EF || AD and EF || BC. A trapezoid with equal lateral sides (AB = CD) is called an *isosceles* trapezoid. *In an isosceles trapezoid angles by each base, are equal* ($\angle A = \angle D$, $\angle B = \angle C$)..

Area of a trapezoid = $\frac{\text{Sum of parallel sides}}{2} \times \text{height} = \frac{AD + BC}{2} \times BM$

In a trapezium ABCD with bases $\overline{AB}$ and $\overline{CD}$, the sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the non-parallel sides and twice the product of the lengths of the parallel sides: $AC^2 + BD^2 = AD^2 + BC^2 + 2 \cdot AB \cdot CD$

Here is one more polygon, a regular hexagon:

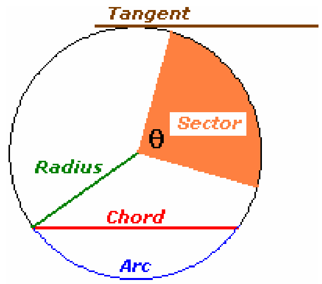
Regular Hexagon: A regular hexagon is a closed figure with six equal sides.



If we join each vertex to the centre of the hexagon, we get 6 equilateral triangles. Therefore, if the side of the hexagon is a, each equilateral triangle has a side a. Hence, area of the regular hexagon = $6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$.

CIRCLE

A circle is a set of all points in a plane that lie at a constant distance from a fixed point. The fixed point is called the center of the circle and the constant distance is known as the radius of the circle.



Arc: An arc is a curved line that is part of the circumference of a circle. A **minor arc** is an arc less than the semicircle and a **major arc** is an arc greater than the semicircle.

Chord: A chord is a line segment within a circle that touches 2 points on the circle.

Diameter: The longest distance from one end of a circle to the other is known as the diameter. It is equal to twice the radius.

Circumference: The perimeter of the circle is called the circumference. The value of the circumference = $2\pi r$, where r is the radius of the circle.

Area of a circle: Area = π x (radius)^2 = πr^2.

Sector: A sector is like a slice of pie (a circular wedge).

Area of Circle Sector: (with central angle θ) Area = $\frac{\theta}{360} \times \pi \times r^2$

Length of a Circular Arc: (with central angle θ) The length of the arc = $\frac{\theta}{360} \times 2\pi \times r$

Tangent of circle: A line perpendicular to the radius that touches ONLY one point on the circle

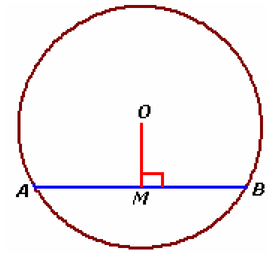
If 45° arc of circle A has the same length as 60° arc of circle B, find the ratio of the areas of circle A and circle B.

Answer: Let the radius of circle A be r_1 and that of circle B be r_2.

$\Rightarrow \frac{45}{360} \times 2\pi \times r_1 = \frac{60}{360} \times 2\pi \times r_2 \Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow$ Ratio of areas = $\frac{\pi r_1^2}{\pi r_2^2} = \frac{16}{9}$.

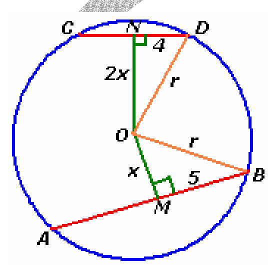
RULE!

The perpendicular from the center of a circle to a chord of the circle bisects the chord. In the figure below, O is the center of the circle and
OM ⊥ AB. Then, AM = MB.



Conversely, the line joining the center of the circle and the midpoint of a chord is perpendicular to the chord.

In a circle, a chord of length 8 cm is twice as far from the center as a chord of length 10 cm. Find the circumference of the circle.

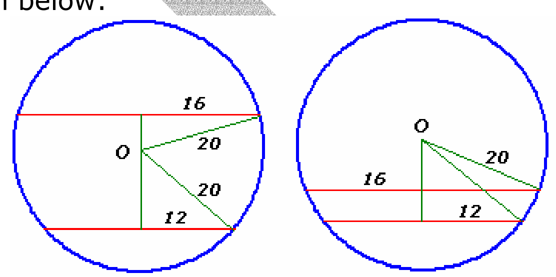


Answer: Let AB and CD be two chords of the circle such that AB = 10 and CD = 8. Let O be the center of the circle and M and N be the midpoints of AB and CD. Therefore OM $\perp$ AB, ON $\perp$ CD, and if ON = 2x then OM = x.
$BM^2 + OM^2 = OB^2$ and $DN^2 + ON^2 = OD^2$.
OB = OD = r → $(2x)^2 + 4^2 = r^2$ and $x^2 + 5^2 = r^2$.
Equating both the equations we get, $4x^2 + 16 = x^2 + 25$
Or $x = \sqrt{3}$ → $r = 2\sqrt{7}$. Therefore circumference = $2\pi r = 4\pi\sqrt{7}$.

What is the distance in cm between two parallel chords of length 32 cm and 24 cm in a circle of radius 20 cm? (CAT 2005)

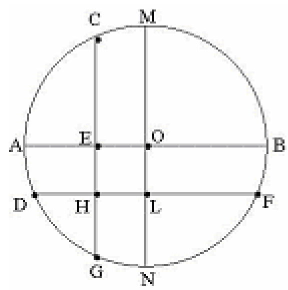
1. 1 or 7 2. 2 or 14 3. 3 or 21 4. 4 or 28

Answer: The figures are shown below:



The parallel chords can be on the opposite side or the same side of the centre O. The perpendicular (s) dropped on the chords from the centre bisect (s) the chord into segments of 16 cm and 12 cm, as shown in the figure. From the Pythagoras theorem, the distances of the chords from the centre are $\sqrt{20^2 - 16^2} = 12$ and $\sqrt{20^2 - 12^2} = 16$, respectively. Therefore, the distances between the chords can be 16 + 12 = 28 cm or
16 – 12 = 4 cm.

In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE:EB = 1:2, and DF is perpendicular to MN such that NL:LM = 1:2. The length of DH in cm is (CAT 2005)



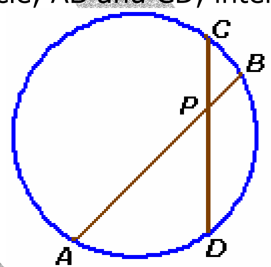
Answer: In the above figure, AB = MN = 3 cm and AE: EB = NL: LM = 1: 2
⇒ AE = NL = 1 cm. Now AO = NO = 1.5 cm ⇒ OE = HL = OL = 0.5 cm. Join O and D
⇒ $OD^2 = OL^2 + DL^2 \Rightarrow DL^2 = \sqrt{OD^2 - OL^2} = \sqrt{1.5^2 - 0.5^2} = \sqrt{2} \Rightarrow DH = DL - HL = \sqrt{2} - \frac{1}{2} = \frac{2\sqrt{2} - 1}{2}$

RULE!

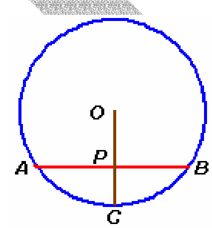
Equal chords are equidistant from the center. Conversely, if two chords are equidistant from the center of a circle, they are equal.

RULE!

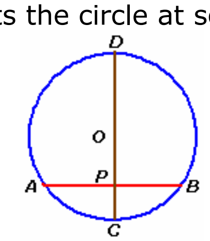
In the following figure, two chords of a circle, AB and CD, intersect at point P. Then, AP × PB = CP × PD.



In the following figure, length of chord AB = 12. O-P-C is a perpendicular drawn to AB from center O and intersecting AB and the circle at P and C respectively. If PC = 2, find the length of OB.



Answer: Let us extend OC till it intersects the circle at some point D.



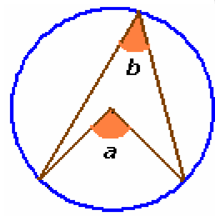
CD is the diameter of the circle. Since OP is perpendicular to AB, P is the midpoint of AB. Hence, AP = PB = 6. Now DP × PC = AP × PB
→ DP = 18. Therefore, CD = 20 → OC = 10. OB = OC = radius of the circle = 10.

RULE!

In a circle, equal chords subtend equal angles at the center.

RULE!

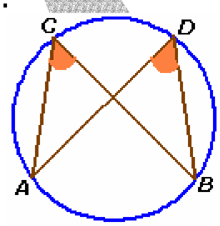
The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circumference.



In the figure shown above, a = 2b.

RULE!

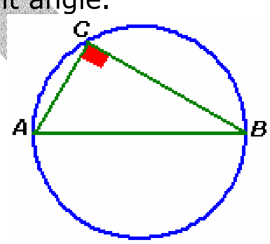
Angles inscribed in the same arc are equal.



In the figure angle ACB = angle ADB.

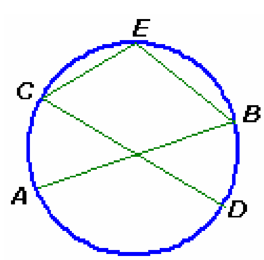
RULE!

An angle inscribed in a semi-circle is a right angle.

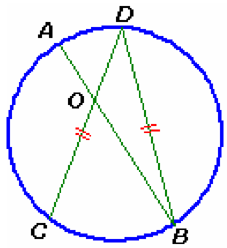


Let angle ACB be inscribed in the semi-circle ACB; that is, let AB be a diameter and let the vertex C lie on the circumference; then angle ACB is a right angle.

In the figure AB and CD are two diameters of the circle intersecting at an angle of 48°. E is any point on arc CB. Find angle CEB.



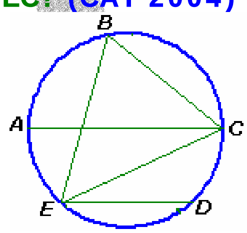
Answer: Join E and D. Since arc BD subtends an angle of 48° at the center, it will subtend half as many degrees on the remaining part of circumference as it subtends at the center. Hence, angle DEB = 24°. Since angle CED is made in a semicircle, it is equal to 90°. Hence, angle CEB = angle CED + angle DEB = 90° + 24° = 114°.



In the above figure, AB is a diameter of the circle and C and D are such points that CD = BD. AB and CD intersect at O. If angle AOD = 45°, find angle ADC.

Answer: Draw AC and CB. CD = BD $\Rightarrow \angle DCB = \angle DBC = \theta$ (say). $\angle ACB = 90^\circ \Rightarrow \angle ACD = 90^\circ - \theta$. $\angle ABD = \angle ACD = 90^\circ - \theta \Rightarrow \angle ABC = \theta - (90^\circ - \theta) = 2\theta - 90$. In ΔOBC, $45^\circ + 2\theta - 90 + \theta = 180^\circ \Rightarrow 3\theta = 225^\circ \Rightarrow \theta = 75^\circ$.
$\angle ADC = \angle ABC = 2\theta - 90 = 60^\circ$.

In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If angle CBE = 65°, then what is the value of angle DEC? (CAT 2004)

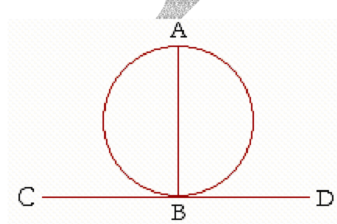


1. 35° 2. 55° 3. 45° 4. 25°

Answer: $\angle ABC = 90^\circ \Rightarrow \angle ABE = 90 - \angle EBC = 25^\circ$. $\angle ABE = \angle ACE = 25^\circ$. $\angle ACE = \angle CED = 25^\circ$ (alternate angles)

RULE!

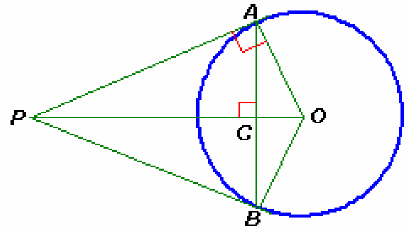
The straight line drawn at right angles to a diameter of a circle from its extremity is tangent to the circle. Conversely, If a straight line is tangent to a circle, then the radius drawn to the point of contact will be perpendicular to the tangent.



Let AB be a diameter of a circle, and let the straight line CD be drawn at right angles to AB from its extremity B; then the straight line CD is tangent to the circle.

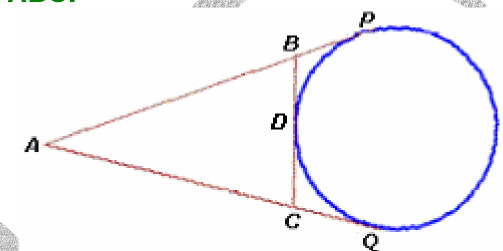
RULE!

If two tangents are drawn to a circle from an exterior point, the length of two tangent segments are equal. Also, the line joining the exterior point to the centre of the circle bisects the angle between the tangents.



In the above figure, two tangents are drawn to a circle from point P and touching the circle at A and B. Then, PA = PB. Also, ∠APO = ∠BPO. Also, the chord AB is perpendicular to OP.

In the following figure, lines AP, AQ and BC are tangent to the circle. The length of AP = 11. Find the perimeter of triangle ABC.

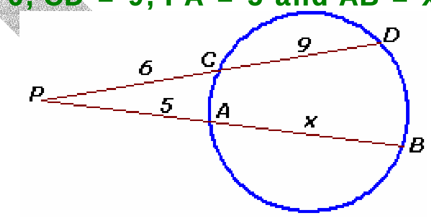


Answer: let AB = x and BP = y. Then, BD = BP because they are tangents drawn from a same point B. Similarly CD = CQ and AP = AQ. Now perimeter of triangle ABC = AB + BC + CA = AB + BD + DC + AC = AB + BP + CQ + AC = AP + AQ = 2AP = 22.

RULE!

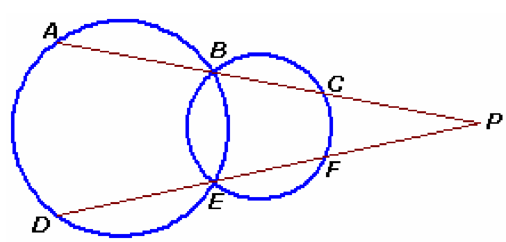
From an external point P, a secant P-A-B, intersecting the circle at A and B, and a tangent PC are drawn. Then, PA × PB = PC^2.

In the following figure, if PC = 6, CD = 9, PA = 5 and AB = x, find the value of x



Answer: Let a tangent PQ be drawn from P on the circle. Hence, PC × PD = PQ^2 = PA × PB → 6 × 15 = 5 × (5 + x) → x = 13

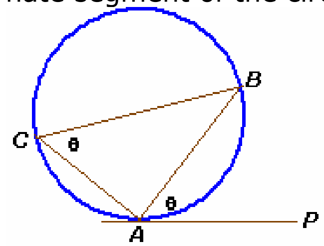
In the following figure, PC = 9, PB = 12, PA = 18, and PF = 8. Then, find the length of DE.



Answer: In the smaller circle PC × PB = PF × PE → PE = $12 \times \frac{9}{8} = \frac{27}{2}$. In the larger circle, PB × PA = PE × PD → PD = 12 × 18 × $\frac{2}{27}$ = 16. Therefore, DE = PD − PE = 16 − 13.5 = 2.5

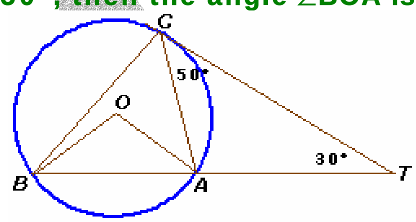
RULE!

The angle that a tangent to a circle makes with a chord drawn from the point of contact is equal to the angle subtended by that chord in the alternate segment of the circle.



In the figure above, PA is the tangent at point A of the circle and AB is the chord at point A. Hence, angle BAP = angle ACB.

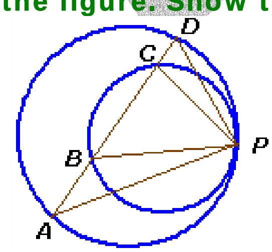
In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then the angle ∠BOA is (CAT 2003)



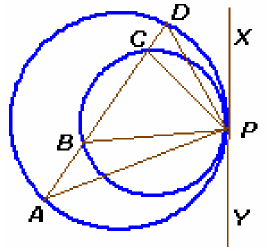
1. 100° 2. 150° 3. 80° 4. not possible to determine

Answer: Tangent TC makes an angle of 50° with chord AC. Therefore, ∠TBC = 50°. In triangle TBC, ∠BCT = 180° – (30° + 50°) = 100°. Therefore, ∠BCA = ∠BCT – ∠ACT = 100° – 50° = 50°. ∠BOA = 2∠BCA = 100°.

Two circles touch internally at P. The common chord AD of the larger circle intersects the smaller circle in B and C, as shown in the figure. Show that , ∠APB = ∠CPD.



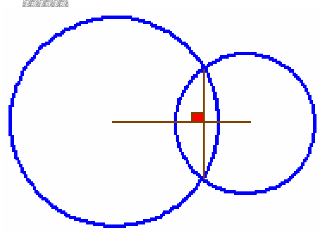
Answer: Draw the common tangent XPY at point P.



Now, for chord DP, ∠DPX = ∠DAP, and for chord PC, ∠CPX = ∠CBP
⇒ ∠CPD = ∠CPX – ∠DPX = ∠CBP – ∠DAP. In triangle APB, ∠CBP is the exterior angle
⇒ ∠CBP = ∠CAP + ∠APB
⇒ ∠CBP – ∠CAP = ∠APB
⇒ ∠CPD = ∠CPX – ∠DPX = ∠CBP – ∠DAP = ∠APB

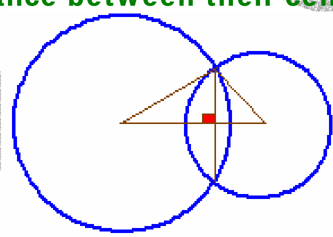
RULE!

When two circles intersect each other, the line joining the centers bisects the common chord and is perpendicular to the common chord.



In the figure given above, the line joining the centers divides the common chord in two equal parts and is also perpendicular to it.

Two circles, with diameters 68 cm and 40 cm, intersect each other and the length of their common chord is 32 cm. Find the distance between their centers.

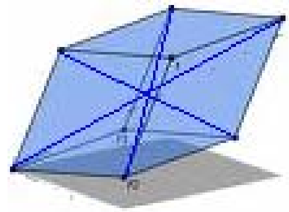


Answer: In the figure given above, the radii of the circles are 34 cm and 20 cm, respectively. The line joining the centers bisects the common chord. Hence, we get two right triangles: one with hypotenuse equal to 34 cm and height equal to 16 cm, and the other with hypotenuse equal to 20 cm and height equal to 16 cm. Using Pythagoras theorem, we get the bases of the two right triangles equal to 30 cm and 12 cm. Hence, the distance between the centers = 30 + 12 = 42 cm.

SOLIDS

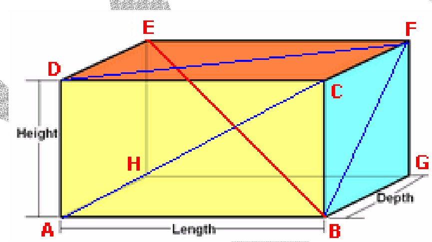
A solid figure, or **solid**, is any portion of space bounded by one or more surfaces, plane or curved. These surfaces are called the **faces** of the solid, and the intersections of adjacent faces are called **edges**. Let's have a look at some common solids and their properties.

Parallelepiped: A parallelepiped is a solid bounded by three pairs of parallel plane faces.



- Each of the six faces of a parallelepiped is a parallelogram.
- Opposite faces are congruent.
- The four diagonals of a parallelepiped are concurrent and bisect one another.

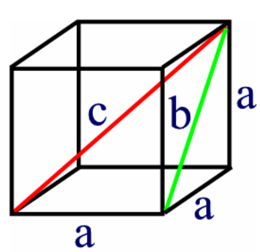
Cuboid: A parallelepiped whose faces are rectangular is called a cuboid. The three dimensions associated with a cuboid are its length, breadth and height (denoted as l, b and h here.)



- The length of the three pairs of face diagonals are BF = $\sqrt{b^2+h^2}$, AC = $\sqrt{l^2+h^2}$, and DF = $\sqrt{l^2+b^2}$.
- The length of the four equal body diagonals AF = $\sqrt{l^2+b^2+h^2}$.
- The total surface area of the cuboid = 2(lb + bh + hl)
- Volume of a cuboid = lbh
- The radius of the sphere circumscribing the cuboid = $\frac{\text{Diagonal}}{2} = \frac{\sqrt{l^2+b^2+h^2}}{2}$.
- Note that if the dimensions of the cuboid are not equal, there cannot be a sphere which can be inscribed in it, i.e. a sphere which touches all the faces from inside.

Euler's Formula: the number of faces (*F*), vertices (*V*), and edges (*E*) of a solid bound by plane faces are related by the formula *F* + *V* = *E* + 2 gives here 6 + 8 = 12 + 2.

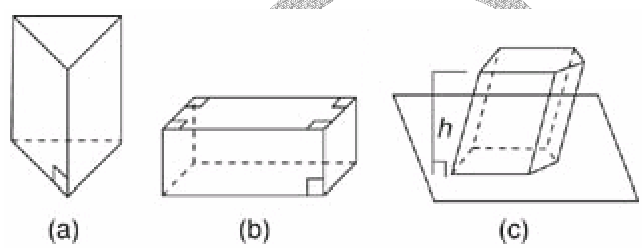
Cube: A cube is a parallelepiped all of whose faces are squares.



- Total surface area of the cube = $6a^2$
- Volume of the cube = a^3

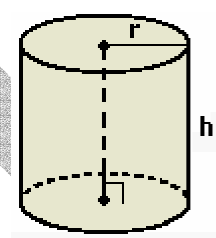
- Length of the face diagonal b = $\sqrt{2}a$
- Length of the body diagonal c = $\sqrt{3}a$
- Radius of the circumscribed sphere = $\frac{\sqrt{3}a}{2}$
- Radius of the inscribed sphere = $\frac{a}{2}$
- Radius of the sphere tangent to edges = $\frac{a}{\sqrt{2}}$

Prism: A prism is a solid bounded by plane faces, of which two, called the ends, are congruent figures in parallel planes and the others, called side-faces are parallelograms. The ends of a prism may be triangles, quadrilaterals, or polygons of any number of sides.



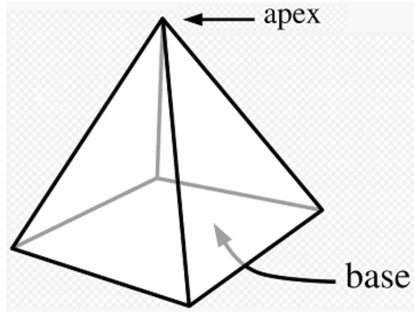
- The side- edges of every prism are all parallel and equal.
- A prism is said to be right, if the side-edges are perpendicular to the ends: In this case the side faces are rectangles. Cuboids and cubes are examples.
- Curved surface area of a right prism = perimeter of the base × height
- Total surface area of a right prism = perimeter of the base × height + 2 × area of the base
- Volume of a right prism = area of the base × height

Right Circular Cylinder: A right circular cylinder is a right prism whose base is a circle. In the figure given below, the cylinder has a base of radius r and a height of length h.



- Curved surface area of the cylinder = $2\pi rh$
- Total surface area of the cylinder = $2\pi rh + \pi r^2$
- Volume of the cylinder = $\pi r^2 h$

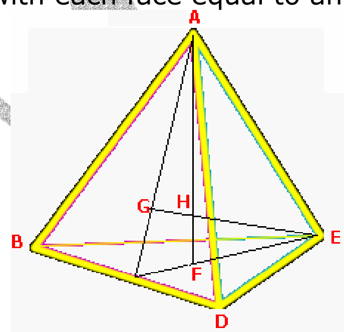
Pyramid: A pyramid is a solid bounded by plane faces, of which one, called the base, is any rectilinear figure, and the rest are triangles having a common vertex at some point not in the plane of the base. The slant height of a pyramid is the height of its triangular faces. The height of a pyramid is the length of the perpendicular dropped from the vertex to the base.



In a pyramid with n sided regular polygon as its base,

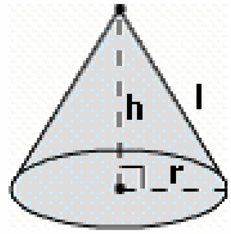
- Total number of vertices = n + 1
- Curved surface area of the pyramid $= \frac{\text{Perimeter}}{2} \times \text{slant height}$.
- Total surface area of the pyramid = $\frac{\text{Perimeter}}{2} \times \text{slant height} + \text{area of the base}$.
- Volume of the pyramid = $\frac{\text{Base area}}{3} \times \text{height}$

Tetrahedron: A tetrahedron is a pyramid which has four congruent equilateral triangles as it four faces. The figure below shows a tetrahedron with each face equal to an equilateral triangle of side a.



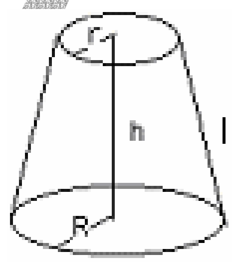
- Total number of vertices = 4
- The four lines which join the vertices of a tetrahedron to the centroids of the opposite faces meet at a point which divides them in the ratio 3: 1. In the figure. AH: HF = 3: 1.
- Curved surface area of the tetrahedron $= \frac{3\sqrt{3}a^2}{4}$.
- Total surface area of the tetrahedron $= \sqrt{3}a^2$.
- Height of the tetrahedron = $\frac{\sqrt{6}a}{3}$
- Volume of the tetrahedron = $\frac{\sqrt{2}a^3}{12}$

Right Circular Cone: a right circular cone is a pyramid whose base is a circle. In the figure given below, the right circular cone has a base of radius r and a height of length h.



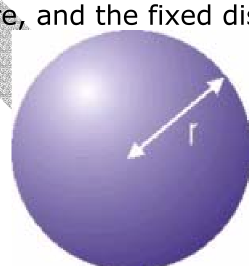
- Slant height $l = \sqrt{h^2 + r^2}$
- Curved surface area of the cone = πrl
- Total surface area of the cone = $\pi rl + \pi r^2$
- Volume of the cone = $\frac{\pi r^2 h}{3}$

Frustum of a Cone: When a right circular cone is cut by a plane parallel to the base, the remaining portion is known as the frustum.



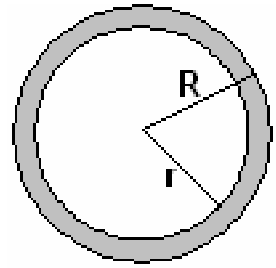
- Slant height $l = \sqrt{h^2 + (R - r)^2}$.
- Curved surface area of the frustum = $\pi(r + R)l$
- Total surface area = $\pi(r + R)l + \pi(r^2 + R^2)$
- Volume of the frustum = $\frac{\pi h(r^2 + R^2 + Rr)}{3}$

Sphere: A sphere is a set of all points in space which are at a fixed distance from a given point. The fixed point is called the centre of the sphere, and the fixed distance is the radius of the sphere.



- Surface area of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

Spherical Shell: A hollow shell with inner and outer radii of r and R, respectively.



- Volume of the shell = $\frac{4}{3}\pi(R^3 - r^3)$

1. A regular polygon with *n* sides has interior angles measuring 178^{o}. What is the value of $\frac{180}{n}$?

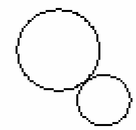
2. A regular hexagon is inscribed in a circle of radius 6. What is the area of the hexagon?

3. A rhombus has a perimeter of 52 cm and a diagonal measuring 24 cm. What is the length, in centimeters, of the other diagonal?

4. A rhombus has diagonals measuring 6 cm and 10 cm. What is its area in square centimeters?

(a) 30 (b) 32 (c) 60 (d) 64 (e) None of these

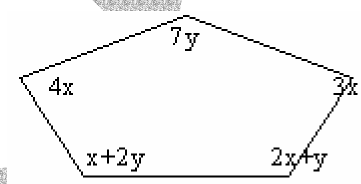
5. Two gears are circular, and the circles are tangent as shown. If the centers are fixed and the radii are 30 cm and 40 cm, how many revolutions will the larger circle have made when the smaller circle has made 4 revolutions?



6. A regular hexagon has a perimeter of 12 cm. What is its area ?

(a) $6\sqrt{3}$ (b) $72\sqrt{3}$ (c) $144\sqrt{3}$ (d) $216\sqrt{3}$ (e) None of these

7. If the expressions shown are the degree measures of the angles of the pentagon, find the value of $x+y$.



8. One angle of a regular polygon measures $177°$. How many sides does it have?

(a) 89 (b) 120 (c) 177 (d) 183 (e) None of these

9. Octagon *ABCDEFGH* is similar to octagon *JKLMNOPQ*. If $AB=10$, $JK=8$, and $m\angle A=120°$, what is $m\angle J$ in degrees?

(a) $96°$ (b) $120°$ (c) $135°$ (d) $150°$ (e) None of these

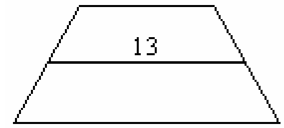
10. Find the sum of the measures of one interior and one exterior angle of a regular 40-gon.

(a) $168°$ (b) $174°$ (c) $180°$ (d) $186°$ (e) None of these

11. One angle of a parallelogram measures $(2x+y)°$. Another angle of the same quadrilateral (but not the opposite angle) measures $(x+2y)°$. What is $(x+y)$?

(a) 30 (b) 60 (c) 90 (d) 120 (e) None of these

12. An isosceles trapezoid has a mid segment measuring 13 cm and an area of 52 cm^2. If one base has length 10 cm, find the perimeter of the trapezoid in centimeters.



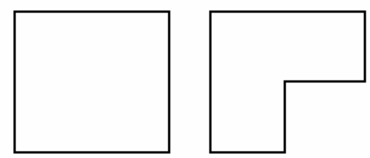
13. A right triangle with legs measuring 12 cm and 16 cm is inscribed in a circle. What is the circumference of the circle in centimeters?

(a) 14π (b) 16π (c) 20π (d) 28π (e) None of these

14. A square has a diagonal measuring 8 cm. When its area is expressed as 2^K square centimeters, what is *K*?

(a) 4 (b) 5 (c) 6 (d) 7 (e) None of these

15. One-fourth of the area of a square with each side measuring $2x$ cm is sectioned off and removed. ("Before And After" pictures of the procedure appear to the right.) The area removed is itself square-shaped. What is the perimeter of the resultant figure in centimeters?



(a) $6x$ (b) $7x$ (c) $8x$ (d) $9x$ (e) None of these

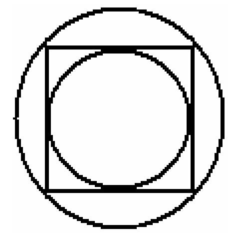
16. A central angle measuring M° intercepts an arc in a circle of radius *r* cm. The length of the subtended arc is 8π cm. The area of the sector formed by (and including) the angle is 48π cm^2. Evaluate $\left(\frac{M}{r}\right)$.

(a) 5 (b) 10 (c) 20 (d) 40 (e) None of these

17. A regular hexagon with a perimeter of $12\sqrt{2}$ units lies in the Cartesian plane in such a way that its center is on the origin, two of the vertices lie on the *x*-axis, and the midpoints of two of its sides lie on the *y*-axis. If the portion of the hexagon that lies in Quadrant I is completely revolved around the *x*-axis, a solid whose volume is *X* cubic units results. If the same portion is completely revolved around the *y*-axis, a solid with a volume of *Y* cubic units results. Evaluate $\left(\frac{X}{Y}\right)^2$.

(a) $\frac{48}{49}$ (b) 1 (c) $\frac{4}{3}$ (d) $\frac{16}{9}$ (e) None of these

18. A circle is inscribed inside a square. The square is inscribed inside another circle. If the area of the small circle is π cm^2, what is the area of the large circle, in square centimeters?



(a) $\pi\sqrt{2}$ (b) 2π

(c) $2\pi\sqrt{2}$ (d) 4π (e) None of these

19. A circle is inscribed in a triangle with sides measuring 4 cm, 6 cm, and 8 cm. What is the area of the circle in square centimeters?

(a) $\frac{7\pi}{6}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{3}$ (d) $\frac{7\pi}{4}$ (e) None of these

20. An equilateral triangle T_1 has area $100\sqrt{3}$ sq. cm. A second triangle, T_2, is drawn with vertices on the midpoints of the sides of T_1. The midpoints of the sides of T_2 are the vertices of triangle T_3, and so on. What is the sum of the perimeters, in centimeters, of all the triangles, T_1, T_2, $T_{3\ldots}$ etc.?

21. In a 30°- 60°- 90° triangle, the longest side and the shortest side differ in length by 2002 units. What is the length of the longest side?

22. What is the area of a triangle with sides of lengths 7, 8 and 9?

23. The base of an isosceles triangle is 80 cm long. If the area of the triangle cannot exceed 1680 square centimeters, what is the maximum number of centimeters in the perimeter of the triangle?

24. A triangle has sides measuring 41 cm, 41 cm and 18 cm. A second triangle has sides measuring 41 cm, 41 cm and x cm, where x is a whole number not equal to 18. If the two triangles have equal areas, what is the value of x?

25. In a triangle ABC, AB = 16 units, $\angle$CAB = 30°, and $\angle$ACB = 45°. What is the length of BC?

26. You have 6 sticks of lengths 10, 20, 30, 40, 50 and 60 cm. The number of non-congruent triangles that can be formed by choosing three of the sticks to make the sides is

(a) 3
(b) 6
(c) 7
(d) 10
(e) 12

27. A triangle has sides of lengths 10, 24 and n, where n is a positive integer. The number of values of n for which this triangle has three acute angles is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

ABC forms an equilateral triangle in which B is 2 km from A. A person starts walking from B in a direction parallel to AC and stops when he reaches a point D directly east of C. He, then, reverses direction and walks till he reaches a point E directly south of C.

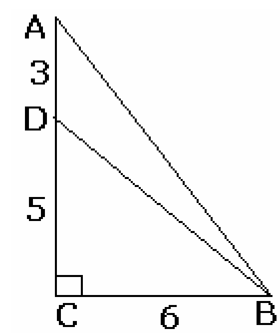
28. Then D is
(a) 3 km east and 1 km north of A
(b) 3 km east and $\sqrt{3}$ km north of A
(c) $\sqrt{3}$ km east and 1 km south of A
(d) $\sqrt{3}$ km west and 3 km north of A

29. The total distance walked by the person is
(a) 3 km
(b) 4 km
(c) $2\sqrt{3}$ km
(d) 6 km

30. How many non-congruent triangles with perimeter 7 have integer side lengths?
(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

31. When the base of a triangle is increased by 10% and he altitude to this base is decreased by 10%, the area is
(a) increased by 10%
(b) decreased by 10%
(c) increased by 1%
(d) decreased by 1%
(e) unchanged

32. In the figure given below, triangle ABC is right-angled. What is the area of triangle ABD?

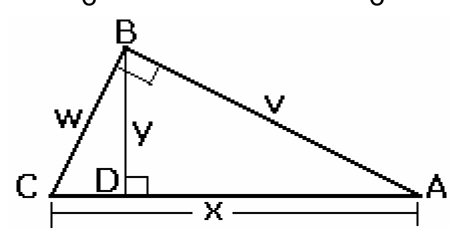


(a) 6
(b) 7
(c) 8
(d) 9
(e) 10

33. Let ABC be an equilateral triangle with sides x. Let P be the point of intersection of the three angle bisectors. What is the length of AP?

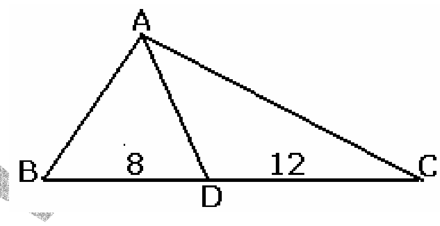
(a) $\frac{x\sqrt{3}}{3}$ (b) $\frac{x\sqrt{3}}{6}$ (c) $\frac{5\sqrt{3}}{6}$ (d) $\frac{2x\sqrt{3}}{6}$ (e) $\frac{4x\sqrt{3}}{6}$

34. In the figure below, ∠ABC and ∠BDA are both right angles. If v + w = 35 and x + y = 37, then what is the value of y?



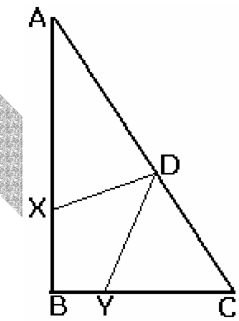
(a) 11
(b) 12
(c) 13
(d) 14
(e) 15

35. The area of ΔABC is 60 square units. If BD = 8 units and DC = 12 units, what is the area of ΔABD?



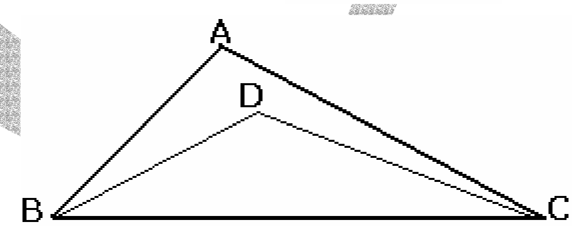
(a) 24
(b) 40
(c) 48
(d) 36
(e) 6

36. In right triangle ABC, AX = AD and CY = CD, as shown in the figure below. What is the measure of ∠XDY?



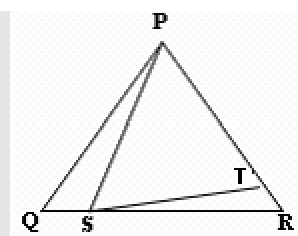
(a) 35°
(b) 40°
(c) 45°
(d) 50°
(e) cannot be determined

37. In triangle ABC, ∠A is equal to 120°. A point D is inside the triangle such that ∠DBC = 2∠ABD and ∠DCB = 2∠ACD. What is the measure of ∠BDC?



(a) 135
(b) 140
(c) 145
(d) 150
(e) 155

In the figure shown, PQR is an isosceles triangle with PQ = PR. S is a point on QR such that PS = PT. Also, ∠QPS = 30°.

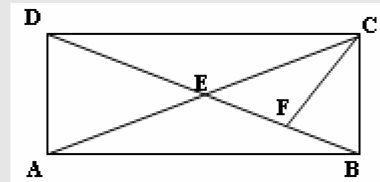


38. What is the measure of ∠RST?
(a) 7.5°
(b) 15°
(c) 20°
(d) 45°

39. Two sides of a triangle are of length 15 and 7 centimeters. If the length of the third side is an integer value, what is the sum of all the possible lengths of the third side?
(a) 253
(b) 231

(c) 210
(d) 195

An agriculturist is conducting an experiment in a rectangular field ABCD. He sows seeds of a crop evenly across the field, but he uses a new variety of manure in the area CEF (E and F are midpoints of BD and BE, respectively) whereas he uses the old variety in the rest of the field. The yield per unit area of the crop in $\triangle$CEF is 3 times the yield in rest of the field.

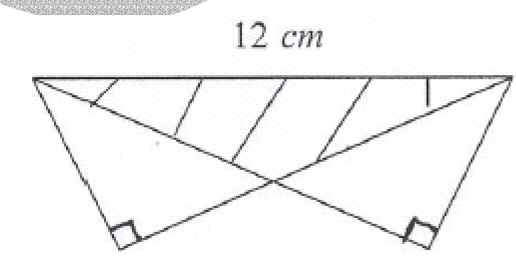


40. What is the ratio of the amount of crop produced in $\triangle$CEF to that produced in the rest of the field?
(a) 3: 4
(b) 1: 2
(c) 3: 7
(d) 3: 8

41. In triangle ABC, the longest side BC is of length 20 and the altitude from A to BC is of length 12. A rectangle DEFG is inscribed in ABC with D on AB, E on AC, and both F and G on BC. The maximum possible area of rectangle DEFG is
(a) 60
(b) 100
(c) 120
(d) 150
(e) 200

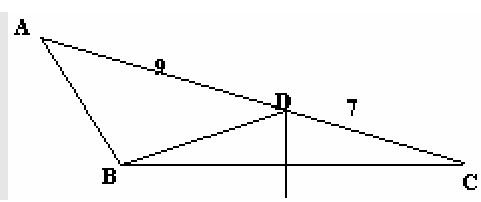
42. The degree measure of an angle whose complement is eighty percent of half the angle's supplement is
(a) 60
(b) 45
(c) 30
(d) 15

43. Two congruent 90°-60°-30° triangles are placed, as shown, so that they overlap partly and their hypotenuses coincide. If the hypotenuse is 12 *cm*, find the area common to both triangles



(a) $6\sqrt{3}$ cm^2
(b) $8\sqrt{3}$ cm^2
(c) $9\sqrt{3}$ cm^2
(d) $12\sqrt{3}$ cm^2
(e) 24 cm^2

In triangle ABC, side AC and the perpendicular bisector of side BC meet in point D, and BD bisects $\angle$ABC. If AD = 9, and DC = 7.

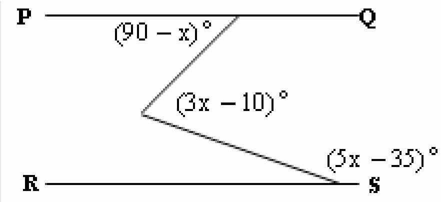


44. What is the area of triangle ABD?
(a) $14\sqrt{5}$
(b) 21

(c) 28

(d) $21\sqrt{5}$

Between two parallel lines PQ and RS, two transversals are drawn, as shown in the figure. The angles which the two transversals make with each other and with their respective lines are also shown.



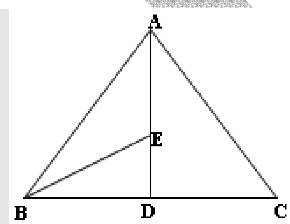
45. What is the value of x?

(a) 20°
(b) 25°
(c) 30°
(d) 35°

46. The perimeter of an isosceles right triangle is 2a. Then its area is

(a) $4a^2\sqrt{2}$
(b) $3a^2/2$
(c) $a^2(3 - 2\sqrt{2})$
(d) $4a^2(1 + \sqrt{3})$

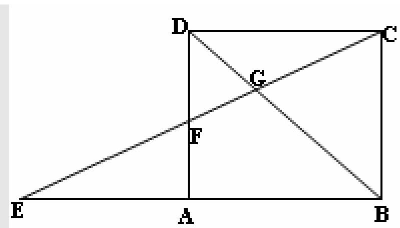
Triangle ABC (shown in the figure) is equilateral with side length of 16. Also, AD ⊥ BC and AE ≅ ED.



47. What is the value of BE?

(a) $4\sqrt{3}$
(b) $\frac{8}{\sqrt{3}}$
(c) $\frac{16}{\sqrt{3}}$
(d) $4\sqrt{7}$

In the square ABCD, AB is extended and E is a point on AB such that CE intersects AD at F and BD at G. The length of FG and GC are 3 and 5 units, respectively.



48. What is the length of EF?

(a) 4
(b) 5
(c) $\frac{13}{3}$
(d) $\frac{16}{3}$

49. Two telegraph poles of height *a* and *b* meters are on opposite sides of a road. Wires are drawn from top of one pole to the bottom of the other. If the wires are completely taut, then how many feet above the ground will the wires cross each other?

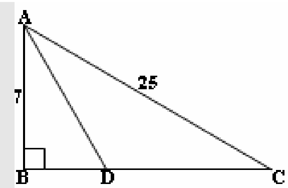
(a) $\frac{2ab}{a+b}$

(a) $\frac{ab}{2(a+b)}$

(b) $\frac{ab}{a+b}$

(c) $\frac{a+b}{2}$

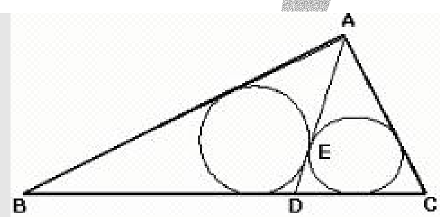
Triangle ABC is right angled at B. AB = 7, AC = 25 and D is a point on BC such that AD is the bisector of angle A, as shown in the figure.



50. What is the length of AD?

(a) 9.00
(a) 8.75
(b) 12.5
(c) 13.0

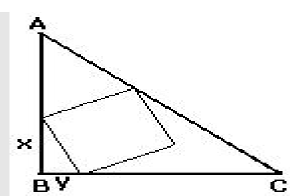
Sides of triangle ABC are AB = 12, BC = 18, and AC = 10. There is a point D, on BC, such that both incircles of triangles ABD and ACD touch AD at a common point E, as shown in the adjacent figure.



51. The length of CD

(a) is 8
(a) is 12
(b) is 16
(c) cannot be determined

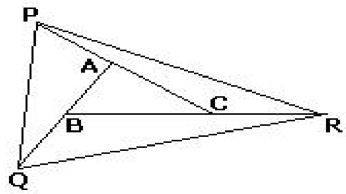
ABC is an isosceles triangle right angled at B. A square is inscribed inside the triangle with three vertices of the square on three sides of the triangle as shown in the adjacent figure. It is known that the ratio x to y is equal to 2 to 1.



52. The ratio of the area of the square to the area of the triangle is equal to

(a) 2: 5
(a) 1: 10
(b) 1: 3
(c) 2: 3

In triangle ABC, sides AB, AC, and BC are extended till Q, P and R such that AC = AP, BC = CR, and AB = BQ, as shown in the adjacent figure. It is known that the area of triangle ABC is 10 square centimetres.



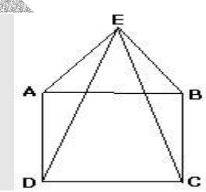
53. What is the area of triangle PQR?

(a) 40 cm^2
(a) 70 cm^2
(b) 80 cm^2
(c) 90 cm^2

54. A water lily with a rigid straight stem extends one meter above the surface of the water. When it bends at the bottom of its stem, it disappears under the water at a distance three meter from where the stem originally came out of the water. How deep is the lake?

(a) 6
(a) 3
(b) 4
(c) 5

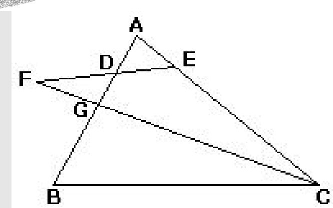
In the adjacent figure, ABCD is a square and ABE is an equilateral triangle.



55. Angle DEC is equal to

(a) 15°
(a) 30°
(b) 45°
(c) 20°

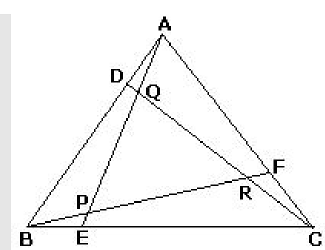
In triangle ABC, D and E are any points on AB and AC such AD = AE. The bisector of angle C meets DE at F. It is known that angle B = 60º.



56. What is the degree measure of angle DFC?

(a) 25°
(a) 30°
(b) 45°
(c) 60°

In the adjacent figure, triangle ABC is equilateral and D, E, and F are points on AB, BC, and AC such AD = BE = CF = $\frac{AB}{3}$. BF, CD, and AE intersect to form triangle PQR inside ABC.



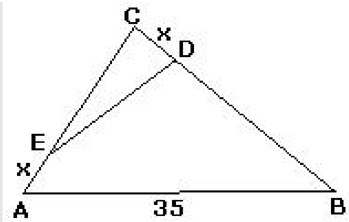
57. What is the ratio of the area of triangle PQR to that of triangle ABC?

(a) 1: 9
(a) 1: 7

(b) 1: 8
(c) 1: 12

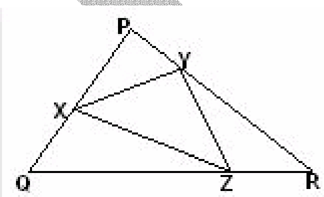
58. Four lines parallel to the base of a triangle divide each of the other sides into five equal segments and the area into five distinct parts. If the area of the largest of these parts is 27, then what is the area of the original triangle?
(a) 135
(a) 75
(b) 225
(c) 175

In the diagram AB = 35, AE = CD = x, EC = 8, ED = 7. Also, angle DEC = angle ABC.



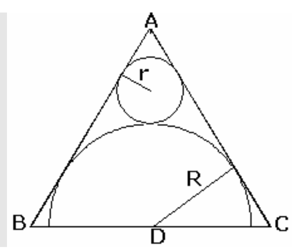
59. What is the value of x?
(a) 1
(a) 2
(b) 3
(c) 4

In triangle PQR, points X, Y and Z are on PQ, PR and QR, respectively, such that PX = XQ, $\frac{RY}{YP} = \frac{a}{b}$, and $\frac{QZ}{ZR} = 3$. Also (area ΔPXY)^2 = (area ΔQXZ) $\times$ (area ΔRYZ)



60. The ratio a: b is
(a) $\frac{3+\sqrt{105}}{6}$
(a) $\frac{2+\sqrt{35}}{6}$
(b) $\frac{3+\sqrt{31}}{3}$
(c) $\frac{\sqrt{105}-3}{6}$

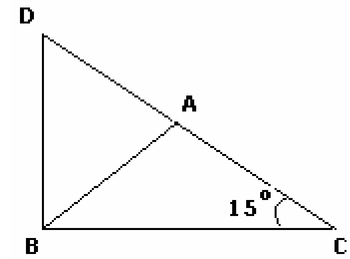
Triangle ABC is equilateral. D, the midpoint of BC, is the centre of the semicircle whose radius is R which touches AB and AC, as well as a smaller circle with radius r which also touches AB and AC.



61. What is the value of $\frac{R}{r}$?

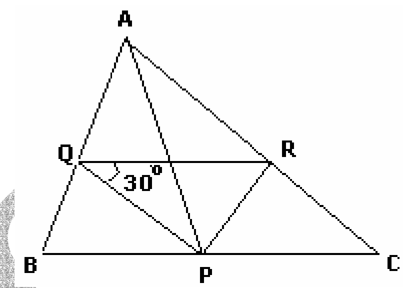
62. In the figure, AB = AC = AD and ∠BCA = 15°. Find ∠BDA.

(a) 30°
(b) 40°
(c) 60°
(d) 70°
(e) 75°



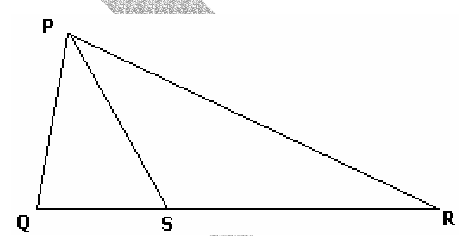
63. In the adjacent figure (not drawn to scale) ∠ PQR = 30°. Find the other two angels of ΔPQR if PQ and PR are the angle bisector of ∠APB and ∠APC, respectively.

(a) 30° and 120°
(b) 60° and 90°
(c) 70° and 80°
(d) 50° and 100°
(e) 75° and 75°



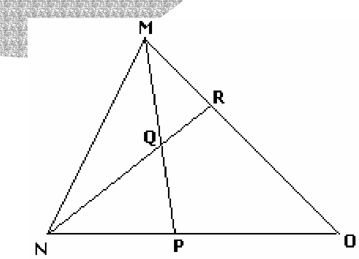
64. In the following figure, PS bisects ∠ QPR. The area of Δ PQS = 40 sq. cm. and PR is 2.5 times of PQ. Find the area of Δ PQR.

(a) 35 sq. cm.
(b) 70 sq. cm.
(c) 105 sq. cm.
(d) 140 sq. cm.
(e) 175 sq. cm.



65. In Δ MNO, MP is a median. NQ bisects MP and meets MO in R. Find the length of MR if MO = 30 cm.

(a) 5 cm.
(b) 6 cm.
(c) 10 cm.
(d) 15 cm.
(e) 20 cm.



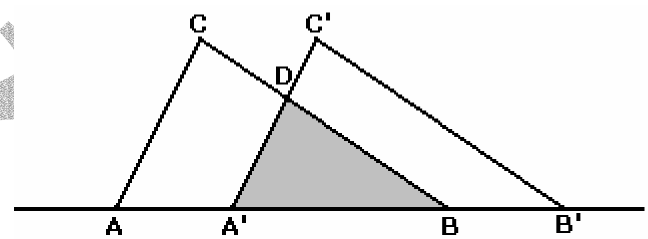
66. S is the point on the side QR of a triangle PQR such that ∠ PSR = ∠ QPR. The length of the side PR is 8cm. Find the maximum possible length of QS if it is known that both QR and SR take integral values greater than one.

(a) 16 cm.
(b) 18 cm.
(c) 30 cm.
(d) 32 cm.
(e) none of the above

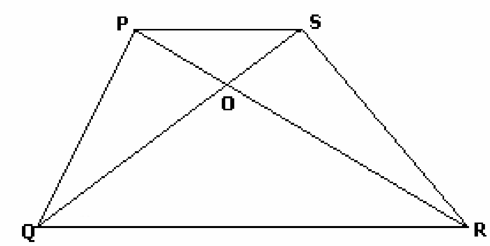
67. In a right angled triangle with sides p, q, r (where $p < q < r$), $2p + 7r = 9q$. If p =12cm, find the value of r

(a) 25 cm
(b) 25.5 cm
(c) 26 cm
(d) 26.5 cm
(e) none of the above

68. In triangle ABC, AB = 10, AC = 7, and BC = 8. How do we need to slide it along AB so that the area of the overlapping region (the shaded triangle A'BD) is one-half the area of the triangle ABC?

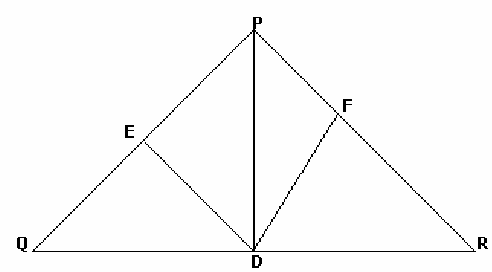


69. In the figure, PS and QR are parallel lines. If PO: OR = 1:4 and the length of QO = 12cm, find out the length of SO.



(a) 2 cm.
(b) 3 cm.
(c) 4 cm.
(d) 4.5 cm.
(e) 5.5

70. In the figure, PD is the median of ΔPQR. The bisectors of ∠PDQ and ∠PDR meet the sides PQ and PR at E and F, respectively. If EQ = 4EP then find the length of PR, given that PF is 2cm.



(a) 8 cm
(b) 9 cm
(c) 10 cm
(d) 11 cm
(e) None of the above

71. D is the mid point of the side QR of a triangle PQR. O is a point on PD such that PO is 4 times OD. QO and RO produced meet PR and PQ in E and F, respectively. Find the length of the side PQ if FQ = 3cm.

(a) 3 cm
(b) 6 cm
(c) 8 cm
(d) 9 cm
(e) 12 cm

72. In Δ MNO, the bisector of ∠ NMO intersects NO at P. MN = 9cm, MO = 12cm. PO = PN + 1. Find the length of PO.

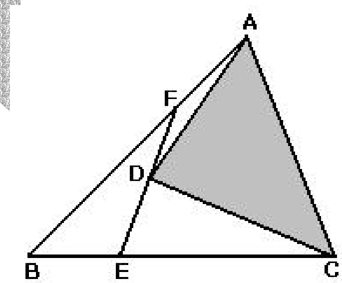
(a) 2 cm
(b) 3 cm
(c) 4 cm

(d) 5 cm
(e) 6 cm

73. In Δ PQR, the line segment MN intersects PQ in M and PR in N such that MN is parallel to QR. Find the ratio of QM: QP if it is known that the area of the Δ PMN is half of the Δ PQR.

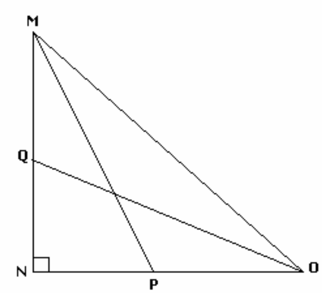
(a) $\frac{\sqrt{2}}{\sqrt{2}-1}$
(b) $\frac{\sqrt{2}-1}{\sqrt{2}}$
(c) $\frac{\sqrt{2}+1}{\sqrt{2}}$
(d) $\frac{\sqrt{2}+2}{\sqrt{2}}$
(e) None of the above

74. ABC is a triangle with area 1. AF = AB/3, BE = BC/3 and ED = FD. Find the area of the shaded figure.



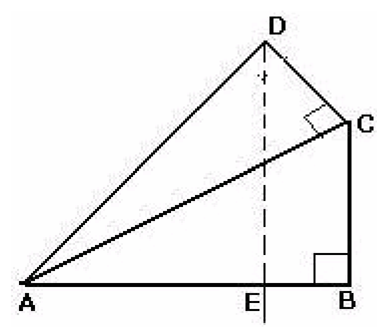
(a) $\frac{5}{9}$
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{13}{16}$
(e) $\frac{1}{5}$

75. Find the sum of squares of the medians MP and OQ drawn from the two acute angled vertices of a right angled triangle MNO. The longest side of Δ MNO is 20cm.



(a) 200 sq. cm
(b) 300 sq. cm
(c) 400 sq. cm
(d) 500 sq. cm
(e) cannot be determined

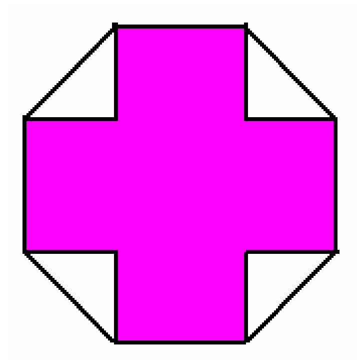
76. Right triangle ABC, with AB = 48, and BC = 20, is kept on a horizontal plane. Another right-triangle ADC (right-angled at C) is kept on the triangle with DC = 39 cm. A vertical line is drawn through point D, intersecting AB at E. Then the length of BE is equal to



(a) 15
(b) 21
(c) 12
(d) 18

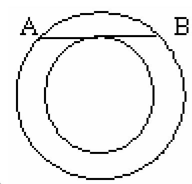
(e) cannot be determined

77. A concave dodecagon (the cross shown to the right) is inside of, and shares four nonconsecutive sides with, a regular octagon. Each reflex angle of the dodecagon measures $270°$, and every other interior angle of the dodecagon is a right angle. If the octagon has a perimeter of 16 cm, what is the area of the dodecagon in square centimeters?

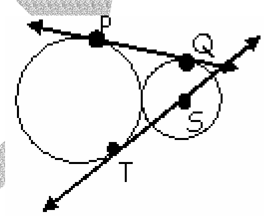


(a) $4+8\sqrt{2}$
(b) 16
(c) $4+16\sqrt{2}$
(d) 20
(e) None of these

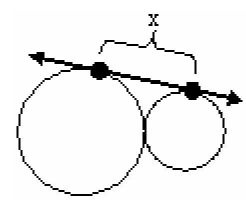
78. Two concentric circles are drawn so that the tangent segment (shown) to the smaller circle is a chord of the larger circle. If the area of the annulus (region outside the smaller circle and inside the larger one) is 100π, find the length of the chord shown ($\overline{AB}$).



79. Two circles are externally tangent, and have a common external tangent line, with points of tangency P and Q. $PQ = 10$ cm, and the radii of the circles are R cm and $R-1$ cm. The center of the smaller circle (the one which contains point Q) is S, and a line through S is drawn tangent to the larger circle. The point of tangency of this line is T, as shown. If ST= $2\sqrt{19}$ cm, find the length of the larger radius R.

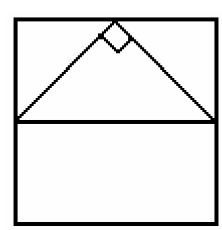


80. Two circles are externally tangent with a common external tangent. If the radii of the circles are 9 and 16, what is the distance (x) between points of tangency?



81. In the rectangle ABCD, the perpendicular bisector of AC divides the longer side AB in a ratio 2:1. Then the angle between AC and BD is
(a) 30°
(b) 45°
(c) 60°
(d) 90°

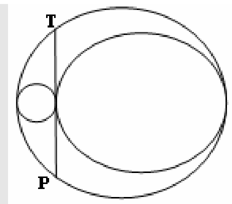
82. An isosceles right triangle is inscribed in a square. Its hypotenuse is a midsegment of the square. What is the ratio of the triangle's area to the square's area?



(a) $\frac{1}{4}$ (b) $\frac{\sqrt{2}}{5}$ (c) $\frac{\sqrt{2}}{4}$ (d) $\frac{1}{2}$ (e) None of these

83. When each side of a square is increased by 4 centimeters, the area is increased by 120 square centimeters. By how many centimeters should each side of the original square be decreased in order to decrease the area of the original square by 120 square centimeters?
(a) 5
(b) 6
(c) 7
(d) 8

Three circles are drawn touching each other, their centers lying on a straight line. The line PT is 16 units long and is tangent to the two smaller circles, with points P and T lying on the larger circle.



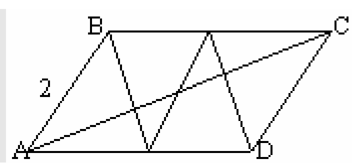
84. The area inside the largest circle but outside the smaller two circles is equal to
(a) 4π
(b) 8π
(c) 16π
(d) 32π

The sum of the number of sides of two regular polygons S_1 and S_2 is 12 and the sum of the number of diagonals of S_1 and S_2 is 19.

85. Then S_1 and S_2 are
(a) square and octagon
(b) heptagon and pentagon
(c) both hexagons
(d) triangle and nonagon

86. If an arc of 45^o on circle A has the same length as an arc of 30^o on circle B, then the ratio of the area of circle A to the area of circle B is
(a) 2: 3
(b) 3: 2
(c) 4: 9
(d) 9: 4

Monty is playing with geometrical shapes made of paper. He cuts four equilateral triangles of side length 2 and joins them together to form a parallelogram ABCD, as shown in the figure.

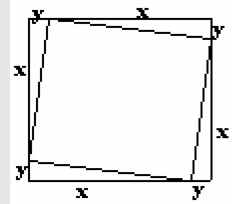


87. What is the length of the diagonal AC?
(a) $\sqrt{8}$
(b) $\sqrt{14}$
(c) $2\sqrt{7}$
(d) 6.5

88. In a circle, chords AB and CD intersect perpendicularly at P. if AP = 20, PB = 36 and CP = 24, then the perimeter of the circle is

(a) $2\pi\sqrt{119}$
(b) $2\pi\sqrt{793}$
(c) $2\pi\sqrt{65}$
(d) $2\pi\sqrt{484}$

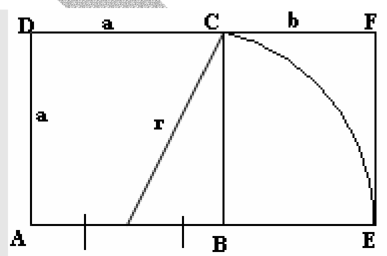
A square is inscribed inside another square, as shown in the figure. Each vertex of the inner square divides the side of the outer square in the ratio $x : y$. The area of the inside square is 4/5th of the area of the bigger square.



89. The value of x/y is equal to

(a) $1+\sqrt{5}$
(b) $\frac{\sqrt{5}}{2}$
(c) $4-\sqrt{15}$
(d) $4+\sqrt{15}$

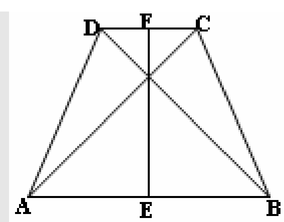
From petal arrangements of roses to shape of our galaxy, the number phi, or the 'golden ratio', is present in many natural phenomena, even in the structure of human body. To find the value of golden ratio, a square of side unity is drawn and midpoint of a side is joined to an opposite vertex, as shown in the figure. Then, an arc of radius r, meeting AB at E is drawn, and the rectangle BEFC is constructed. The value of a + b gives the golden ratio.



90. The value of the golden ratio is

(a) $\frac{1+\sqrt{3}}{2}$
(b) $\frac{\sqrt{3}-1}{2}$
(c) $\frac{\sqrt{5}-1}{2}$
(d) $\frac{\sqrt{5}+1}{2}$

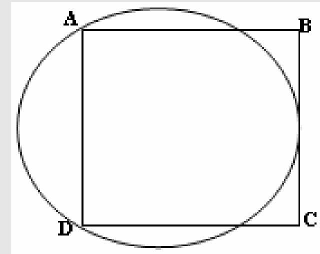
ABCD is an isosceles trapezoid with
AB = 10 and CD = 6. The length of the altitude EF = 8.



91. Then the perimeter of ABCD is

(a) $16 + 2\sqrt{11}$
(b) $16 + 8\sqrt{15}$
(c) $16 + 4\sqrt{17}$
(d) $16 + 4\sqrt{13}$

ABCD is a square with side length 10. A circle is drawn through A and D so that it is tangent to BC.

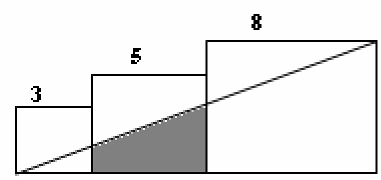


92. What is the radius of circle?
- (a) 5
- (b) 6
- (c) 6.25
- (d) 6.75

93. On a circle with center O, ten points $A_1, A_2, A_3 \ldots A_{10}$ are equally spaced. The value of $\angle A_1A_5O$ is
- (a) 180°
- (b) 72°
- (c) 36°
- (d) 18°

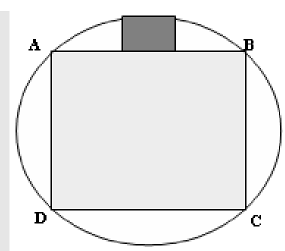
Use the information given below to answer the question that follows.

Three squares of side lengths 3, 5, and 8 are kept side by side. A corner of the smallest square is joined to a corner of the biggest square, as shown in the figure.



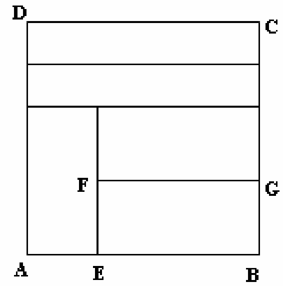
94. What is the area of the shaded figure?
- (a) 10
- (b) 12.5
- (c) 13.75
- (d) 15

Square ABCD is inscribed inside a circle. Another square is inscribed between square ABCD and the circle such that its two vertices are on the circle and one side lies along AB, as shown in the figure.



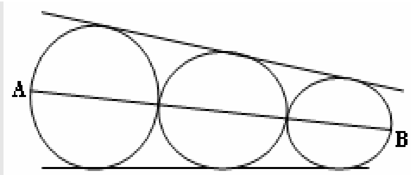
95. The ratio of the length of the sides of the smaller square and bigger square is
- (a) 1/5
- (b) 2/7
- (c) 3/8
- (d) 4/9

ABCD is a square with side length 2 cm. It is divided into five rectangles of equal areas, as shown in the figure.



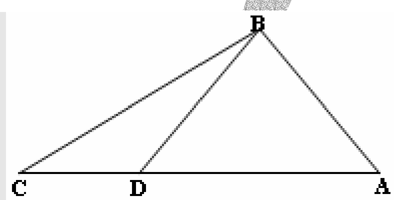
96. The perimeter of the rectangle BEFG is
(a) 51/16
(b) 36/11
(c) 58/15
(d) 47/13

Three circles are in a row touching each other such that all three of them have two common tangents. The radii of the largest and the smallest circle are 9 and 4 respectively. Line segment AB passes through the centres of the circles and lies on the two outer circles.



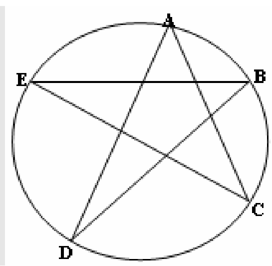
97. What is the length of AB?
(a) 38
(b) 26 + 3√32
(c) 26 + 2√38
(d) 19

In triangle ABC the length of the sides AB, BC, and AC are 12, 18 and 20 units, respectively. D is a point on AC such that AB = DB.



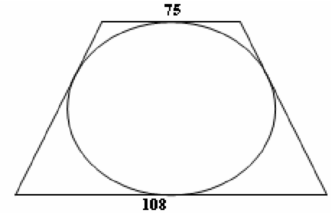
98. The value of the ratio AD: DC is
(a) 3: 2
(b) 11: 9
(c) 7: 3
(d) 3: 1

A star is formed from five points A, B, C, D and E lying on a circle, as shown in the figure.



99. What is the sum of the angles at these five points?
(a) 180°
(b) 270°
(c) 360°
(d) 540°

A circle is inscribed inside an isosceles trapezoid with lengths of its parallel sides as 75 and 108 units, as shown in the figure.



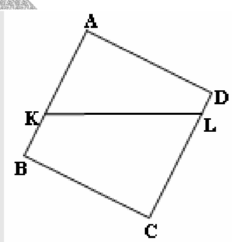
100. The diameter of the inscribed circle is

(a) 87.5
(b) 90
(c) 91.5
(d) 100

101. A square and an equilateral triangle have the same perimeter. What is the ratio of the area of the circle circumscribing the square to the area of the circle inscribed in the triangle?

(a) 16: 9
(b) 18: 5
(c) 24: 7
(d) 27: 8

A cubic container of edge 16 cm is 5/8 full of liquid. It is tilted along an edge. The diagram shows the cross section of the container and the liquid in it. The ratio of length of line segment LC to length of line segment BK is 3: 2 exactly.



102. The length of line segment LC is

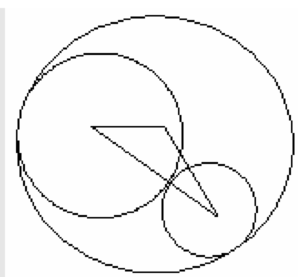
(a) 6

(a) 9
(b) 12
(c) 15

103. Hexagon ABCDEF is inscribed in a circle. The sides AB, CD, and EF are each x units in length whereas the sides BC, DE, and FA are each y units in length. Then, the radius of the circle is

(a) $[(x^2 + y^2 + xy)/3]^{1/2}$
(b) $[(x^2 + y^2 + xy)/2]^{1/2}$
(c) $[(x^2 + y^2 - xy)/3]^{1/2}$
(d) $[(x^2 + y^2 - xy)/2]^{1/2}$

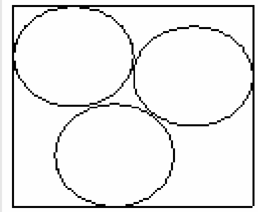
Two circles touch each other externally and also touch a bigger circle of diameter 10 cm internally, as shown in the figure. A triangle is formed by joining the centers of the three circles.



104. The perimeter of the triangle, in cm, is

(a) 5 (b) 10 (c) 15 (d) 20

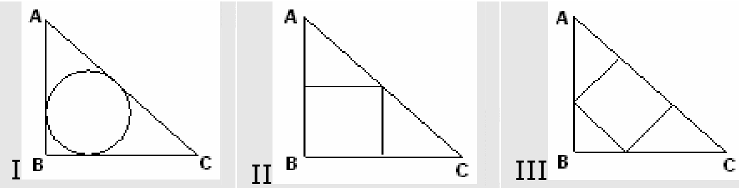
Three circles, each of diameter 4 cm, are kept touching each other. The smallest square circumscribing these circles is drawn, as shown in the figure.



105. What is the length of the side of the square?

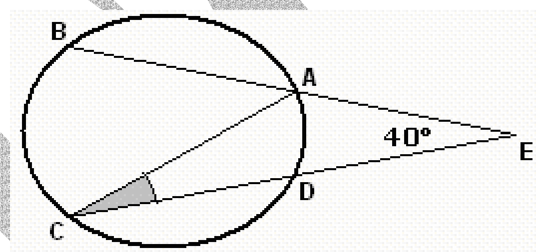
(a)7.14
(b)7.58
(c)7.86
(d)7.92

In a right-angled isosceles triangle ABC, a circle and two separate squares are inscribed, as shown in the figure.



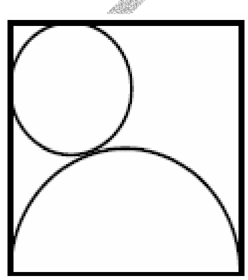
106. The increasing order of the areas of the inscribed figures in the three cases is

(a) I > III > II
(b) II > I > III
(c) I > II > III
(d) II > III > I



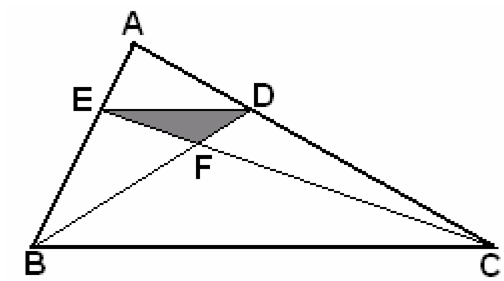
107. In the above figure, arcs AB, BC and CD are equal. Then the value of $\angle ECA$ is equal to

108. A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, **A** and **B**, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. **A** jogs along the rectangular track, while **B** jogs along the two circular tracks in a figure of eight. Approximately, how much faster than **A** does **B** have to run (in percentage), so that they take the same time to return to their starting point?



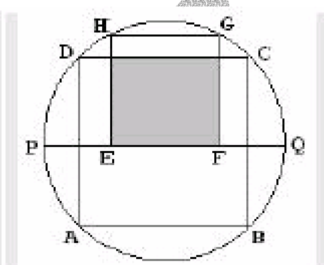
109. The length of the side of the square is 2. Find the radius of the smaller circle.

110. A cone of volume V is cut into three pieces by planes parallel to the base. If the planes are at heights $\frac{h}{3}$ and $\frac{2h}{3}$ above the base, the volume of the piece of the cone between the two planes is



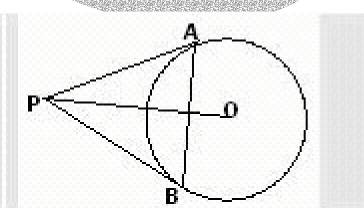
111. In triangle ABC, D and E are points on AC and AB such that DE // BC and length of DE is one-third of BC. If the area of triangle ABC is 216 square units, find the area of the shaded triangle.

ABCD is a square inscribed inside a circle. PQ is a diameter of the circle and is parallel to AB. EFGH is a square inscribed inside the semicircle with diameter PQ. The radius of the circle is 5 cm.

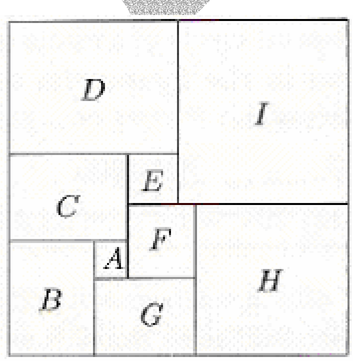


112. What is the value of the shaded area, common to both the squares?

To a circle with center O, tangents PA and PB are drawn from an external point P touching the circle at A and B, respectively, as shown in the figure. Also, $\frac{1}{AO^2} + \frac{1}{PA^2} = \frac{1}{25}$.

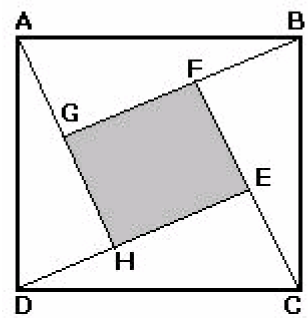


113. The length of the chord AB is



114. 9 squares are arranged as shown in the figure above. If the area of square A is 1cm2 and that of square B is 81 cm2, find the area of square I.

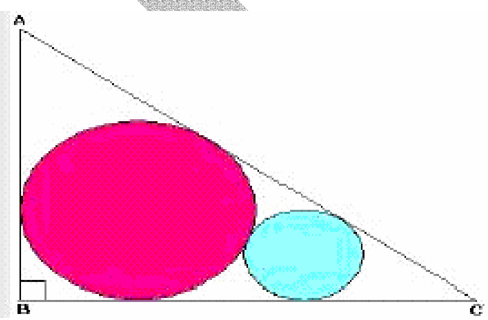
115. Four points **A**, **B**, **C**, and **D** lie on a straight line in the X-Y plane, such that **AB** = **BC** = **CD**, and the length of **AB** is 1 metre. An ant at **A** wants to reach a sugar particle at **D**. But there are insect repellents kept at points **B** and **C.** The ant would not go within one metre of any insect repellent. The minimum distance in metres the ant must traverse to reach the sugar particle.



116. In the figure, ABCD is a square with side length 17 cm. Triangles AGB, BFC, CED and DHA are congruent right triangles. If EC = 8, find the area of the shaded figure.

117. A cow is tied with a 50 m rope to a corner of a 20 m by 30 m rectangular field. The field is completely fenced and the cow can graze on the outside only. What area of the land can the cow graze?

Triangle ABC is a right angled triangle, right-angled at B. The pink circle, of radius 4 cm touches the three sides of the triangle and the blue circle, with radius 1 cm, touches the pink circle and the two sides of the triangle, as shown in the figure.

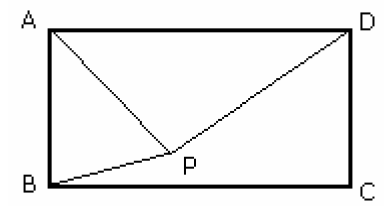


118. Find the length of the side AB.

119. Rectangular tiles each of size 70 cm by 30 cm must be laid horizontally on a rectangular floor of size 110 cm by 130 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should overshoot any edge of the floor. The maximum number of tiles that can be accommodated on the floor is

Carmen is ascending a staircase where every stair is 1ft. high and 1 ft. wide. There is one stair from the ground floor to the first floor, two stairs from the first floor to the second floor…, 100 stairs from the 99^{th} floor to the 100^{th} floor. The stairs take a sharp right turn after every floor. The building has exactly 100 floors.

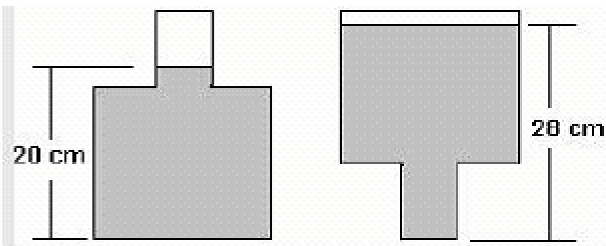
120. At the top of the 100^{th} floor, how far is Carmen from the bottom of the staircase?



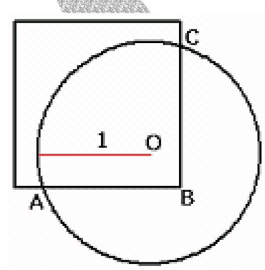
121. P is a point inside rectangle ABCD such that AP = 4 units, BP = 3 units and PD = 5 units. Find the length of PC.

122. All three sides of a triangle have integer side lengths of 11, 60 and ***n*** cm. For how many values of ***n*** is the triangle acute-angled?

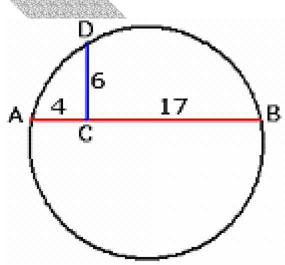
An airtight medicine bottle has been made by fixing a cylinder of radius 3 cm with a cylinder of radius 1 cm. The bottle contains a liquid medicine. When the bottle is up the right way the height of the liquid inside is 20 cm. When the bottle is inverted the height of the liquid inside is 28 cm, as shown in the figure.



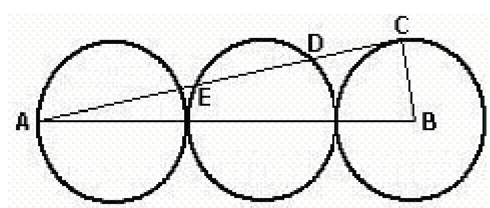
123. What is the total height of the bottle?



124. In the figure, the square and the circle are intersecting each other such that AB = BC. If the radius of the circle, with the centre as O, is 1 unit and OB = $\frac{1}{2}$, then find the length of AB.



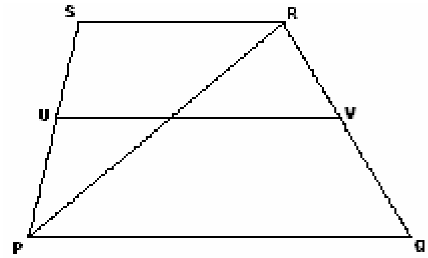
125. In the figure, CD is perpendicular to chord AB. Find the radius of the circle.



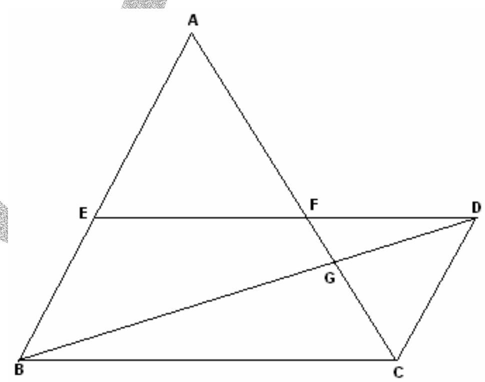
Three circles, each of radius 1 cm, are touching each other. AB is the line passing through the centres of the three circles, with A lying on one of the outermost circle and B being the centre of the other outermost circle, as shown in the figure. AC is tangent to the circle with centre B and cuts chord DE on the middle circle.

126. Find the length of DE.

127. In triangle ABC, AB = 5 cm, BC = 6 cm, and CA = 7 cm. There is a point P inside the triangle such that P is at a distance of 2 cm from AB and 3 cm from BC. How far is P from CA?



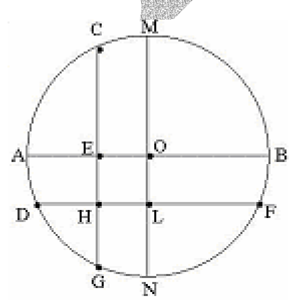
128. In a trapezium PQRS, PQ is parallel to RS and 2PQ = 3RS. UV is drawn parallel to PQ and cuts SP in U & RO in V such that SU: UP = 1: 2. Find the ratio UV : SR

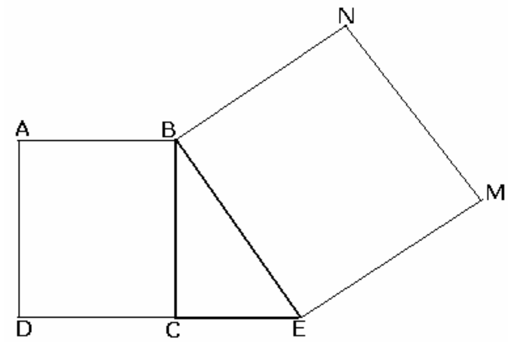


129. In the given figure BCDE is a parallelogram and F is the midpoint of the side DE. Find the length of AG, If CG = 3cm.

130. **P**, **Q**, **S**, and **R** are points on the circumference of a circle of radius *r*, such that **PQR** is an equilateral triangle and **PS** is a diameter of the circle. What is the perimeter of the quadrilateral **PQSR**?

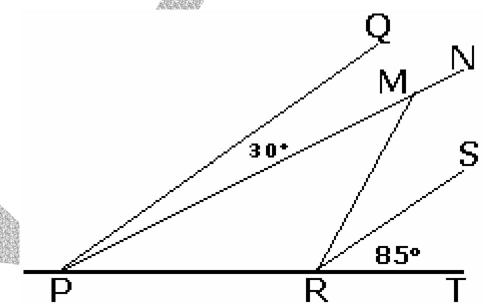
131. In the following figure, the diameter of the circle is 3 cm. **AB** and **MN** are two diameters such that **MN** is perpendicular to **AB**. In addition, **CG** is perpendicular to **AB** such that **AE**:**EB** = 1:2, and **DF** is perpendicular to **MN** such that **NL**:**LM** = 1:2. The length of **DH** in cm is





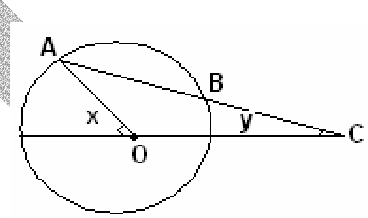
132. In the given figure, ABCD is a square of side 3cm. If BEMN is another square of side 5cm and BCE is a triangle right-angled at C, then the length of CN will be

(a) $\sqrt{56}cm$ (b) $\sqrt{57}cm$ (c) $\sqrt{58}cm$ (d) $\sqrt{59}cm$



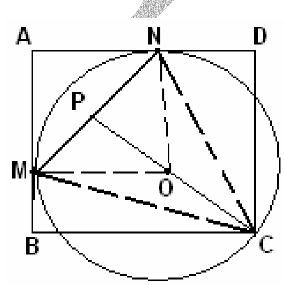
133. In the given figure, if PQ || RS, $\angle QPM = 30°, \angle SRT = 85°$ and RM = RP, then $\angle RMN$ is equal to

(a) 90° (b) 110° (c) 115° (d) 125°



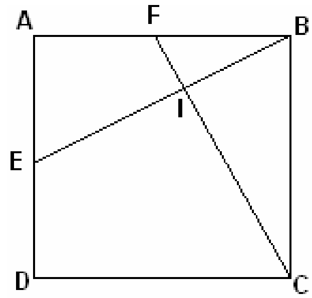
134. If O is the centre of the given circle and BC = AO, then

(a) 2x = y (b) x = 3y (c) 3x = y (d) x = 2y



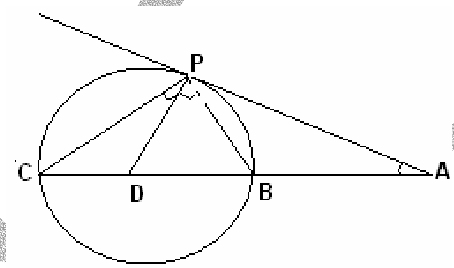
135. A circle passes through the vertex C of a rectangle ABCD and touches its sides AB and AD at M and N respectively. If the distance from C to the line segment MN is equal to 5 units find the area of rectangle ABCD.

(a) $25\ units^2$ (b) $30\ units^2$ (c) $35\ units^2$ (d) $40\ units^2$



136. In the given figure, ABCD is a square of side 1 unit. E and F are midpoints of AD and AB respectively. I is the point of intersection of BE and CF. Then the area of quadrilateral IEDC is

(a) $\frac{11}{16}unit^2$ (b) $\frac{11}{18}unit^2$ (c) $\frac{11}{20}unit^2$ (d) $\frac{7}{24}unit^2$



137. In the adjoining figure, AP is tangent to the circle at P, ABC is a secant and PD is the bisector of $\angle BPC$. Also, $\angle BPD = 25^o$ and ratio of angle $\angle ABP$ and $\angle APB$ is 5:3. Find $\angle APB$.

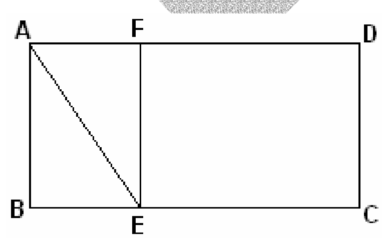
(a) $75°$ (b) $125°$ (c)) $65°$ (d) None of these

138. In a triangle ABC, the internal bisector of the angle A meets BC at D. If AB=4, AC=3 and $\angle A = 60°$, then the length of AD is

(a) $2\sqrt{3}$ (b) $\frac{12\sqrt{3}}{7}$ (c) $\frac{15\sqrt{3}}{8}$ (d) $\frac{6\sqrt{3}}{7}$

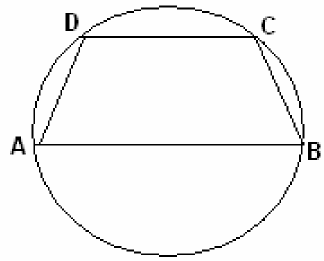
139. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is (in cm)

(a) 24 (b) 25 (c) 15 (d) 20

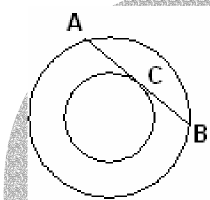


140. In the figure given above, ABCD is a rectangle. The area of the isosceles right triangle ABE= $7cm^2$; EC=3(BE). The area of ABCD $\left(in\ cm^2\right)$ is

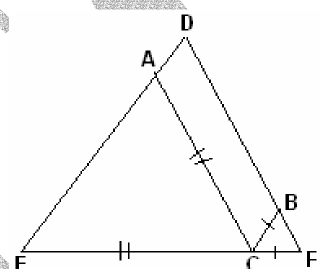
(a) 21 (b) 28 (c) 42 (d) 56



141. In the given figure, AB is a diameter of the circle and points C and D are on the circumference such that $\angle CAD = 30°$ and $\angle CBA = 70°$. What is the measure of $\angle ACD$?
(a) $40°$ (b) $50°$ (c) $30°$ (d) $90°$

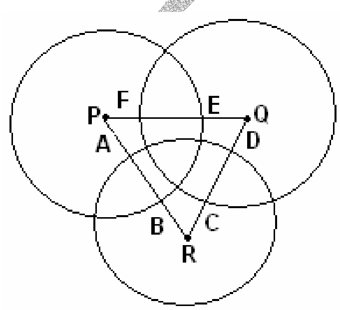


142. The line AB is 6metres in length and tangent to the inner one of the two concentric circles at point C. It is known that the radii of the two circles are integers. The radius of the outer circle is
(a) 5 m (b) 10 m (c) 6 m (d) 4 m



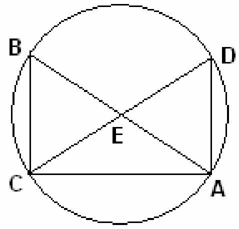
143. In triangle DEF shown below, points A, B, and C are taken on DE, DF and EF respectively such that EC=AC and CF = BC. IF angle D=40 degrees then what is angle ACB in degrees?
(a) 140 (b) 70 (c) 100 (d) None of these

144. If a, b and c are the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$, then the triangle is
(a) equilateral (b) isosceles (c) right-angle (d) obtuse-angled



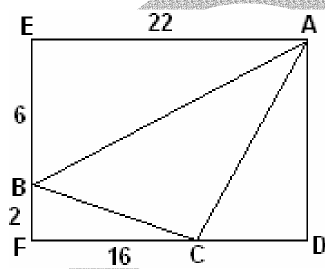
145. Shown above are three circles, each of radius 20 and centres at P, Q and R; further AB=5, CD=10 and EF=12. What is the perimeter of the triangle PQR?

(a) 120 (b) 66 (c) 93 (d) 87



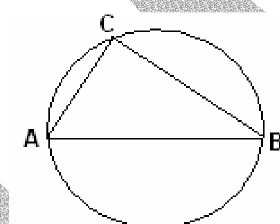
146. In the given figure, points A, B, C and D lie on the circle. AD=24 and BC=12. What is the ratio of the area of ΔCBE to that of ΔADE?

(a) 1:4 (b) 1:2 (c) 1:3 (d) Data insufficient

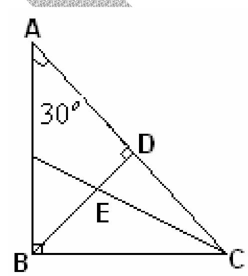


147. In the given figure, EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of EADF. AE = 22, BE = 6, CF = 16 and BF = 2. Find the length of the line joining the midpoints of the sides AB and BC.

(a) $4\sqrt{2}$ (b) 5 (c) 3.5 (d) None of these

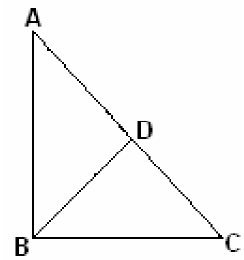


148. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5cm long, find the area of ΔABC.



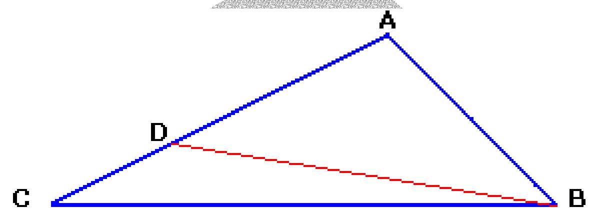
149. AB ⊥ BC, BD ⊥ AC and CE bisects $\angle C, \angle A = 30°$. Then what is $\angle CED$?

(a) 30^o (b) 60^o (c) 45^o (d) 65^o

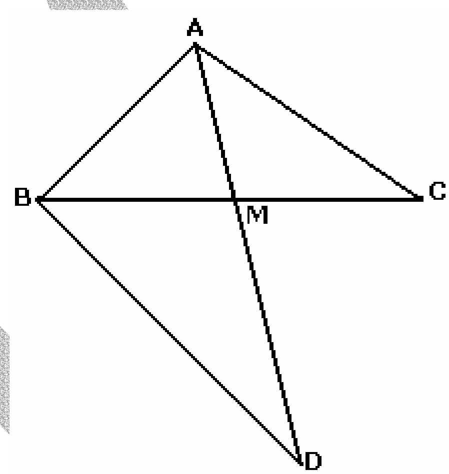


150. In “AABC, $\angle B$ is a right angle, AC=6cm, and D is the midpoint of AC. The length of BD is

(a) 4cm (b) $\sqrt{6}cm$ (c) 3cm (d) 3.5cm

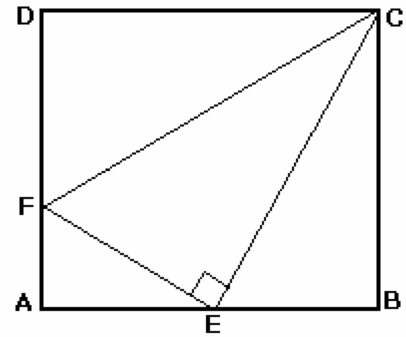


151. In ∆ABC, a point D is on AC such that AB = AD and ∠ABC – ∠ACB = 40°. Then the value of ∠CBD is equal to

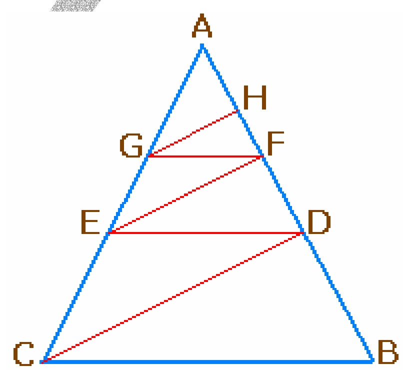


152. In ∆ABC, median $\overline{AM}$ is such that m ∠BAC is divided in the ratio 1:2, and $\overline{AM}$ is extended through to D so that ∠DBA is a right angle, then the ratio AC: AD is equal to

(a) 1: 2
(b) 1: 3
(c) 1: 1
(d) 2: 3
(e) 3: 4

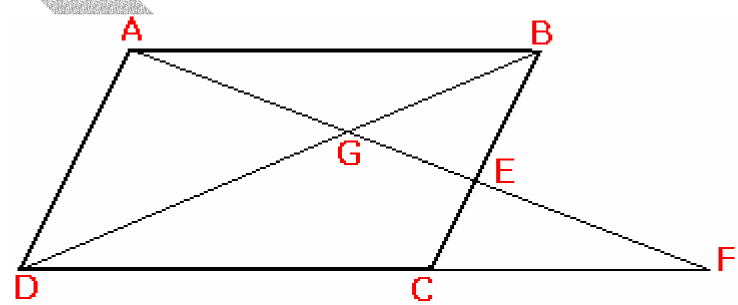


153. In square ABCD, E is the midpoint of $\overline{AB}$. A line perpendicular to $\overline{CE}$ at E meets $\overline{AD}$ at F. What fraction of the area of square ABCD is the area of triangle CEF?



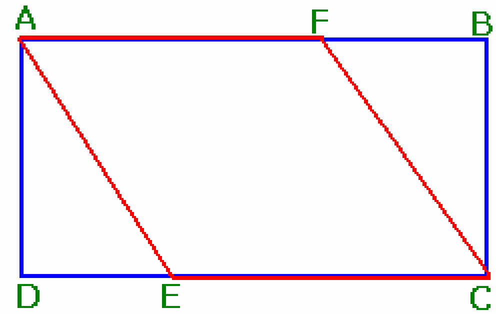
154. In △ABC, $\overline{DE} \parallel \overline{BC}, \overline{FE} \parallel \overline{DC}$, AF = 8, and FD = 12. Find DB.

155. The measure of the longer base of a trapezoid is 97. The measure of the line segment joining the midpoints of the diagonals is 3. Find the measure of the shorter base.



156. In ▱$ABCD$, E is on $\overline{BC}$. $\overline{AE}$ cuts diagonal $\overline{BD}$ at G and $\overleftrightarrow{DC}$ at F. If AG = 6 and GE = 4, find EF.

157. In △ABC, median $\overline{AD}$ is perpendicular to median $\overline{BE}$. Find AB if BC = 6 and AC = 8.



158. On sides $\overline{AB}$ and $\overline{DC}$ of rectangle ABCD, points F and E are chosen so that AFCE is a rhombus. If AB = 16 and BC = 12, find EF.

159. The radius of a cylinder is increased by 16.67%. By what percent should the height of the cylinder be reduced to maintain the volume of the cylinder?
(a) 10% (b) 14.28% (c) 16.67% (c) 20% (e) 25%

160. The ratio of the radius and height of a cylinder is 2:3. Further the ratio of the numerical of value of its curved surface area to its volume is 1:2. Find the total surface area of the cylinder.
(a) 3π units (b) 6π units (c) 12π units (d) 24π units (e) 48π units

161. An ant starts from a point on the bottom edge of a circular cylinder and moves in a spiral manner along the curved surface area such that it reaches the top edge exactly as it completes two circles. Find the distance covered by the ant if the radius of the cylinder is $\frac{12}{\pi}$ and height is 20 units?

(a) $2\sqrt{61}$ units (b) $4\sqrt{61}$ units (c) 24 units (d) 52 units (e) 60 units

162. A sphere is carved out of a cone with height 15cm and radius of base circle 12cm. What is the maximum volume of the cylinder?

163. A right circular cone of volume 'P', a right circular cylinder of volume 'Q' and sphere of Volume 'R' all have the same radius, and the common height of the cone and the cylinder is equal to the diameter of the sphere. Then :
(a) P − Q + R = 0
(b) P + Q = R
(c) 2P = Q + R
(d) $P^2 - Q^2 + R^2 = 0$
(e) 2P + 2Q = 3R

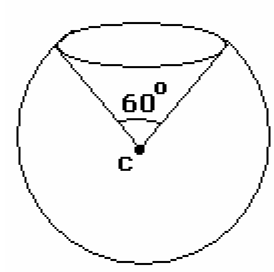
164. The volume of the solid generated by the revolution of an isosceles right angled triangle about its hypotenuse of length 3x is:
(a) $\frac{8\pi x^3}{3}$
(b) $8\pi x^3$
(c) $\frac{9}{4}\pi x^3$
(d) $\frac{27\pi x^3}{3}$

(e) $\frac{32\pi x^3}{3}$

165. A sphere of 10.5cm radius is melted and cast into a cuboid of maximum volume. The total surface area of such cuboid is approximately :

(a) 1720 (b) 1650 (c) 1810 (d) 1932 (e) 1680

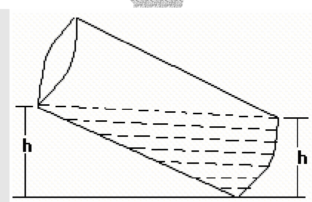
166. One day sanjeev planned to make lemon tea and used a portion of the spherical lemon as shown in the figure. Find out the volume of the remaining lemon. Radius of the lemon is 6cm.



167. A solid metallic cylinder of base radius 3 cm and height 5 cm is melted to make 'n' solid cones of height 1 cm and base radius 1 mm. Find the value of 'n'.

168. Three cubes of volumes, 1 cm^3, 216 cm^3 and 512 cm^3 are melted to form a new cube. What is the diagonal of the new cube?

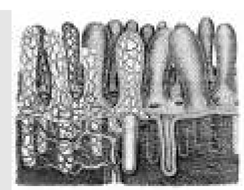
A cylinder with height and radius in a ratio of 2: 1 is full of soft drink. It is tilted so as to allow the soft drink to flow off till the point where the level of soft drink just touches the lowest point of the upper mouth and the highest point of the base, as shown in the figure.



169. If 2.1 L soft drink is retained in the cylinder, what is the capacity of the cylinder?

(a) 3.6 L (b) 4 L (c) 1.2 L (d) 4.2 L (e) 5 L

To facilitate the absorption of food, the inside walls of the small intestine are covered with finger-like tiny projections called villi, as shown in the figure. Every villi can be assumed to be a cylinder of length 1.5×10^{-3} m and radius 1.3×10^{-4} m. It can be assumed that villi cover the walls of the intestine completely.



170. By what fraction is the absorption area of the intestinal wall increased (approximately) because of the villi?

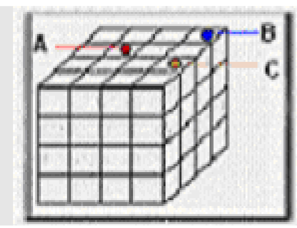
(a) 25 (b) 30 (c) 35 (d) 40 (e) 45

A human head can be assumed a sphere with average diameter of 14 cm. Five-eighths of a human head is covered with hair, with 300 hairs per square centimeters. The average daily hair loss for humans is one hair per fifteen hundred hairs.

171. According to the given data, how many hairs, on average, does a person lose daily?

(a) 77 (b) 80 (c) 87 (d) 115 (e) 308

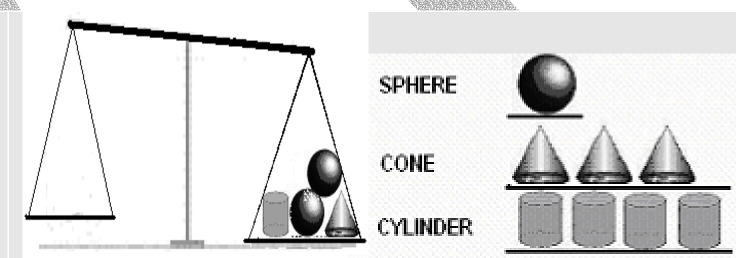
A large 4 cm × 4 cm × 4 cm cube is made up of 64 unit cubes, as shown in the figure. Three unit cubes, A, B, and C (also shown) are marked on the larger cube. The three cubes are taken out all three at a time.



172. What is the area of the remaining solid?

(a) 100 (b) 101 (c) 102 (d) 104 (e) 105

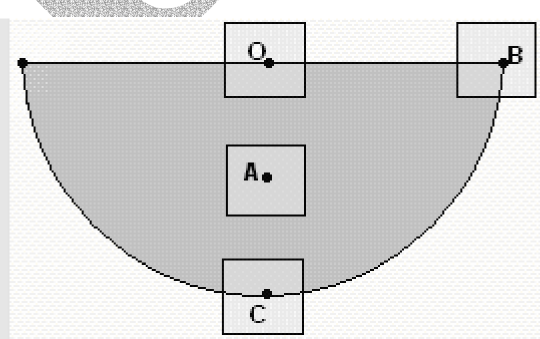
There are three types of solids, spheres, cones and cylinders. The diameter and heights of all the objects are equal. All the solids are made of the same material. Four of the objects, a cylinder, a cone, and two spheres are kept on one pan of a beam balance, as shown in the figure. To balance the beam, 8 solids of the same dimensions and material, a sphere, three cones and four cylinders, are available.

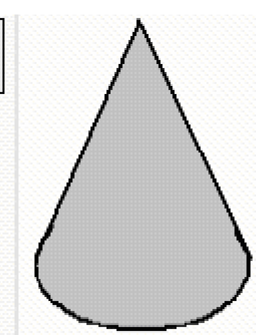


173. In how many ways can you balance the beam?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

A semi circular strip of paper, with radius 10 cm, has O as its centre, and A and C as the midpoints of OC and the semicircular part, respectively. The strip is rolled to form a perfect right circular cone without overlap of any two surfaces.





174. Which of the four points shown forms the apex of the cone?

(a) A (b) B (c) C (d) O (e) None of these

175. What is the volume of the cone?

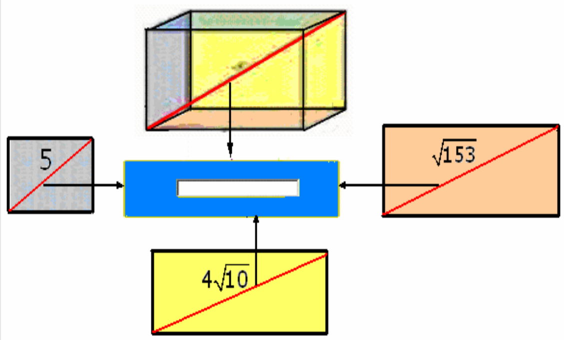
(a) $\frac{250\pi}{3}$

(b) $\frac{125\pi}{\sqrt{3}}$

(c) $\frac{1000\pi}{\sqrt{3}}$

(d) $\frac{1000\pi}{3}$

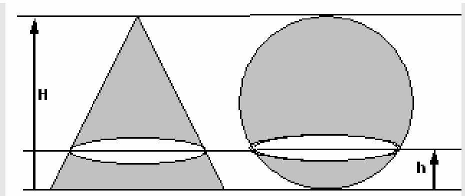
In a cuboidal box of unknown dimensions, a rod is kept along the body diagonal as shown in the figure. The length of the rod when observed from the side, front and bottom of the box, appears to be 5, $4\sqrt{10}$ and $\sqrt{153}$ cm respectively, as shown. Te correct length of the rod is to be filled in the space provided inside the box in the figure



176. The length of the rod is

(a) 13 (b) $2\sqrt{29}$ (c) $3\sqrt{13}$ (d) 15

A right circular cone of height H cm and base diameter H cm, and a sphere of diameter H cm are kept on a horizontal plane. If a horizontal plane slices both the solids, both the cross-sections will be circles. A horizontal plane at height h gives cross-sections of equal areas with both the cone and the sphere, as shown in the figure.



177. The value of height h is

(a) $\frac{H}{3}$ (b) $\frac{H}{4}$ (c) $\frac{H}{5}$ (d) $\frac{H}{6}$

178. If the height of a cylinder is decreased by 10% and the radius of the cylinder is increased by 10%, then the volume of the cylinder

(a) remains unchanged

(b) decreases by 8.9%

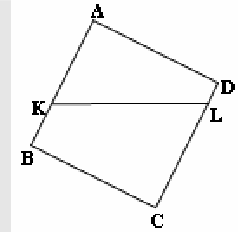
(c) increases by 8.9%

(d) increases by 10.9%

179. A sphere is inscribed in a cone whose radius and height are 12 and 16 units, respectively. Then, the volume of the sphere is

(a) 216π (b) 256π (c) 288π (d) 312π

A cubic container of edge 16 cm is 5/8 full of liquid. It is tilted along an edge. The diagram shows the cross section of the container and the liquid in it. The ratio of length of line segment LC to length of line segment BK is 3: 2 exactly.



180. The length of line segment LC is

(a) 6 (b) 9 (c) 12 (d) 15

181. The volume of the circumscribed sphere of a cube C_1 is twice the volume of the inscribed sphere of another cube C_2. Then, the ratio of the surface area S_1 of the inscribed sphere of the cube C_1 and the surface area S_2 of the circumscribed sphere of the cube C_2 is

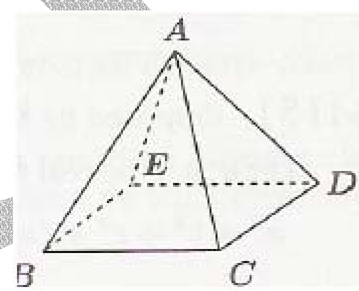
(a) $\frac{2^{\frac{2}{3}}}{9}$ (b) $2^{\frac{2}{3}}$ (c) $\frac{2^{\frac{2}{3}}}{3}$ (d) $\frac{1}{3\times 2^{\frac{2}{3}}}$

182. For two cubes S_1 and S_2, the sum of their volumes is numerically equal to the sum of the lengths of their edges. Then the ratio of the lengths of the edges of S_1 and S_2 is

(a) 2: 1 (b) 3: 1 (c) 3: 2 (d) 3: 4

183. The pyramid ABCDE has a square base, and all four triangular faces are equilateral. Find the measure of the angle BAD (in degrees).

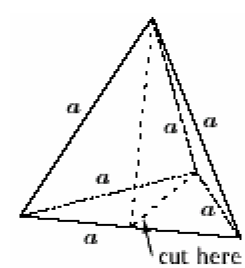
(a) 30° (b)45° (c)60°
(d) 75°



184. A mixing bowl is hemispherical in shape, with a radius of 12 inches. If it contains water to half its depth, then the angle through which it must be tilted before water will begin to pour out is

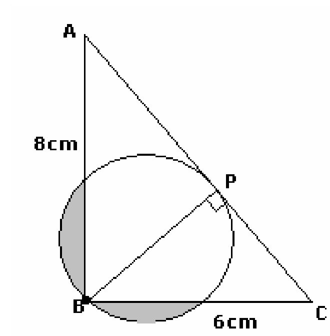
(a) 15° (b)30° (c) 45° (d) 60°

185. The 6 edges of a regular tetrahedron are of length a. The tetrahedron is sliced along one of its edges to form two identical solids. Find the area of the slice.



186. An hourglass is formed from two identical cones. Initially, the upper cone is filled with sand and the lower one is empty. The sand flows at a constant rate from the upper cone to the lower cone. It takes exactly one hour to empty the upper cone. How long does it take for the depth of the sand in the lower cone to be half the depth of sand in the upper cone? (Assume that the sand stays level in both cones at all times)

187. In a right angle triangle ABC, a perpendicular is dropped from vertex B to hypotenuse AC. Now taking BP as diameter a circle is drawn as shown in the figure. Find out the area of shaded portion.
(a) 49.37 cm^2
(b) 52.12 cm^2
(c) 55 cm^2
(d) 47.5 cm^2
(e) 46 cm^2



188. The area of the circle circumscribing the three circles of radius a, as shown in the figure is :

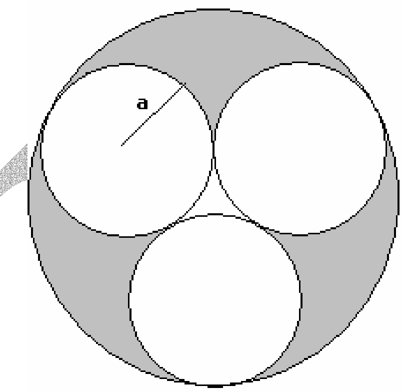
(a) $\frac{\pi(2+\sqrt{3})^2 a^2}{3}$

(b) $6\pi(2+\sqrt{3})^2 a^2$

(c) $3\pi(2+\sqrt{3})^2 a^2$

(d) $\frac{\pi(2+\sqrt{3})^2 a^2}{6}$

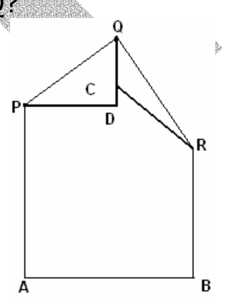
(e) $(2+\sqrt{3})^2 a^2$



189. In the above problem, Find out the area of the shaded portion.

190. If the curved surface area of a cone is twice that of another cone and slant height of the second cone is twice that of the first, find the ratio of the area of their bases.
(a) 2 : 3 (b) 1 : 4 (c) 4 : 1 (d) 8 : 1 (e) 16 : 1

191. The figure when unfolded, becomes a square ABCD with Q lies on CD. IF 2(CQ) = 5(DP) & 4RB=AB=60 cm. What is the area of ΔPDQ?



a.150.2 **b.**151.2 **c.** 152.2 **d.** 153.2 **e.** None of these

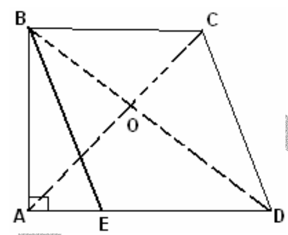
192. A quadrilateral is obtained by joining the midpoints of the adjacent sides of the rhombus ABCD with angle A=60 degrees. This process of joining midpoints of the adjacent sides is continued definitely. If the sum of the areas of all the above said quadrilateral including the rhombus ABCD is $64\sqrt{3}$ sq.cm., what is the sum of the perimeters of all the quadrilaterals including the rhombus ABCD? (in cm).

a. 16(5+√3) **b.** 16(5-√3) **c.** 16(3+√5) **d.** 16(2+√3) **e.** None of these

193. ABCD is a parallelogram in which AB=6√3 cm & BC=6 cm & angle ABC=120 degrees. The bisector of angles A, B, C & D from a quadrilateral PQRS. The area of PQRS in sq.cm. is

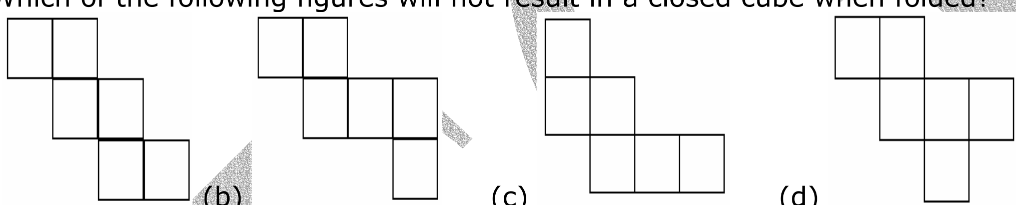
a. 18√3(2-√3) **b.** 18√3 **c.** 36/√3 **d.** 18(2-√3) **e.** None of these

194. In the given figure, EB is parallel & equal in length to DC, the length of ED is equal to the length of DC, the area of triangle ADC=8 units, the area of triangle BDC is 3 units. And angle DAB is right angle. Then the area of triangle AEB is



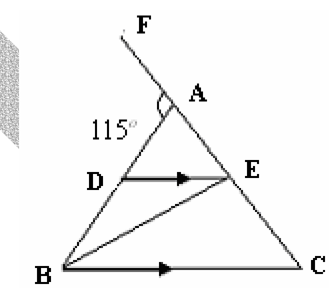
a. 3 units **b.** 5 units **c.** 2 units **d.** 8 units **e.** None of these

195. Which of the following figures will not result in a closed cube when folded?



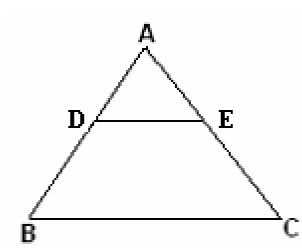
(a) (b) (c) (d)

196. In the given figure AB = AC & EB = BC. If the measure of $\angle BAF = 115^o$ and if DE is parallel to BC, then find the measure of the $\angle DEB$.



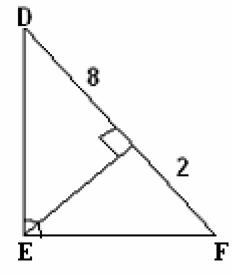
a. 65^o **b.** 75^o **c.** 57.5^o **d.** 35^o **e.** Data inconsistent

197. In ΔABC, DE || BC and the area of the quadrilateral DBCE = 45 sq. cm. If AD : DB = 1 : 3 then find the area of ΔADE.



a. 2 sq. cm **b.** 3 sq. cm **c.** 4 sq. cm **d.** 6 sq. cm **e.** None of these

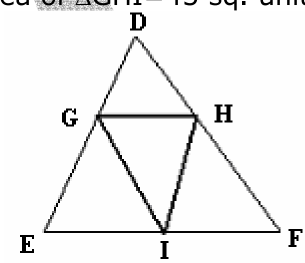
198. ΔDEF is right-angled triangle right angled at E. EG ⊥ DF. If DG = 8cm and GF= 2cm, then find the ratio of DE : EF.



a. 3:2 **b.** 4:3 **c.** 5:4 **d.** 2:1 **e.** None of these

199. A ladder of 85 m length is resting against a wall. If it slips 7 m down the wall, then how far is the bottom from the wall if it was initially 40 m away from it.
a. 34 m **b.** 51 m **c.** 17 m **d.** 26 m **e.** None of these

200. In ΔDEF, points G, I and H are on sides DE, EF and DF respectively such that DG : GE = 3 : 5, EI : IF = 5 : 3, HF : HD = 3 : 2. If the area of ΔGHI=45 sq. units, then find the area of ΔDEF.



a. 128 sq. cm **b.** 64 sq. cm **c.** 192 sq. cm **d.** 256 sq. cm **e.** None of these

201. In ΔPQR right angled at Q, A & B are mid points of the sides PQ & QR respectively. Which of the following is true?

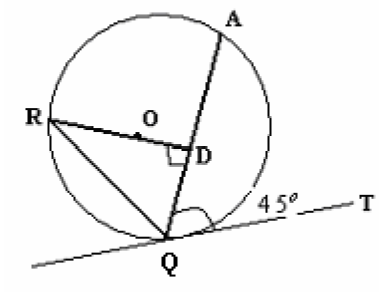
a. $PB^2 + AR^2 = PR^2$ **b.** $PB^2 + AR^2 = \frac{6}{5}PR^2$ **c.** $PB^2 + AR^2 = \frac{7}{6}PR^2$

D. $PB^2 + AR^2 = \frac{5}{4}PR^2$ **e.** None of these

202. In a rectangle ABCD, AB = 8 cm and BC = 6 cm. If CT⊥BD, then find the ratio of BT : DT.
a. 16:9 **b.** 25:9 **c.** 9:16 **d.** 1:4 **e.** None of these

203. ABCD is a parallelogram with AB = 21 cm, BC = 13 cm and BD = 14 cm. Find $AC^2 + BD^2$
a. 1200 **b.** 1220 **c.** 1240
d. 1260 **e.** None of these

204. In the given figure, O is the centre of the circle and QT is a tangent. If the measure of $\angle AQT = 45°$ and AQ = 20 cm, then find the area of the ΔQDR.



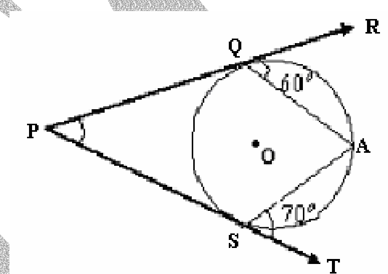
a. $50\left(\sqrt{2}+1\right)cm^2$ **b.** $50\left(\sqrt{2}-1\right)cm^2$ **c.** $50\sqrt{2}cm^2$

d. $50\left(1+\frac{1}{\sqrt{2}}\right)cm^2$ **e.** $50\left(1-\frac{1}{\sqrt{2}}\right)cm^2$

205. In a regular hexagon of side a cm, the mid-points of the three alternate sides are joined in order to form a triangle. What is the ratio of the area of the triangle such formed to area of the hexagon?

a. 2:5 **b.** 7:8 **c.** 3:8 **d.** 1:2 **e.** 2:7

206. In the figure given below, PQR and PST are tangents to the circle whose centre is 'O', touching the circle at Q and S respectively. Also the measure $\angle RQA = 60°$ and $\angle AST = 70$. Find the measure of $\angle QPS$.

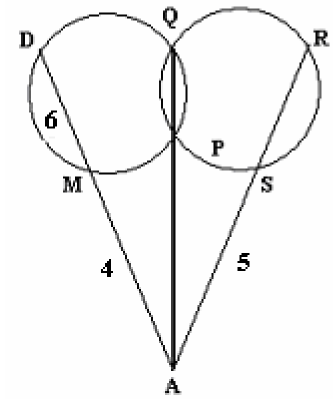


a. 50^o **b.** 70^o **c.** 80^o **d.** 60^o **e.** 40^o

207. A Square is inscribed in a circle of radius 'a' units and an equilateral Δ is inscribed in a circle of radius '2a' units. Find the ratio of the length of the side of square to the length of the side of equilateral triangle.

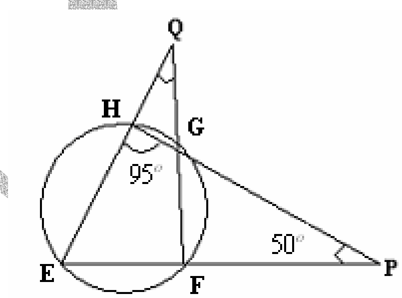
a. 1:3 **b.** $\sqrt{2}:\sqrt{3}$ **c.** $1:\sqrt{3}$ **d.** 2:3 **e.** $1:\sqrt{6}$

208. In the given figure AMD, APQ and ASR are secants to the given circles. IF AM=4 cm, MD=6 cm and AS=5 cm, then find the length of line segment SR.



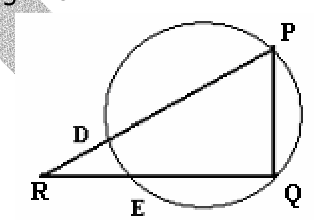
a. 6 cm **b.** 5 cm **c.** 4 cm **d.** 3 cm **e.** 2 cm

209. The sides EF and GH of a cyclic quadrilateral are produced to meet at P, the sides EH and FG are produced to meet at Q. If the measure of $\angle EHG = 95°$ and $\angle GPF = 50°$, then find the measure of $\angle GQH$.



a. 55° **b.** 45° **c.** 35° **d.** 60° **e.** 40°

210. In the following figure, the measure of $\angle PRQ = 45°$ and ΔPQR is right angled at Q. If RD = 3 units and $QE = 5\sqrt{2}$ units, then find the length of PR.



a. 17 units **b.** 20 units **c.** $20\sqrt{2}$ units **d.** 15 units **e.** 16 units

211. O is the centre of circle of radius 'r' units. AOB is a diameter and circles are drawn on OA and OB as diameters. If a circle is drawn to touch these three circles, then its radius will be

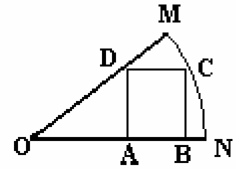
a. $\frac{2r}{3} units$ **b.** $\frac{r}{2} units$ **c.** $\frac{r}{4} units$ **d.** $\frac{r}{3} units$ **e.** $\frac{3r}{4} units$

212. The area of a parallelogram PQRS is B sq. cm. The distance between PQ and SR is 'a_1', cm and the distance between QR and PS is 'a_2' cm. Find the perimeter of the parallelogram PQRS.

a. $\frac{2B(a_1 a_2)}{a_1 + a_2}$ **b.** $\frac{2B(a_1 + a_2)}{a_1 a_2}$ **c.** $\frac{B(a_1 a_2)}{a_1 + a_2}$

d. $\frac{B(a_1 + a_2)}{a_1 a_2}$ **e.** $\frac{B(a_1 + a_2)}{2a_1 a_2}$

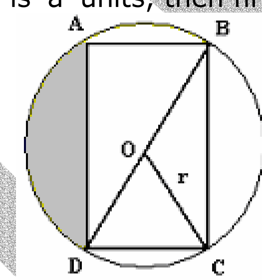
213. In the figure given below, OMN is an octant of a circle having center at O. ABCD is a rectangle with AD = 6 cm and AB = 2 cm. Find the area of the octant of the circle.



a. $8\pi cm^2$ **b.** $8.5\pi cm^2$ **c.** $12\pi cm^2$

d. $10\pi cm^2$ **e.** $12.5\pi cm^2$

214. In the following figure, ABCD is a rectangle and the measure of $\angle ODC$ is $60°$. If the radius of the circle circumscribing the rectangle ABCD is 'a' units, then find the area of the shaded region.



a. $\frac{a^2}{2}\left(2\sqrt{3} - \pi\right) sq.units$ **b.** $\frac{a^2}{2}\left(\pi - \sqrt{3}\right) sq.units$ **c.** $\frac{a^2}{2}\left(2\pi - \sqrt{3}\right) sq.units$

d. $a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) sq.units$ **e.** $a^2\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) sq.units$

215. The areas of the three faces of a cuboid are in the ratio of 1:3:4 and its volume is 144 cu. Cm. Find the length of its longest diagonal.

a. 17 cm **b.** 21 cm **c.** 12 cm **d.** 24 cm **e.** 13 cm

216. A regular prism of length 15 cm is cut into two equal halves along its length, So that the cutting pane passes through two opposite vertices of the hexagonal base. Find the surface area of one of the resultant solids if one side of the hexagon measures 6 cm.

a. 580 sq.cm **b.** 543 sq.cm **c.** 486 sq.cm **d.** 593 sq.cm **e.** 523 sq.cm

217. Sum of the radius of the base and the height of a solid cylinder is 12cm. If the total surface area of the cylinder is $96\pi sq.cm$, then find the height of the cylinder

a.6 cm **b.**8 cm **c.**10 cm **d.**4 cm **e.** None of these

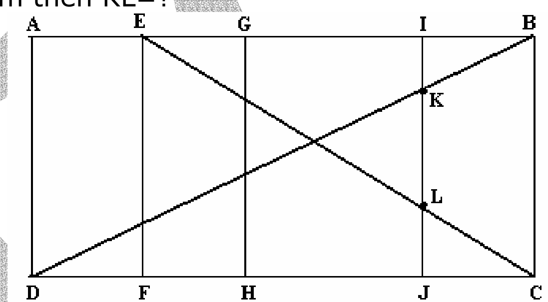
218.A cone has a height h and base radius r. The volume of the cone is bisected by a plane parallel to the base which is at a distance of k from the base. Find the value of k.

a. $\frac{1}{2}h$ **b.** $\left(\frac{1}{2}\right)^{1/2} h$ **c.** $\left(\frac{1}{2}\right)^{1/3} h$ **d.** $\left(1-\left(\frac{1}{2}\right)^{1/3}\right)h$ **e.** None of these

219. A hollow conical flask of base radius 12cm and height 8cm is filled with water. If that flask is emptied into a hemispherical bowl of same base-radius, then what portion of the bowl will remain empty?

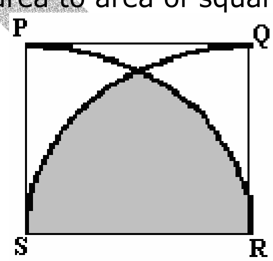
a. 65% **b.** 33.33% **c.** 35% **d.** 66.66% **e.** 75%

220. In the given figure, ABCD is a rectangle which is divided into four equal rectangles by EF, GH, and IJ. If BC =3 cm and AB=8 cm then KL=?



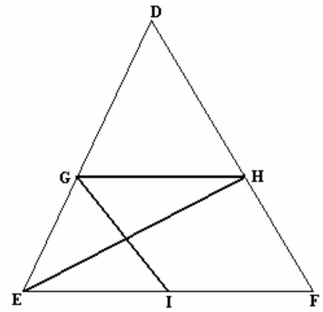
a. 1 cm **b.** 1.25 cm **c.** 1.5 cm **d.** 2 cm **e.** None of these

221. PQRS is a square. Arc PR and QS are drawn on the square PQRS with centre at S and R respectively. Find the ratio of shaded area to area of square PQRS.



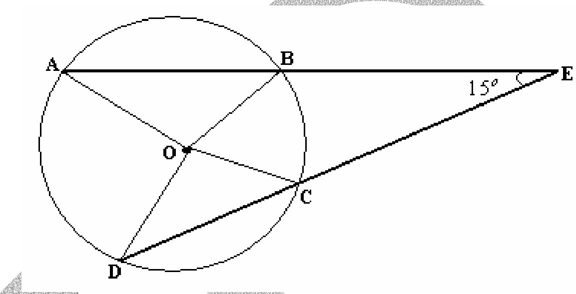
a. $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ **b.** $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ **c.** $\frac{2\pi}{3}-\frac{2\sqrt{3}}{4}$ **d.** $\frac{\pi}{4}-\frac{\sqrt{3}}{8}$ **e.** None of these

222. Area of ΔDEF=10 square units. Given EI : IF = 2 : 3 and area of □GHFI = Area of Δ EFH. What is the area of Δ EFH.



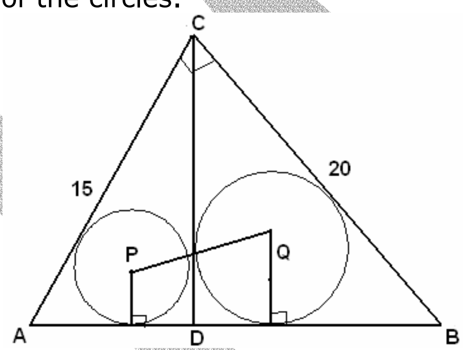
a. 3 **b.** 4 **c.** 5 **d.** 6 **e.** None of these

223. $\angle AOD - \angle BOC = ?$



a. 15 **b.** 22.5 **c.** 30 **d.** 37.5 **e.** None of these

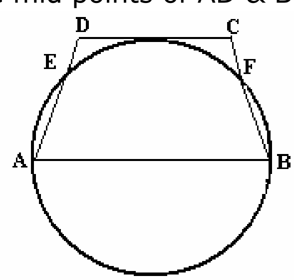
224. In the above figure, ACB is a right angle Δ. CD is the altitude. Circles are inscribed within ΔACD and ΔBCD. P and Q are centres of the circles.



(i) What is ratio of radius of circles inscribed within ΔACD and ΔBCD?
(a) 2:3 (b) 3:4 (c) 4:3 (d) 3:2 (e) None of these

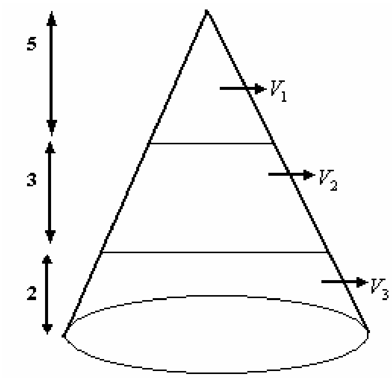
(ii) What is the distance PQ.
(a) 5 (b) $\sqrt{50}$ (c) $\sqrt{30}$ (d) 7 (e) 8

225. ABCD is a trapezium with CD || AB and CD is a tangent to the circle. As shown in the figure. AB is a diameter of the circle. E & F are mid points of AD & BC respectively. What is the $\angle ABC = ?$



a. 55^o **b.** 65^o **c.** 75^o **d.** 85^o **e.** None of these

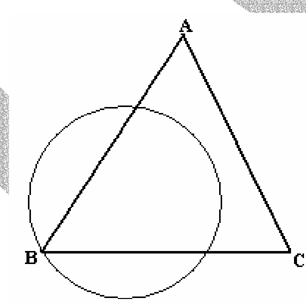
226. A right circular cone of height 10 cm is divided into three parts by cutting the cone by two plane parallel to the base at a height of 2 cm & 5 cm from the base respectively. Find the ratio of $V_1 : V_2 : V_3 = ?$



a. 125 : 383 : 487 **b.** 125 : 387 : 428 **c.** 5 : 3 : 2
d. 5 : 8 : 10 **e.** None of these

Direction for questions 263 and 264: Answer the questions on the basis of the information given below.

A punching machine is used to punch a circular hole of diameter 4 units from a sheet of steel that is in the form of an equilateral triangle of side $4\sqrt{3}$ units as shown in the figure given below. The hole is punched such that it passes through one vertex B of the triangular sheet and the diameter of the hole originating at B is in line with the median of the sheet drawn from the vertex B.



227. Find the area (in square units) of the part of the circle (round punch) falling outside the triangular sheet.

a. $\frac{8\pi}{3} - \sqrt{3}$ **b.** $\frac{4\pi}{3} - \sqrt{3}$ **c.** $2\left(\pi - \sqrt{3}\right)$ **d.** $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ **e.** $\frac{8\pi}{3} - 2\sqrt{3}$

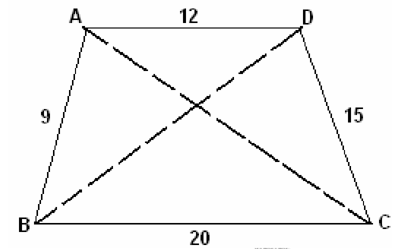
228. The proportion of the sheet area that remains after punching is

a. $\frac{17\sqrt{3} - 2\pi}{18\sqrt{3}}$ **b.** $\frac{15\sqrt{3} - 3\pi}{12\sqrt{3}}$ **c.** $\frac{11\sqrt{3} - 2\pi}{9\sqrt{3}}$ **d.** $\frac{8\sqrt{3} - 3\pi}{6\sqrt{3}}$ **e.** $\frac{15\sqrt{3} - 2\pi}{18\sqrt{3}}$

229. If two parallel sides of a trapezium are 60 cm & 77 cm & other non-parallel sides are 25 cm & 26 cm then the area of the trapezium is

a. 1724 **b.** 1550 **c.** 1475 **d.** 1644 **e.** None of these

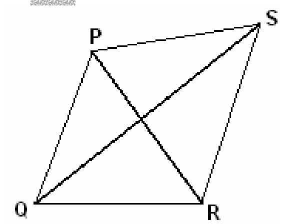
230. ABCD is a trapezium. Find $AC^2 + BD^2$



a. 686 **b.** 786 **c.** 784 **d.** 886 **e.** None of these

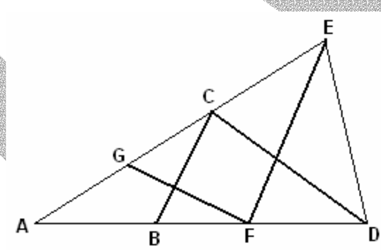
Some Unsolved Problems

231. In the fig below, ∆PQR is equilateral, PQRS is a quadrilateral in which PQ = PS. Find angle QSR in degrees.



a. 60 **b.** 30 **c.** 45 **d.** 15 **e.** None of these

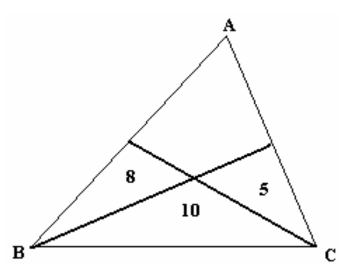
232. In the fig. below,



AB=BC=CD=DE=EF=FG=GA. Then angle DAE is approximately.

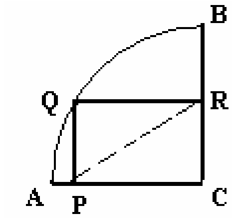
a. 15 **b.** 20 **c.** 30 **d.** 25 **e.** 22

231. ∆ABC is divided into four parts by straight lines from two of its vertices. The area of the three triangular part are 8 sq unit, 5 sq unit & 10 sq unit. What is the area of the remaining part (in sq units)?



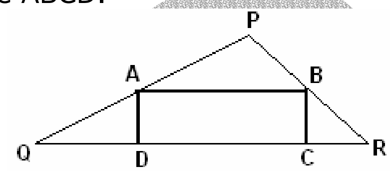
a. 32 **b.** 40 **c.** 54 **d.** 22 **e.** None of these

232. The quarter circle has centre C & radius=10. If the perimeter of rectangle CPQR is 26, then the perimeter of APRBQA is



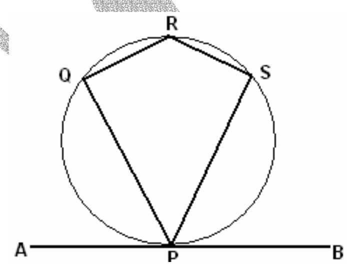
a.112+5pie **b.** 17+5pie **c.** 15+7pie **d.** 13+7pie **e.** None of these

233. A square ABCD is constructed inside a triangle PQR having sides PR = 10 cm, PQ=17 cm & QR=21 cm. Find the perimeter of the square ABCD.



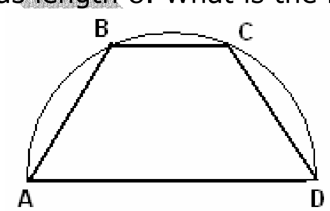
a. 28 **b.** 23.2 **c.** 25.4 **d.** 28.8 **e.** None of these

234. In the following figure, AB touches the circle at P. Also $\angle BPS = 70^o$ and $\angle APQ = 80^o$. If PR bisects $\angle QPS$, then $\angle PQR$ is equal to



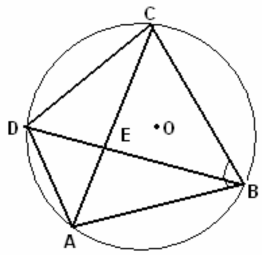
a. 70^o **b.** 85^o **c.** 95^o d. 65^o e. None of these

235. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC?



a. 7.5 **b.** 7 **c.** 7.75 **d.** 7.25 **e.** None of these

236. In the adjoining figure ABCD is a cyclic quadrilateral with AC ⊥ BD and AC meets BD at E. Given that $EA^2 + EB^2 + EC^2 + ED^2 = 100cm$, find the radius of the circumscribed circle.



a. 4 cm **b.** 5 cm **c.** $6\frac{2}{3}$ cm **d.** $8\frac{1}{3}$ cm **e.** None of these

237. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 $A_2, A_3,$......... be the areas and $P_1, P_2, P_3,$..... be the perimeters of S_1, S_2, S_3....., respectively, then the ratio $\frac{P_1 + P_2 + P_3 + ...}{A_1 + A_2 + A_3 + ...}$ equals.

a. $2\left(1+\sqrt{2}\right)/a$

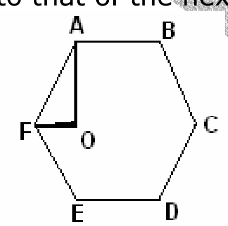
b. $2\left(2-\sqrt{2}\right)/a$

c. $2\left(2+\sqrt{2}\right)/a$

d. $2\left(1+2\sqrt{2}\right)/a$

e. None of these

238. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90^o$, FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?

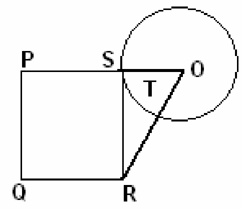


a. $\frac{1}{12}$ **b.** $\frac{1}{6}$ **c.** $\frac{1}{24}$ **d.** $\frac{1}{18}$ **e.** None of these

239. Consider a circle with unit radius. There are seven adjacent sectors, $S_1, S_2, S_3.....S_7$, in the circle such that circle. Further, the area of the jth sector is twice that of the (j-1)th sector, for j=2,…,7. What is the angle, in radius, subtended by the arc of S_1 at the centre of the circle?

a. $\frac{\pi}{508}$ **b.** $\frac{\pi}{2040}$ **c.** $\frac{\pi}{1016}$ **d.** $\frac{\pi}{1524}$ **e.** None of these

240. PQRS is a square. SR is a tangent (at point S) to the circle with centre O and TR = OS. Then, the ratio of the area of the circle to that of the square is

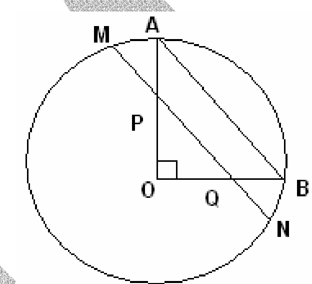


a. $\frac{\pi}{3}$ **b.** $\frac{11}{7}$ **c.** $\frac{3}{\pi}$ **d.** $\frac{7}{11}$ **e.** None of these

241. Let C be a circle with centre P_o and AB be a diameter of C. Suppose P_1 is the midpoint of the line segment P_oB, P_2 is the mid point of the line segment P_1B and so on. Let $C_1, C_2, C_3, \ldots$ be circles with diameters $P_oP_1, P_1P_2, P_2P_3 \ldots$ respectively. Suppose the circles $C_1, C_2, C_3, \ldots$ are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle C is
a. 8:9 **b.** 9:10 **c.** 10:11 **d.** 11:12 **e.** None of these

242. A right triangle contains a 60^o angle. If the measure of the hypotenuse is 4, find the distance from the point of intersection of the 2 legs of the triangle to the point of intersection of the angle bisectors.

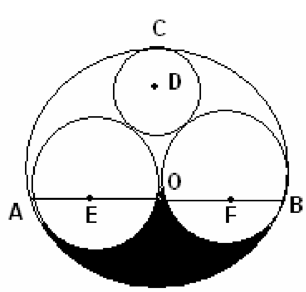
243. Square ABCD is inscribed in a circle. Point E is on the circle. If AB = 8. Find the value of $(AE)^2 + (BE)^2 + (CE)^2 + (DE)^2$.



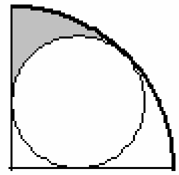
244. Radius $\overline{AO}$ is perpendicular to radius $\overline{OB}, \overline{MN}$ is parallel to $\overline{AB}$ meeting $\overline{AO}$ at P $\overline{OB}$ at Q, and the circle at M and N. If MP = $\sqrt{56}$, and PN = 12, find the measure of the radius of the circle.

245. In circle O, perpendicular chords $\overline{AB}$ and $\overline{CD}$ intersect at E so that AE = 2, EB = 12, and CE = 4. Find the measure of the radius of circle O.

246. A circle with radius 3 is inscribed in a square. Find the radius of the circle that is inscribed between two sides of the square and the original circle.

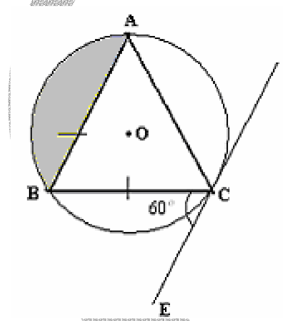


247. $\overline{AB}$ is a diameter of circle O, as shown in Fig. 24. Two circles are drawn with $\overline{AO}$ and $\overline{OB}$ as diameters. In the region between the circumferences, a circle D is inscribed, tangent to the three previous circles. If the measure of the radius of circle D is 8, find AB.



248. A circle is inscribed in a quadrant of a circle of radius 1 unit. What is the area of the shaded region?

249. In the figure given below, ΔABC is circumscribed by a circle with center O. A tangent is drawn touching the circle at C, such that $\angle BCE = 60°$. If AB = BC = 4cm, then find the area of shaded portion.



a. $\frac{16\pi}{3} - 4\sqrt{3}$ **b.** $\frac{(16\pi - 12\sqrt{3})}{9}$ **c.** $\frac{(16\pi - 10\sqrt{3})}{9}$ **d.** $\frac{16\pi}{3} - 5\sqrt{3}$ **e.** None of these

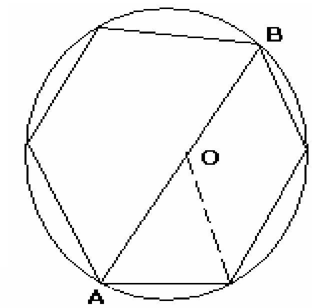
1. Exterior angle = 180 – 178 = 2°

$\Rightarrow \frac{360°}{n}$ = 2 so $\frac{180°}{n}$ = 1°

2. As all the sides of a regular hexagon makes an equilateral Δ with the centre of hexagon so AB = 2a, where a is the side of each hexagon.

$\Rightarrow$ 2a = 2r = 2×6 $\Rightarrow$ a = 6.

So area of the hexagon = $\frac{3\sqrt{3}}{2}6^2 = 54\sqrt{3}.$

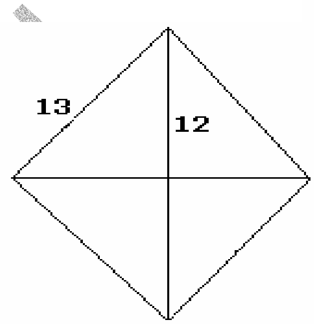


3. Perimeter of the rhombus = 4a = 52

$\Rightarrow$ a= 13cm. one diagonal = 24cm.

$\Rightarrow \frac{1}{2}$ (other diagonal) = $\sqrt{13^2 - 12^2}$ = 5cm.

So length of the other diagonal = $5 \times 2 = 10cm.$



4. Area of rhombus = $\frac{1}{2}.d_1d_2 = \frac{1}{2}.6.10 = 30sqcm$

5. Distance covered by smaller circle in 4 revolution = $4 \times 2.\pi.30 = 240\pi$

Circumference of bigger circle = $2.\pi.40 = 80\pi$

So revolutions taken = $\frac{240\pi}{80\pi} = 3$

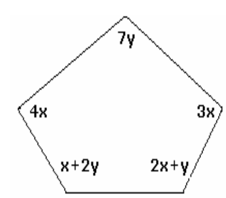
6. Perimeter of the hexagon = 12cm = 6a

$\Rightarrow$ a = 2cms.

So area = $\frac{3\sqrt{3}}{2}.2^2 = 6\sqrt{3}.$

7. Sum of all the angles of a pentagon = (5 – 2) × 180^0 = 540°

$\Rightarrow$ 10x + 10y = 540° $\Rightarrow$ x + y = 54°



8. Each angle of a regular polygon = $\frac{(n-2)180°}{n} = 177°$

$\Rightarrow$ n = 120

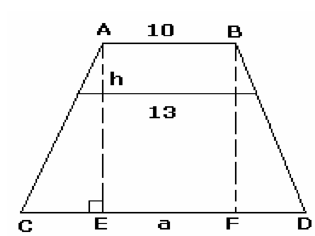
9. Because the two figures are similar, all cones angles would be equal. Hence ∠ J = ∠ A = 120^0

10. Interior ∠ + Exterior ∠ = 180° always.

11. Sum of two consecutive angles of a ∏gm is 180°.

Hence 2x+y+x+2y = 180o $\Rightarrow$ x + y = 60°

12. Mid segment $13 = \frac{10+a}{2}$ $\Rightarrow$ a = 16



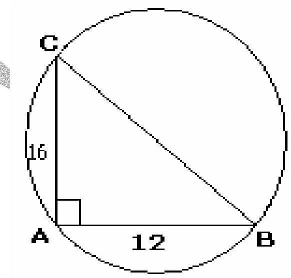
Area of a trapeze = $\frac{1}{2}(10+16)h = 52$

$\Rightarrow$ h = 4cm.

As AB = EF = 10 , CE = FD = $\frac{(16-10)}{2} = 3cm.$

$\therefore AC = \sqrt{AE^2 + CE^2} = \sqrt{4^2 + 3^2} = 5cm.$

Hence perimeter = 10+5+16+5 = 36cms.



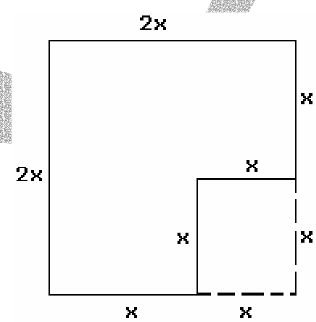
13. BC = $\sqrt{12^2 + 16^2} = 20cm$

$\therefore$ radius of circle = 10cm ($\therefore$ angle in a semi circle is 90°)
$\therefore$ circumference = $2\lambda.10 = 20\lambda cm$

14. Diagonal of a sq with side a = $a\sqrt{2}$

$\therefore a\sqrt{2} = 8$ $\Rightarrow$ a = $\frac{8}{\sqrt{2}} = 4\sqrt{2}$

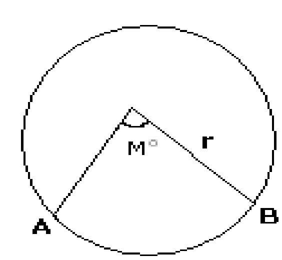
Hence area = $(4\sqrt{2})^2 = 32 = 2^5$. Hence K =5.



15.AS area of original square = $(2x)^2 = 4x^2$.
$\Rightarrow$ area of square cut out = x^2.
So each of side = x cm.
Hence perimeter of rem. Figure = 8x cm.

16. Length of AB = $2\lambda r\frac{M}{360^o} = 8\lambda$

$\Rightarrow$ r, $\frac{M}{360^o} = 4.$



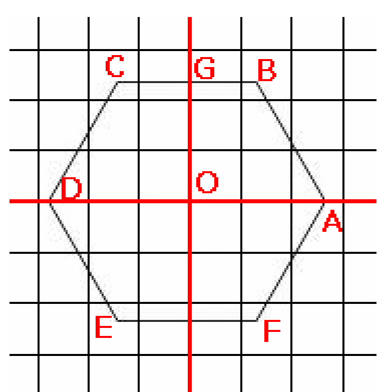
Area of the sector = $\lambda r^2 \frac{M}{360^0} = 48\lambda.$

$\Rightarrow \frac{M}{360^0} = \frac{1}{3}$ $\Rightarrow$ M = 120°.

Hence

$\left(\frac{M}{r}\right) = \frac{120}{12} = 10.$

17. In the figure shown below, ABCDEF is the hexagon, with vertices A and D lying on the x-axis, and midpoints of sides BC and EF lying on the y-axis. If we rotate the portion ABGO about the x-



axis we would get a cylinder with a cone at the top. The radius OG of the cylinder would be the side of the hexagon and height BG would be a/2. The same would be true for the cone.

Therefore, the volume = $\pi a^2 \frac{a}{2} + \frac{1}{3}\pi a^2 \frac{a}{2} = 2\pi \frac{a^3}{3}$

If the portion is rotated about the y-axis, a frustum would be formed of height a, and radii of a and a/2, respectively. The volume = $\frac{\pi a}{3}\left(a^2 + \frac{a^2}{4} + \frac{a}{2}\right) = \frac{7\pi a^3}{12}$. Ratio = $\frac{8}{7} \Rightarrow$ Ratio^2 = $\frac{64}{49}$

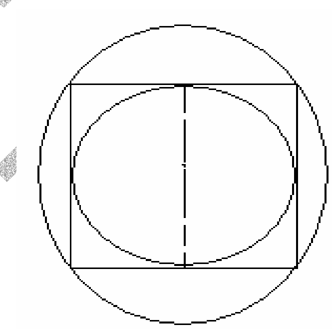
18. Area of smaller circle = $\pi r^2 = \pi$

$\Rightarrow$ r = 1 cm.

So side of square = 2cm.

Diameter of bigger circle = Diagonal of the square = $2\sqrt{2}$.

So are of bigger circle = $\pi\left(\frac{2\sqrt{2}}{2}\right)^2 = 2\pi cm.$



19. If r is the radius of the circle

$\Rightarrow$ A = r × 5, s = $\frac{4+6+8}{2} = 9cm.$, A= $\sqrt{9.5.3.1} = 3\sqrt{15}$

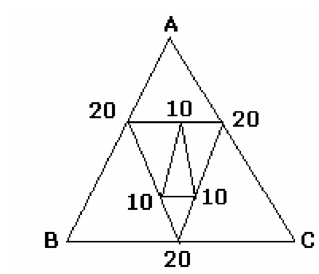
$3\sqrt{15}$ = r × 9 $\Rightarrow$ r = $\sqrt{\frac{5}{3}}cms$

Area of the circle = $\pi.\frac{5}{3}$

20. Area = $\frac{\sqrt{3}}{4}a^2 = 100\sqrt{3} \Rightarrow$ a^2 = 400 $\Rightarrow$ a = 20

The perimeters of every next triangle would be half of its previous triangle. Therefore sum of perimeters = $3 \times 20\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....\right)=$

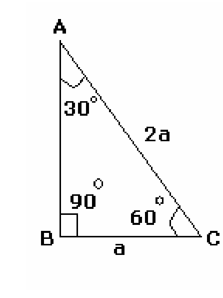
$60 \times \frac{1}{1-\frac{1}{2}}$ = 60×2=120



21. In a 30-60-90 triangle, the side opposite to 30^o is half the hypotenuse.

2a-a= 2002 a= 2002

Therefore longest side= 2a= 4004

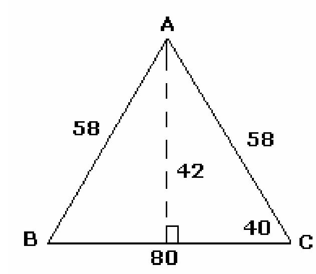


22. Area $\sqrt{s(s-a)(s-b)(s-c)}$ 5= $\frac{7+8+9}{1}$ = 12

Area $\sqrt{12\times5\times4\times3}$ = 12 √5

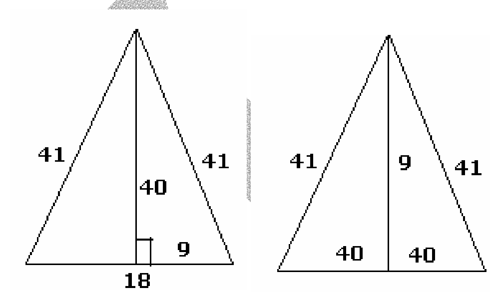
23. Area= 1/2 base X height   1680= $\frac{1}{2}\times80\times$ height

Height= 42. In the given figure AB= AC= 58
Therefore, perimeter= 58+ 58+ 80= 196



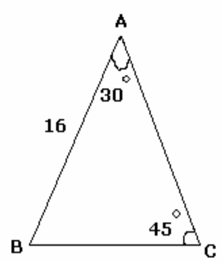
24. Area= $\frac{1}{2}$ base × height= $\frac{1}{2}\times18\times40$

To keep the area constant we can interchange base and height as shown in the second figure. Therefore x= 80



25. $\frac{\sin A}{BC}=\frac{\sin c}{16}\Rightarrow\frac{1}{2BC}=\frac{1}{\sqrt{2}\times16}$

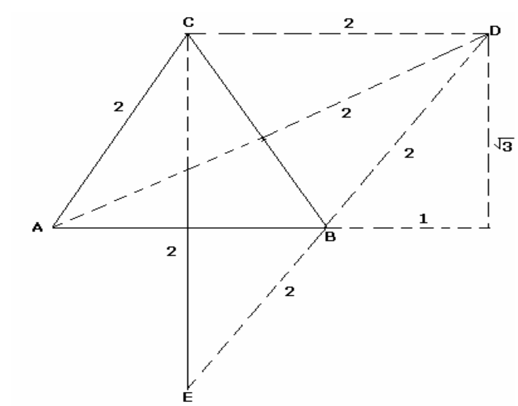
⇒ BC= 8√16



26. In a triangle sum of two sides > third side, keeping this in mind we form the following triangles: (20, 30, 40), (20, 40, 50), (20, 50, 60), (30, 40, 50), (30, 40, 60), (30, 50, 60), (40, 50, 60). Therefore 7 triangles.

27. For an acute- angled triangle, square of any side is less than the sum of the squares of the other two sides. $10^2 + 24^2 > n^2$ and $n^2 + 10^2 > 24^2$....
⇒ n< 26 and n > √476. The values of n satifying the above conditions are 22, 23, 24 and 25. Therefore 4 values.

28. & **29.** as we can see, D is 3km east and √3 north of A.
The total distance walked by the person
= 2×BD+ BE= 6km



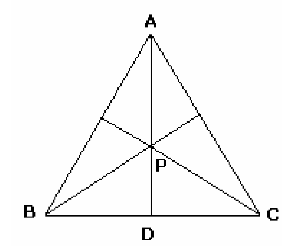
30. The triangles possible are (1, 3, 3), (2, 2, 3). Others triangles are not possible as sum of two sides will not be greater than the third side.

31. The new area= $\frac{1}{2}\times 1.1\times base\times 0.9\times$height = original area $\times$0.99. Therefore the area decreases by 1.1.

32. Area of the triangle ABD= $\frac{1}{2}\times base\times$ height= $\frac{1}{2}\times 3\times 6$= 9 units.

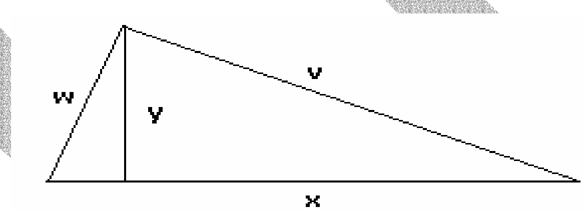
33. In an equilateral triangle, medians, angle bisectors, and altitudes are the some thing

PA= $\frac{2}{3}$ AD= $\frac{2}{3}\times\frac{\sqrt{3}}{2}$x= $\frac{x}{\sqrt{3}}$ = $\frac{2\sqrt{3}}{3}$



34.

Area



$$\frac{1}{2}\times w\times v=\frac{1}{2}\times y\times x\Rightarrow wv=xy$$
$$(w+v)^2=35^2\Rightarrow w^2+v^2+2wv=35^2$$
$$=\Rightarrow x^2+2wv=35^2\ [w^2+v^2=x^2]$$
$$\Rightarrow x^2+2xy=35^2$$
$$\Rightarrow (x+y)^2-y^2=35^2\Rightarrow 37^2-35^2=y^2\Rightarrow y=12$$

35. In the given triangle, as the height for triangle ABD and ADC is the same, the ratio of the areas of the two triangles are in the ratio of their bases.

$$\frac{Area\ \Delta ABD}{Area\ \Delta ADC}=\frac{BD}{DC}=\frac{8}{12}=\frac{2}{3}$$

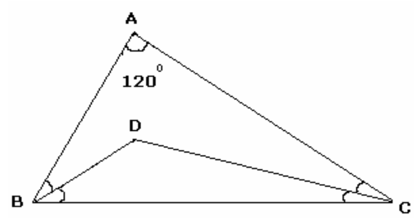
$$Therefore\ area\ \Delta ABD=\frac{2}{5}\times area\Delta ABC=\frac{2}{5}\times 60=24$$

36. Let ∠ADX= ∠AXD= ∝ ⇒ ∠XAD= 180°- 2∝
Let ∠CYD= ∠CDY= β ⇒ ∠DCY= 180°- 2β
∠XAD + ∠DCY= 90°
180°- 2∝ + 180°- 2β= 90° ⇒ ∝ **+**β = 135°
⇒ ∠XDY= 180°- (∝ **+** β) = 45°

37. From the given figure

$3\propto + 3\beta + A = 180^o \quad \propto + \beta = 20^o$

$\angle BDC = 180^o - 2(\propto + \beta) = 140^o$

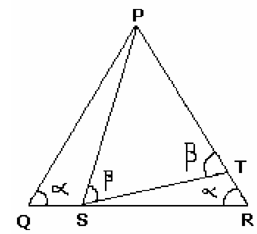


38. Let $\angle PST = \angle PTS = \beta$ and $\angle PQR = \angle PRQ = \propto$

$\angle QPR + \angle PQR + \angle PRQ = 180^o$

$30^o + 180^o - 2\beta + 2\propto = 180^o$

$\beta - \propto = 15^o$. As $\angle STP$ is the external angle of Δ RST $\Rightarrow \beta - \propto = \angle RST = 15^o$



39. Let Length of third side be= x

x < 15 + 7 and 7 + x >15

x < 22 and x> 8. Therefore values of x are 9, 10, 11, 12…., 21 $\Rightarrow$ sum= 195

40. Area Δ CEB= $\frac{1}{2}$(Area Δ BCD) = $\frac{1}{2}\left(\frac{1}{2}\text{area rectangle}ABCD\right)$

Area Δ CEF= $\frac{1}{2}$ Area Δ CEB= $\frac{1}{2} \times \frac{\text{area rectangle ABCD}}{4}$

Area Δ CEF= $\frac{\text{Area of rectangle}}{8} \Rightarrow$ Area Δ CEF: Rest of the area= 1: 7

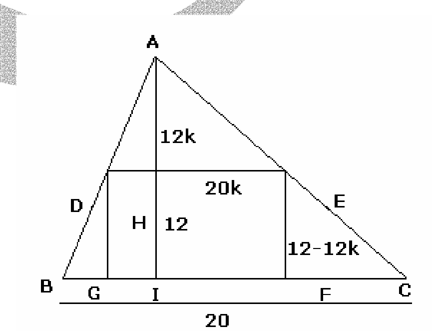
$\Rightarrow$ Amount of crop produced in Δ CEF: Amount of crop produced in rest of the area = 3: 7

41. DE|| BC $\Rightarrow \Delta$ ADE is similar to Δ ABC

Let $\frac{DE}{BC} = k \Rightarrow DE = kBC = 20k$

$\Rightarrow AH = kAI = 12k \Rightarrow EF = HI = 12 - 12k$

Area of DEFG = $20k \times (12 - 12k) = 240k(1 - k)$. This is maximum when k= 1 $\Rightarrow$ area = $240 \times \frac{1}{2} \times \frac{1}{2} = 60$



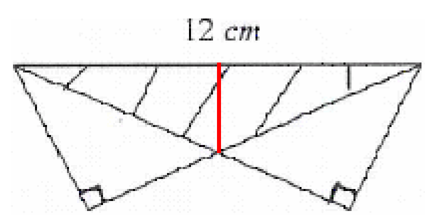
42. Let the angle be θ. $90 - \theta = \frac{80}{100} \times \frac{180 - \theta}{2}$

$\Rightarrow \theta = 30^o$

43. Height of the shaded region (shown by red) = $\frac{6}{\sqrt{3}}$

Therefore, the area of the shaded region = $\frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12 \times \frac{6}{\sqrt{3}} = 12\sqrt{3}$



44. Since DE is perpendicular bisector of BC.

BD = DC = 7

and $\angle DBE = \angle DCE$ (say)

$\Rightarrow \angle ABD = \theta$ (BD is the angle bisector of $\angle B$)

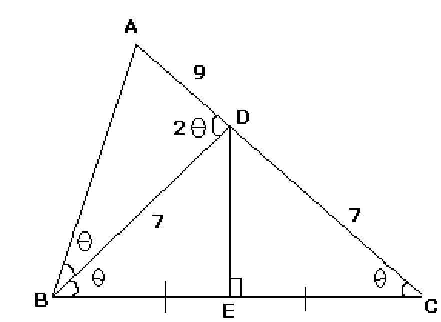
Also $\angle BDA = 2\theta$ (External angle of $\triangle BDC$)

$\Rightarrow \Delta$ ABC is similar to Δ ABD (Three angles equal)

$\Rightarrow \frac{AC}{AB} = \frac{AB}{AD} = \frac{BC}{BD} \Rightarrow AB = AB = \sqrt{AC \times AD} = AB = \sqrt{16 \times 9} =$

12 and $BC = \frac{AB \times BD}{AD} = \frac{12 \times 7}{9} = \frac{20}{3}$

Area $= \sqrt{s(s-a)(s-b)(s-c)} = 14\sqrt{5}$

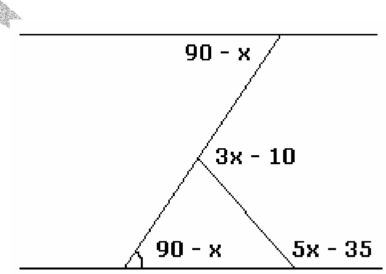


45. From the figure

$5x - 35\ (180 - (3x - 10)) + 90 - x$

$5x - 35 = 180 - 3x + 10 + 90 - x$

$9x = 315 \Rightarrow x = 35^o$

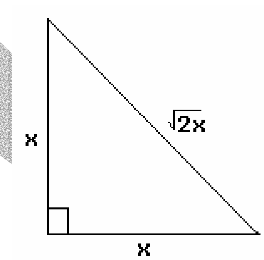


46. Perimeter= $x + x + x\sqrt{2} = 2a$

$\Rightarrow 2x + x\sqrt{2} = 2a \Rightarrow a = \frac{(\sqrt{2}+1)x}{\sqrt{2}}$

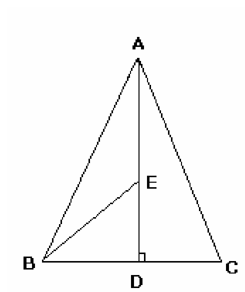
$\Rightarrow x = \frac{\sqrt{2}a}{\sqrt{2}+1} = a\sqrt{2}(\sqrt{2}-1)$

Area= $\frac{1}{2}x^2 = a^2(3 - 2\sqrt{2})$



47. $AD = \frac{\sqrt{3}a}{2} = \frac{16\sqrt{3}}{2} = 8\sqrt{3} \Rightarrow 4\sqrt{3}$

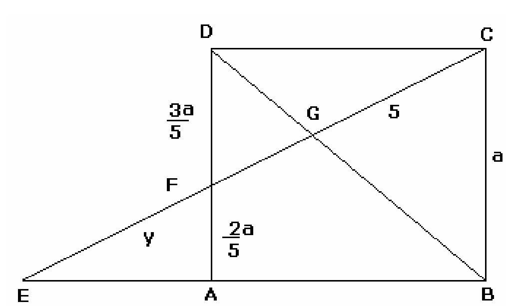
$BE = DE^2 + BD^2 = \sqrt{48+64} = \sqrt{112} = 4\sqrt{7}$



48. ΔDGF and ΔCGB are similar

$\Rightarrow \frac{FG}{GC} = \frac{FD}{BC} \Rightarrow \frac{FD}{BC} = \frac{3}{5} \Rightarrow \frac{FA}{BC} = \frac{2}{5}$

Let EF = y. ΔEFA and ΔECB are similar

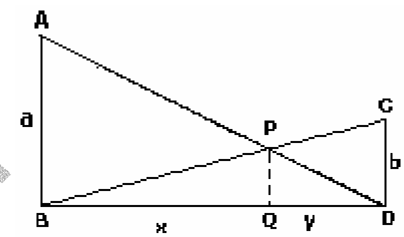


$$\Rightarrow \frac{EF}{EC} = \frac{FA}{BC} \Rightarrow \frac{y}{y+8} = \frac{2}{5} \Rightarrow y = \frac{16}{3}$$

49. Let PQ be the height of the intersection of wires. Let BQ= x and DQ = y.

ΔPQD and ΔABD are similar

$$\Rightarrow \frac{y}{x+y} = \frac{PQ}{a} \quad ---(1)$$



ΔBPQ and ΔBCD are similar

$$\Rightarrow \frac{x}{x+y} = \frac{PQ}{b} \quad ---(2)$$

Now dividing (1) by (2) we get

ΔPQD and ΔABD are similar

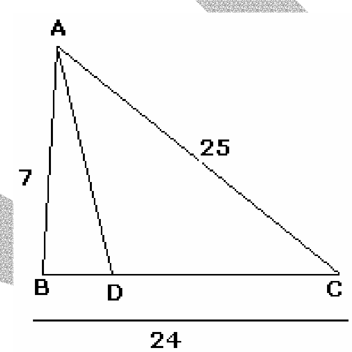
$$\Rightarrow \frac{y}{x} = \frac{b}{a}. \text{ Now } PQ = \frac{bx}{x+y} = \frac{ba}{a+b}$$

50. AD is the angle bisector

$\Rightarrow$ AB = BD $\Rightarrow$ BD = 7 $\Rightarrow$ BD = 7 $\times$ BC= 7 $\times$ 24

$$\Rightarrow AD = \sqrt{AB^2 + BD^2} = \sqrt{49 + \frac{441}{16}}$$

= 8.75



51. Tangents drawn to a circle from the same point are equal in length.

$\Rightarrow$ OE = DG = DF = x (say)

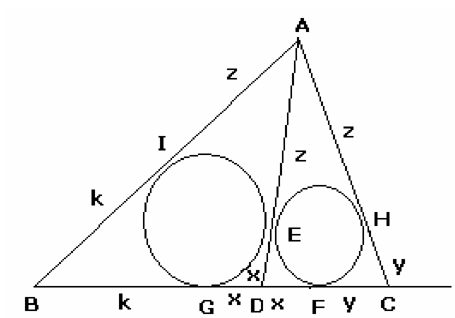
$\Rightarrow$ FC = HC = y (say) AI = AE = AH = z

BG = BI = R (say)

Now AB + BC + CA = k + z+ k+ zx + y +z + y = 2(k+ x+ y +z)

$\Rightarrow$ 40 = 2(k + x + y+ z) $\Rightarrow$ x + y + z + k = 20

z + k = AB = 12 $\Rightarrow$ x + y = CD = 20 - (z + k) = 20 – 12 = 8

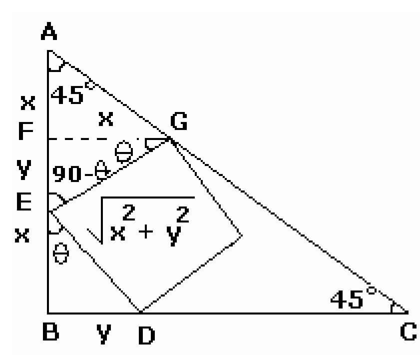


52. We draw a perpendicular GF from the vertex G of the square on the side AB.

ΔEFG and ΔEBD are congruent (three angles equal and one side equal)

$\Rightarrow$ EF = y and FG = x

Now FG = AF ($\angle$FAG = $\angle$AGF = 45°)



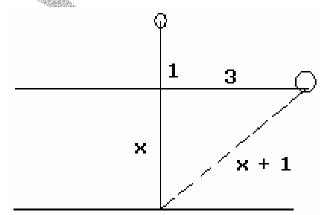
$\Rightarrow$ AF = x
Therefore one side of the right triangle = 2x + y. The other side is also equal to 2x + y $\Rightarrow$ Area of the triangle = $\frac{(2x+y)^2}{2}$

Area of the square = $x^2 + y^2$

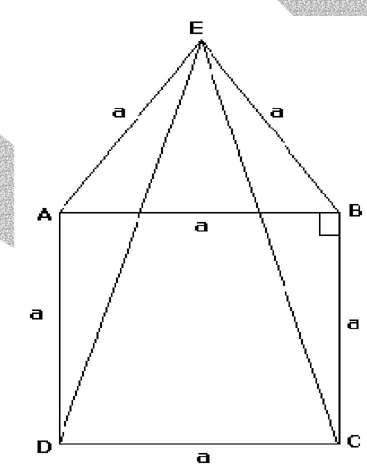
Ratio = $\frac{x^2+y^2}{(2x+y)^2} = 2:5$

53. Let's join A to R, B to P, and C to Q. The median divide the triangle into two equal areas. QC, AR, BP are medians in also ΔQBR, ΔPCR and ΔAPQ, Also PC, BR, AQ are medians in triangle PBR, AQR, PQC. Equating the areas formed by medians, we can see Area ΔPQR = 7area ΔABC = $70cm^2$.

54. From the given figure $(x + 1)^2 = 3^2 + x^2 \Rightarrow x = 4$
Therefore depth of lake = 4m.



55. BE= BC = a $\Rightarrow \angle BEC = \angle BCE$
$\angle CBE = 90^o + 60^o = 150^o \Rightarrow \angle BEC = 15^o$
$\therefore \angle DEC = 30^o$

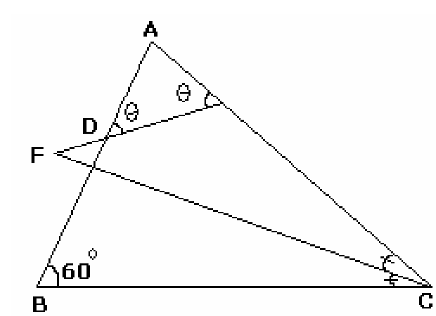


56. $\angle B = 60^o \Rightarrow \angle A + \angle C = 120^o$

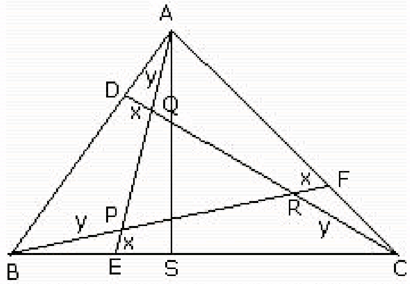
In the figure θ is the external angle of ΔFEC $\Rightarrow \angle EFC + \frac{C}{2}$

$\theta = \frac{180 - A}{2} = 90 - \frac{A}{2} = \angle EFC + \frac{C}{2}$

$\Rightarrow EFC = 90 - (\frac{A}{2} + \frac{C}{2}) = 90 - 60 = 30^o$



57. Here



Have a look at the picture. Since everything is symmetrical, triangle PQR is also equilateral (I can prove it also, but take this for granted right now). Let PE = DQ = RF = x, and PB = AQ = RC = y.

Draw perpendicular AS.

$$BS = \frac{a}{2} \Rightarrow ES = BS - BE = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

$$AS = \frac{\sqrt{3}a}{2}$$

$$\Rightarrow AE = \sqrt{ES^2 + AS^2} = \frac{\sqrt{7}a}{3}$$

ΔBPE and ΔABE are similar [three angles equal]

$$\Rightarrow \frac{BP}{AB} = \frac{PE}{BE} = \frac{BE}{AE}$$

$$\Rightarrow \frac{y}{a} = \frac{x}{a/3} = \frac{a/3}{\sqrt{7}a/3} \Rightarrow y = 3x \text{ and } y = \frac{a}{\sqrt{7}}$$

$$\text{Now } AE = \frac{\sqrt{7}a}{3} = y + x + PQ = \frac{a}{\sqrt{7}} + \frac{a}{3\sqrt{7}} + PQ$$

$$\Rightarrow PQ = \frac{\sqrt{7}a}{3} - \left(\frac{a}{\sqrt{7}} + \frac{a}{3\sqrt{7}}\right) \Rightarrow PQ = \frac{a}{\sqrt{7}}$$

$$\Rightarrow \frac{\text{Area } \Delta PQR}{\text{Area } \Delta ABC} = \frac{PQ^2}{AB^2} = \frac{1}{7}$$

58. ΔAJK, ΔAHI, ΔAFG, ΔAOE and ΔABC are similar

$$\frac{AJ}{JK} = \frac{AH}{HI} = \frac{AF}{FG} = \frac{AD}{DE} = \frac{AB}{BC}$$

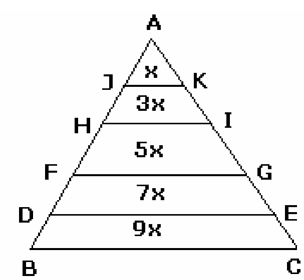
$\Rightarrow$ JK : HE : FG : DE : BC = 1: 2 : 3 : 4: 5

$\Rightarrow$ Area $\triangle$ AJK : $\triangle$AHI: $\triangle$AFG: $\triangle$ADE: ABC = 1: 4: 9: 16: 25

Therefore areas of the parts in between = 1: 3: 5: 7: 9

Are of the largest part = 9x = 27 $\Rightarrow$ x= 3

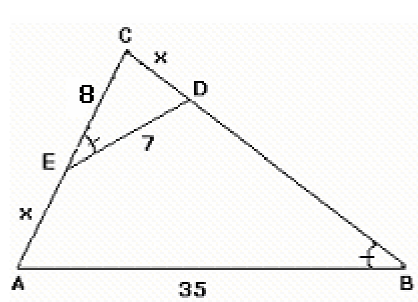
$\Rightarrow$ Area of triangle = 25 × 3= 75



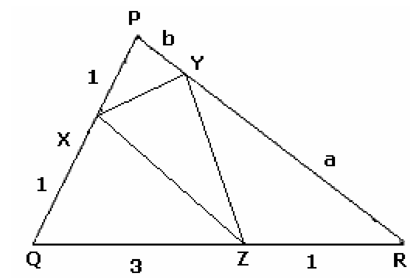
59. ΔCEO and ΔABC are similar (A- A- A)

$$\frac{ED}{AB} = \frac{CD}{AC} = \frac{EC}{BC}$$

$$\frac{7}{35} = \frac{x}{x+8} \Rightarrow x = 2$$



60.



$$\frac{\text{Area } \Delta PXY}{\text{Area } \Delta PQR} = \frac{b}{2(a+b)}$$

$$\frac{\text{Area } \Delta RZY}{\text{Area } \Delta PQR} = \frac{a}{4(a+b)}$$

$$\frac{\text{Area } \Delta QXZ}{\text{Area } \Delta PQR} = \frac{3}{8}$$

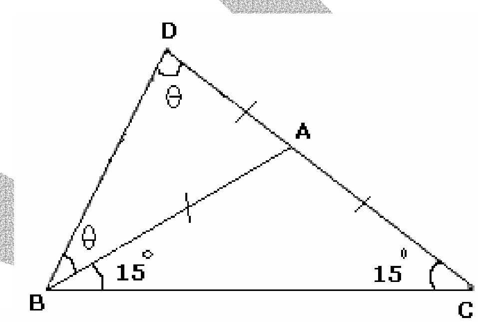
$$\left(\frac{b}{2(a+b)}\right)^2 = \frac{3}{8} \times \frac{a}{4(a+b)}$$

solving we get $\frac{a}{b} = \frac{\sqrt{105}-3}{6}$

61. $\angle OAC = 30^\circ \Rightarrow AO = 2r$

ΔAPR and ΔAOR are similar

$$\frac{PQ}{OR} = \frac{AP}{AO} = \frac{r}{R} = \frac{2R-(r+R)}{2R} \Rightarrow \frac{R}{r} = 3$$



62. Let $\angle BDA = \theta$

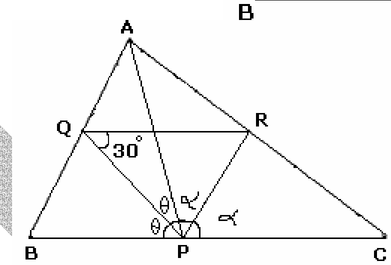
In the given figure, $\theta + \theta + 15^\circ + 15^\circ = 180^\circ$

$\angle\theta = 150^\circ \Rightarrow \theta = 75^\circ$

63. In the given figure, $\theta + \theta + \propto + \propto = 180^\circ$

$\Rightarrow \theta + \propto = 90^\circ \Rightarrow \angle QRP = 60^\circ$

Therefore other two angles are 90° and 60°

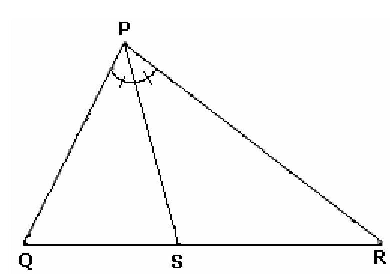


64. PS is the angle bisector of $\angle OPR$

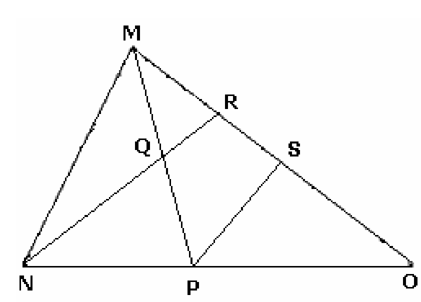
$$\Rightarrow \frac{PQ}{PR} = \frac{QS}{SR} \Rightarrow SR = 2.5QS$$

$\Rightarrow$ Area $\Delta PSR = 2.5 \times$ Area $\Delta PQS = 2.5 \times 40 = 100$

$\Rightarrow$ Area $\Delta PQR = 100 + 40 = 140$ sq. cm



65. We draw a line PS parallel to QR and meeting MO at S.



ΔRNO and ΔPSO are similar

$\Rightarrow \frac{OP}{ON} = \frac{OS}{OR} \Rightarrow \frac{OS}{OR} = \frac{1}{2} \Rightarrow OS = SR$

ΔMQR and ΔMPS are similar

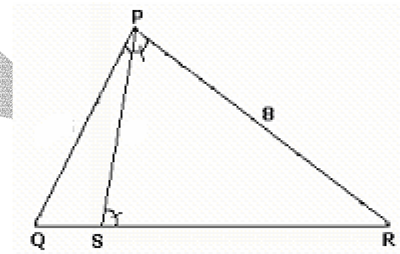
$\Rightarrow \frac{MQ}{MP} = \frac{MR}{MS} \Rightarrow \frac{MR}{MS} = \frac{1}{2} \Rightarrow MR = SR \Rightarrow MR = RS = SO \Rightarrow MR = 10$

66. ΔPSR and ΔQPR are similar

$\Rightarrow \frac{PR}{QR} = \frac{SR}{PR} \Rightarrow PR^2 = SR \times QR = 64$

The values possible are (2, 32) and (4, 16)

QS = 30, 12



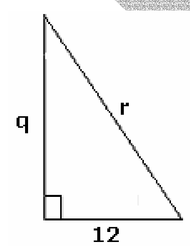
67. $2p + 7r = 9q \Rightarrow 9q - 7r = 24 \Rightarrow q = \frac{24 + 7r}{9}$

Also, $r^2 - q^2 = 144 \Rightarrow r^2 - \frac{(24 + 7r)^2}{9} = 144$

$\Rightarrow 81r^2 - (49r^2 + 576 + 366r) = 81 \times 144$

$\Rightarrow 32r^2 - 576 - 336r = 81 \times 144 \Rightarrow 2r^2 - 36 - 21r = 729$

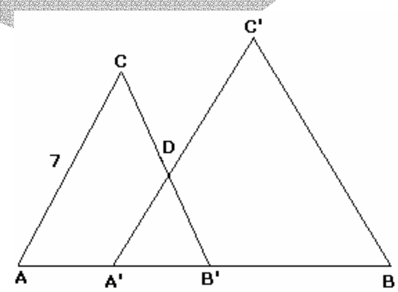
$\Rightarrow 2r^2 - 21r - 765 = 0 \Rightarrow r = 25.5$



68. $\Delta A'DB'$ and ΔACB are similar

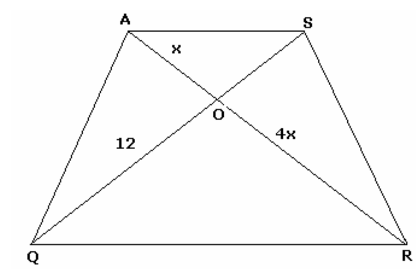
$\Rightarrow \frac{\text{Area } \Delta A'B'D}{\text{Area } \Delta ABC} = \frac{(A'B')2}{(AB)2} \Rightarrow \frac{A'B'}{AB} = \frac{1}{\sqrt{2}}$

$\Rightarrow A'B' = 5\sqrt{2} \Rightarrow AA' = 10 - 5\sqrt{2}$

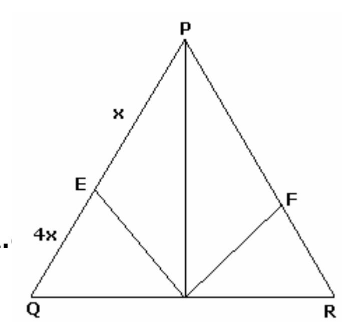


69. ΔDPS is similar to ΔORQ

$\Rightarrow \frac{AO}{OR} = \frac{OS}{OQ} \Rightarrow \frac{OS}{OQ} = \frac{1}{4} \Rightarrow OS = 3$



70. DE is the angle bisector of ΔPDQ



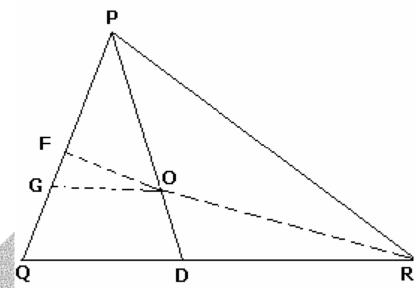
$\Rightarrow \frac{PD}{QD} = \frac{DE}{QE} = \frac{1}{4} \Rightarrow QD = 4PD$

Now QD = DR $\Rightarrow$ DR = 4PD

Now DE is the angle bisector in $\triangle$PDR

$\frac{DR}{DP} = \frac{RF}{FP} \Rightarrow \frac{RF}{FP} = 4 \Rightarrow RF = 10$

71. Draw a line parallel to QR from point O, meting PQ at G.



ΔPGO and ΔPQD are similar

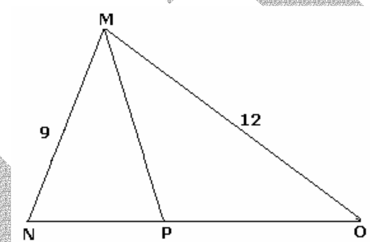
$\Rightarrow \frac{GO}{QD} = \frac{PO}{PD} = \frac{4}{5}$

ΔFGO and ΔFQR are similar

$\Rightarrow \frac{FG}{FQ} = \frac{GO}{QR} = \frac{GO}{2QD} = \frac{4}{2 \times 5} = \frac{2}{5} \Rightarrow FG = \frac{6}{5}$ and $GQ = \frac{9}{5}$

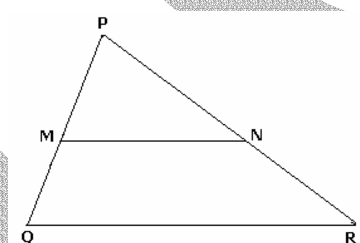
72. $\frac{MO}{MN} = \frac{PO}{PN} \Rightarrow \frac{4}{3} = \frac{PO}{PN} = \frac{PN+1}{PN} = 1 + \frac{1}{PN} \Rightarrow PN = 3$

PO = 4



73. $\frac{\text{Area } \Delta PMN}{\text{Area } \Delta PQR} = \frac{MN^2}{QR^2} \Rightarrow \frac{MN}{QR} = \frac{1}{\sqrt{2}} = \frac{PM}{PQ}$

$\frac{PQ}{PM} = \sqrt{2} \Rightarrow 1 + \frac{QM}{PM} = \sqrt{2}$



74. Join point B to D.

Area ΔBDF = Area ΔBDF = 2a (say)

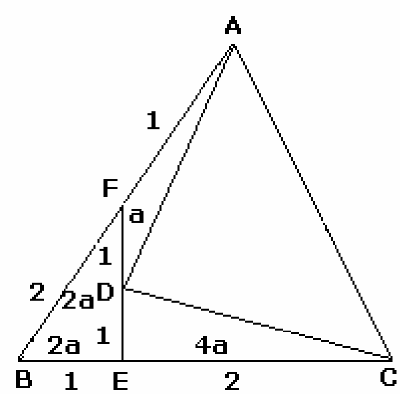
Area ΔCDE = 2 Area ΔBDE = 4a

Area ΔAFD = Area ΔBDF/2 = a

$\frac{\text{Area } \Delta BFE}{\text{Area } \Delta ABC} = \frac{2}{9} \Rightarrow \text{Area } ABC = \frac{9}{2} \times 4a = 18a$

Area ΔADC = 18a - (2a +2a +a +4a) = 9a

$\Rightarrow$ Area ΔADC/ Area ΔABC = 9a/18a = 1/2



75. $MP^2 + QO^2 = MN^2 + NP^2 + QN^2 + NO^2$

$= MN^2 + \frac{NO^2}{4} + \frac{NO^2}{4} + NO^2$

$= 5\,(MN^2 + NO^2)/4$

$= (5 \times 20^2)/4 = 500$

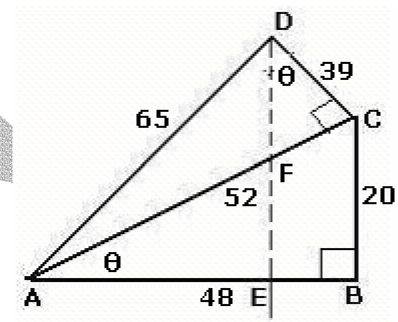
76. In ΔABC, $\tan\theta = \frac{20}{48} = \frac{5}{12}$

In ΔFCD $\tan\theta = \frac{FC}{39} \Rightarrow FC = \frac{5}{12} \times 39$

$\Rightarrow AF = 52 - FC = \frac{13 \times 33}{12}$

$\frac{AE}{EB} = \frac{AF}{FC} = \frac{11}{5}$

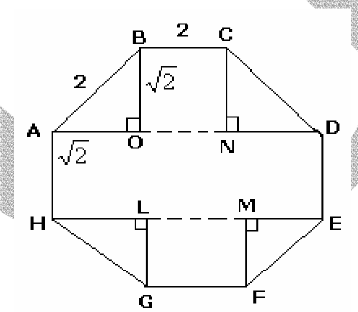
$\Rightarrow EB = 5 \times 48 = 15$



77. Each side of octagon = 2cm.
Area of the shaded region
= Area of ADEH + 2 (Area of BCNO)

$= 2\left(2 + 2\sqrt{2}\right) + 2\left(2\sqrt{2}\right)$

$= 4 + 4\sqrt{2} + 4\sqrt{2} = 4 + 8\sqrt{2}$



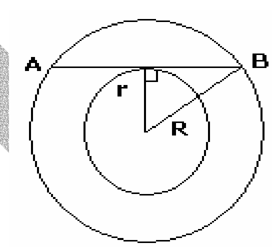
78. Area of Annulus = $\pi\left(R^2 - r^2\right) = 100\pi$

$\Rightarrow R^2 - r^2 = 100$

$\frac{1}{2}(AB) = \sqrt{R^2 - r^2}$.

= 10.

$\Rightarrow AB = 20cms$



79. Let the center of the larger circle be R.
RS = R + R -1 = 2R – 1

$\Rightarrow RS^2 = R^2 + \left(2\sqrt{19}\right)^2$

$(2R - 1)^2 = R^2 + 76$

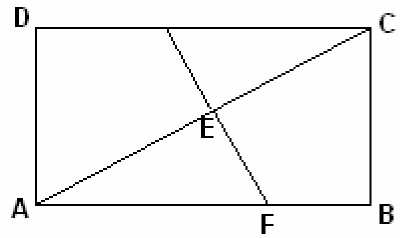
$4R^2 + 1 - 4R = R^2 + 76$

$3R^2 - 4R - 75 = 0$

$\Rightarrow R = \frac{2 + \sqrt{229}}{3} cms.$

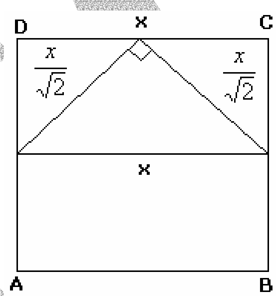
80. $x = \sqrt{(9+16)^2 - (16-9)^2}$ = 24cms.

81. Let us say AE = EC = x



AF : FB = 2:1
Say AF = 2y, FB = y
In ΔAEF & ABC
$\angle AEF = \angle ABC = 90^o$
$\angle A = \angle A$ (Common)
$\Rightarrow$ AEF ~ ABC $\quad \Rightarrow \frac{AF}{AE} = \frac{AC}{AB}$
$\Rightarrow \frac{2y}{x} = \frac{2x}{3y} \quad \Rightarrow 3y^2 = x^2$
As $BC^2 = 4x^2 - 9y^2 = 4x^2 - 3x^2 = x^2$
BC =x
Now in $\Delta ABC \quad \rightarrow \frac{BC}{AC} = Sin\angle A \quad \Rightarrow \angle A = 30^o$
$\angle BAC = \angle ABE = 30^o$
Hence $\angle AEB = 180^o - (30^o + 30^o) = 120^o$
So angle between AC & BD = 120^o or $180^o - 120^o = 60^o$

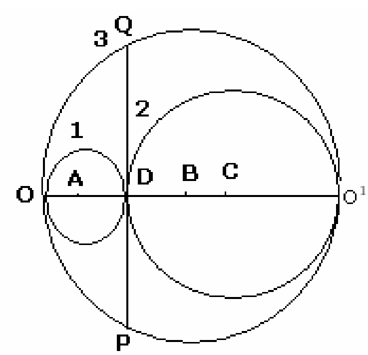
82. Area of the $\Delta = \frac{1}{2}.\frac{x}{\sqrt{2}}.\frac{x}{\sqrt{2}}$



= $\frac{x^2}{4}$
Area of square = x^2
Hence ratio= $\frac{\frac{x^2}{4}}{x^2} = \frac{1}{4}$..

83. Let original length of side of square = x cms.
$(x + 4)^2 - x^2 = 120$
$x^2 + 16 + 8x - x^2 = 120 \quad \Rightarrow x = 13$ cm
Let each side should be decreased by a cm
s.t. $\quad 13^2 - (13 - a)^2 = 120$
a = 6 cms.

84. Let A, C, B be the centers of the smallest to the largest circle respectively.



All O, A, B, C, O’ would be collinear.
Let r_1, r_2, r_3 be the radii of circle 1, 2, 3 respectively.
So r, $2r_1 + 2r_2 = 2r_3$
$r_1 + r_2 = r_3$
As PQ is tangent to both the circles in side and a chord of circle 3 so D will be the midpoint or PQ. Now, OD. DO’ = PD . DQ
$2r_1 . 2r_2 = 8 . 8$
$r_1 r_2 = 16$

Required area = $\pi\left(\left(r_1 + r_2\right)^2 - r_1^2 - r_2^2\right)$

= $\pi\left(2r_1r_2\right)$

= 32 π

85. Let the no. of sides are x & 12 – x.

No. of diagonals = $\frac{x(x-1)}{2} - x + \frac{(12-x)(12-x-1)}{2} - (12-x) = 19$

Solving for x, we get x = 7 or 5.

Hence S_1 & S_2 are heptagon & pentagon.

86. Let radius of circle A & B are r_1 & r_2 respectively

Hence $2\pi r_1 \frac{45^o}{360^o} = 2\pi r_2 \frac{30^o}{360^o}$

$\Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$

Hence ratio of the areas of A & B is $\pi r_1^2 : \pi r_2^2 = 2^2 : 3^2 = 4 : 9$

87. AD = 2 + 2 = 4cm.

CD = 2 cm.

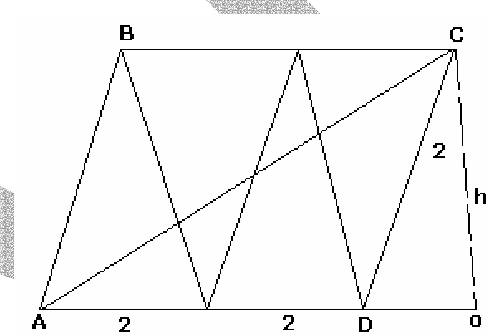
$\angle ADC = 120^o$

Hence Area of 11 gm = 4, 2, Sin 120° = $4\sqrt{3}cm^2$

Also area = 4. h = $4\sqrt{3}$ $\Rightarrow$ h = $\sqrt{3}cm.$

DO = $\sqrt{4-h^2} = 1cm.$

Hence AC = $\sqrt{5^2 + \left(\sqrt{3}\right)^2} = \sqrt{28} = 2\sqrt{7}cms.$



88. AP. PB = DP . PC

PD = 30

Let O be the Center of circle OC, & OB' are $\perp$'s on CD & AB

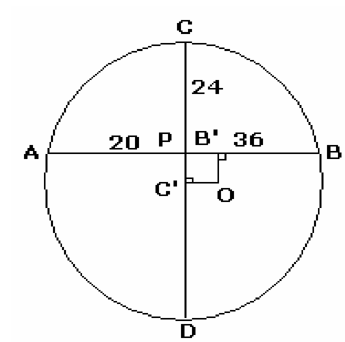
Respect & bisect them.

Hence CC' = C'D = 27

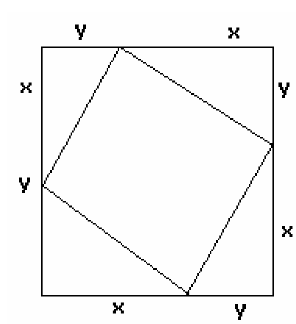
AB' = BB' = 28

PC' = OB' = 30 – 27 = 3

Hence perimeter of the circle = $2\pi\sqrt{793}$



89. Area of the inner square is $\frac{4}{5^{th}}$ of outer square i.e. area of the 4 right angled



Triangles together is $\frac{1}{5^{th}}$ of outer square

$\Rightarrow \frac{1}{5}(x+y)^2 = 4\left(\frac{1}{2}xy\right)$

X $(x+y)^2 = 10\,xy$

Solving we get $\frac{x}{y} = 4 \pm \sqrt{15}$ Hence c or d.

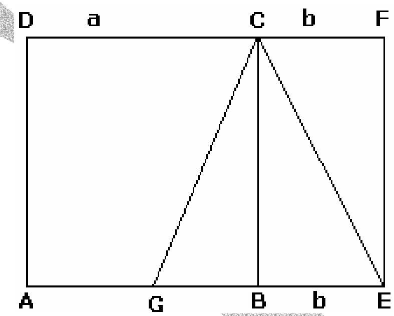
90. a = 1

$$GC = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$$

Hence GE = $b + \frac{1}{2} = \frac{\sqrt{5}}{2}$

b = $\frac{\sqrt{5}-1}{2}$

So golden ratio = $\frac{\sqrt{5}-1}{2} + 1 = \frac{\sqrt{5}+1}{2}$



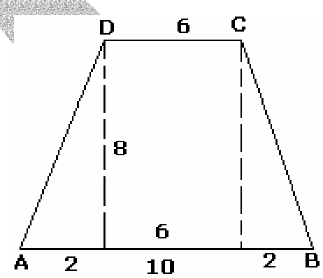
91. $AD = \sqrt{8^2 + 2^2}$

= $\sqrt{68} = 2\sqrt{17}$

Hence BC = $2\sqrt{17}$

So Perimeter = 10 + 6 + $2\left(2\sqrt{17}\right)$

= 16 + $4\sqrt{17}$



92. Let O in the center of circle OE $\perp$ AD

AE = ED = 5

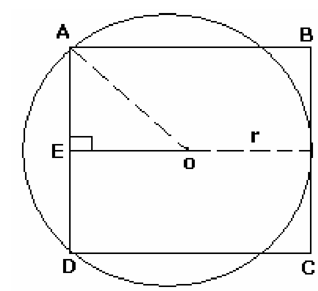
Let r be the radius of circle.

Hence OE = (10-r).

In $\Delta AEO \Rightarrow$ $AE^2 + EO^2 = AO^2$

$5^2 + (10 - r)^2 = r^2$

Solving for r, we get r = 6.25

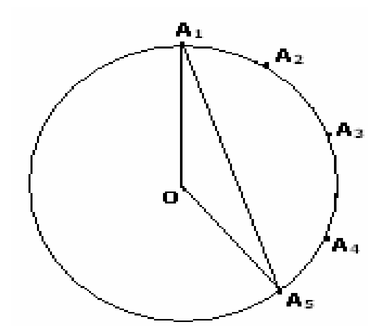


93. As all the points are equally spaced hence angle formed at the centre by

Every two consecutive points is $\frac{360^o}{10} = 36^o$,

Hence $\angle A_1OAS = 36 \times 4 = 144^o$

In ΔOA_1A_5 $OA_1 = OA_5$ (radius)

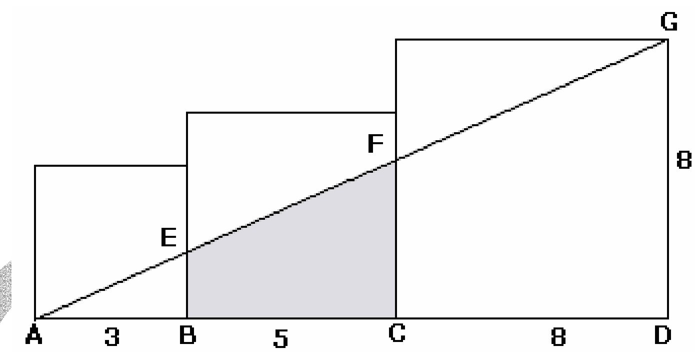


Hence $\angle OA_1A_5 = \angle OA_5A_1$

So $\angle OA_5A_1 = \dfrac{180^o - 144^o}{2} = 18^o$

94. In $\Delta's$ ABE & ADG,

the $\Delta's$ are similar



$\therefore \dfrac{AB}{BE} = \dfrac{AD}{DG}$

$\dfrac{3}{BE} = \dfrac{16}{8}$ $\Rightarrow$ BE = $\dfrac{3}{2} cms.$

Also $\Delta ABE \sim \Delta ACF$

Hence $\dfrac{AB}{BE} = \dfrac{AC}{CF} \Rightarrow \dfrac{3}{3/2} = \dfrac{8}{CF}$

CF = 4

$\therefore$ Area of EBFC = AC of ACF – Area of ABE

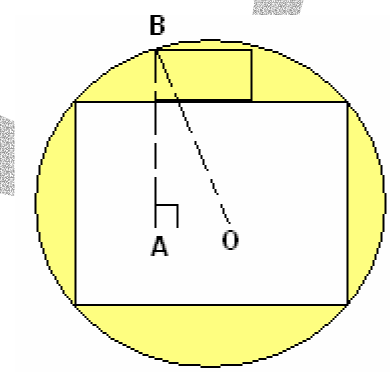
= $\dfrac{1}{2}.8.4 - \dfrac{1}{2}.3.\dfrac{3}{2}$

= $16 - \dfrac{9}{4} = 13.75$

95. Let O be the centre of circle

Let the sides of bigger & smaller squares be x & y cm.

Hence in ΔOAB



AB= y+$\dfrac{x}{2}$, OA= $\dfrac{y}{2}$, OB= $\dfrac{x}{\sqrt{2}}$

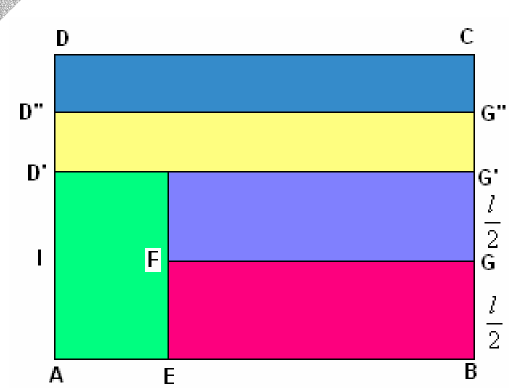
$\Rightarrow \left(\dfrac{y}{2}\right)^2 + \left(\dfrac{x}{2} + y\right)^2 = \left(\dfrac{x}{\sqrt{2}}\right)^2$

Solving we get $\dfrac{x}{y} = \dfrac{5}{1}$

96. Let AD′ = l cm

$\Rightarrow$ G′ G = BG = l cm

($\because$ they both have equal area & lengths are equal for them.)



So l (AE) = $\dfrac{l}{2}$ (EB)

l (AE) = $\dfrac{l}{2}$(2-AE)

Solving we get AE = $\dfrac{2}{3} \Rightarrow$ EB = $\dfrac{4}{3} cm.$

Also area of DCD″G″ = Area of BEFG

2.DD″ = $\frac{4}{3} \cdot \frac{l}{2}$

2. $\frac{(2-l)}{2} = \frac{2}{3}l$

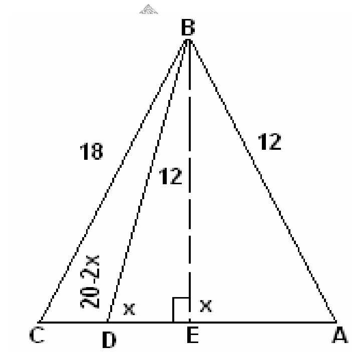
$\rightarrow$ l = $\frac{6}{5}cm.$

Hence Perimeter of BEFG = 2 $\left(\frac{3}{5}+\frac{4}{3}\right) = \frac{58}{15}cm$

97. If three circles touch the other circles in a row & have two direct common targets then their radio are in G.P series

Hence radius of middle circle is $\sqrt{9\times 4} = 6cm.$

$\Rightarrow$ AB = 2(9+4+6) = 38cms.



98. Let BE $\perp$ AC

As ABD is isosceles Δ

$\therefore$ DE = EA = x (say)

In ΔBDE $\rightarrow BE^2 = 144 - x^2$

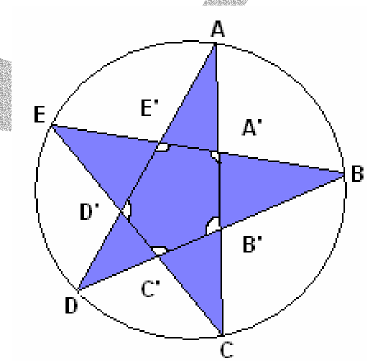
In $\Delta BCE \rightarrow AE^2 = 324 - (20 - 2x + X)^2$

Comparing both the equation's above we get

$144 - x^2 = 324 - (20-x)^2$

$\Rightarrow$ x= 5.5

So AD:DC = 11:9



99. If we add all the angles of Δ's EBC′, ADB′, BDE′, CEA′ & ΔABC we get (A+B+C+D+E) + 2(A′+B′+C′+D′+E′) = 180×5

but A′+B′+C′+D′+E′ = 360

$\Rightarrow$ (A+B+C+D+E) = 900-7200

Hence A+B+C+D+E = 180^o

100. As ABCD is an isosceles trapezium

$\therefore$ DE **=** $CE' = \frac{(108-75)}{2}$

= 16.5

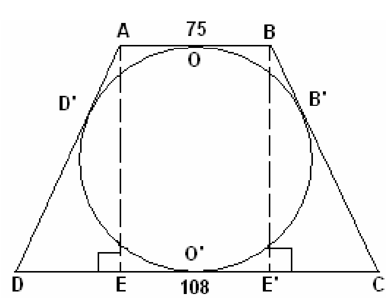
$\Rightarrow$ Also DO′ = DD′ **=** $\frac{108}{2}$ **=** 54 cm

AO = AD′ = $\frac{75}{2}$ = 37.5 cm

So AE = $\sqrt{AD^2 - DE^2}$

= $\sqrt{(91.5)^2 - (16.5)^2}$

= 90 CM.



101. Let the sides of square & equation D are 3x & 4x respectably such that they have equal evens. For the circle circumscribes the square

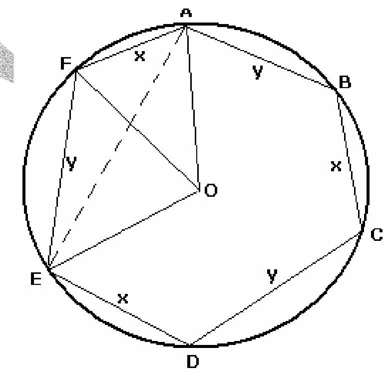
Radius = $\frac{1}{2}(diagonal) = \frac{1}{2}\left(3\sqrt{2}x\right) = \frac{3x}{\sqrt{2}}$

Also in radius of equation D = $\frac{4x}{2\sqrt{3}}$

$\Rightarrow$ Ratio of areas = $\left(\frac{3x}{\sqrt{2}}\right)^2 \lambda : \left(\frac{4x}{2\sqrt{3}}\right)^2 \lambda$

$\Rightarrow$ 27 : 8

102. Answer 12.



103. Let the hexagon ABCDEF be inscribed in a circle. Let the sides x and y subtend angles α and β at the center O, respectively. $3\alpha + 3\beta = 360° \Rightarrow \alpha + \beta = 120°$

$\Rightarrow \angle AOE = 120°$

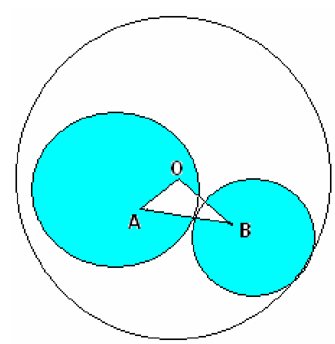
In ΔAOE cos120 = $\frac{r^2 + r^2 - AE^2}{2r^2}$

$\angle AFE = \frac{180 - \alpha}{2} + \frac{180 - \beta}{2} = 120°$

In ΔAFE cos120 = $\frac{x^2 + y^2 - AE^2}{2xy}$

Eliminate AE from both the equation to get the answer.

104. AS the according to the property circles the touching point & centres of circles should be collinear. Hence all O, AB should be collinear



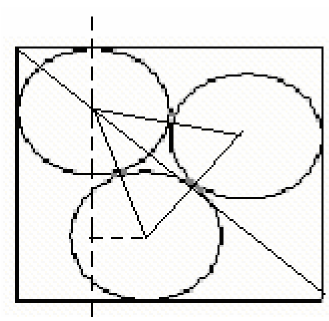
Also $AB = \frac{1}{2}$ (diameter of bigger circle)

= 5 cm

So OAB = OA + OB + AB

= AB + AB = 5 + 5 = 10 cm.

105. side = 2rcos15 + 2r = 7.86



106. Find the radius of each circle and determine the order. Remember that it's an isosceles right triangle with 45- 45- 90 angles.

107. Equal chords subtend equal angles in a circle.

$\Rightarrow \angle BAC = \angle BDA = \angle BDC = \angle CAD = \theta$(say)

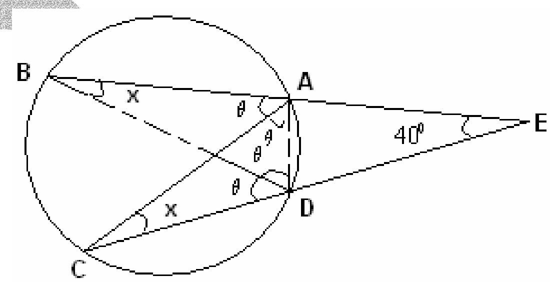
Let $\angle ACD = \angle DBA =$ x

In ΔACD, exterior $\angle ADE$ = x+θ = $\angle DAE$

In ΔBDE, x + θ + 0 + x +40 = 180

$\Rightarrow$ θ + x = 70, Also 30 + x = 180^0 in ΔADC

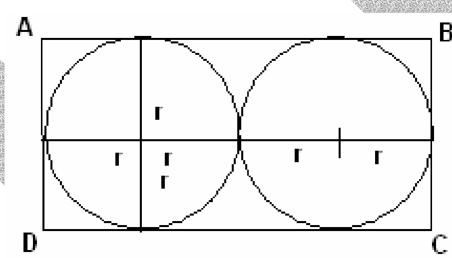
$\Rightarrow$ x = 15^0



108. B travels $4\pi r$ while A travels 12r

$$\therefore \frac{v_B}{v_A} = \frac{4\pi r}{12r} = \frac{\pi}{3} \approx \frac{22}{21}$$

% greater = $\frac{1}{21}$ = 4.76%



109. $(1+r)^2 = (2-r)^2 + (1-r)^2$

$1 + r^2 + 2r = 4 + r^2 - 4r + 1 + r^2 - 2r$

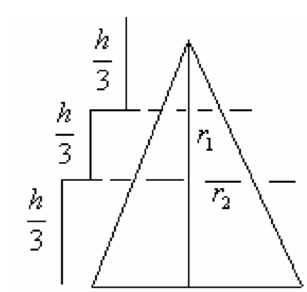
= $r^2 - 8r + 4$

r **=** $\frac{8 \pm \sqrt{48}}{2} = 4 - 2\sqrt{3}$

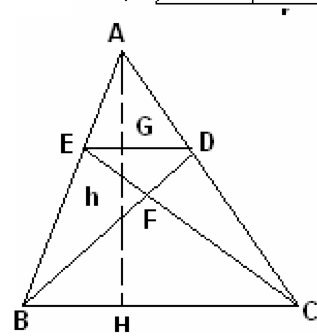
110. By similarity, the radius r_1 and r_2 will be $\frac{1}{3}r$ and $\frac{2}{3}r$. Therefore the volume of the two cones will be $\frac{1}{3}\pi\left(\frac{r}{3}\right)^2 \times \frac{h}{3}$ and $\frac{1}{3}\pi\left(\frac{2r}{3}\right)^2 \times \frac{2h}{3}$

= $\frac{v}{27}$ and $\frac{8v}{27}$

Volume of the piece between the planes = $\frac{8v}{27} - \frac{v}{27} = \frac{7v}{27}$



111. ΔADE and ΔABC are similar



Height of triangle ADE = $\frac{1}{3}$ height of ΔABC

Height GH = $\frac{2}{3}h$

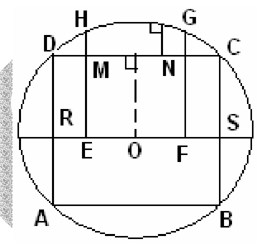
ΔDEF and ΔBFC are similar $\Rightarrow \frac{heightof\ \Delta DEF}{heightof\ \Delta BFC} = \frac{DE}{BC} = \frac{1}{3}$

height of $\Delta DEF = \frac{1}{4} \times \frac{2h}{3} = h$

area $\Delta DEF = \frac{1}{2} \times DE \times \frac{h}{6} = \frac{1}{2} \times \frac{BC}{3} \times \frac{h}{6} = \frac{Area \Delta ABC}{18} = 12$

112. In square ABCD OS = OT = CS = $\frac{1}{2}$ (side of ABCD)

= $\frac{1}{2}$ a (say)



$OC^2 = OS^2 + SC^2$

$\Rightarrow \quad 5^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 \Rightarrow a = 5\sqrt{2}$

Also OF = $\frac{1}{2}$ (FG).

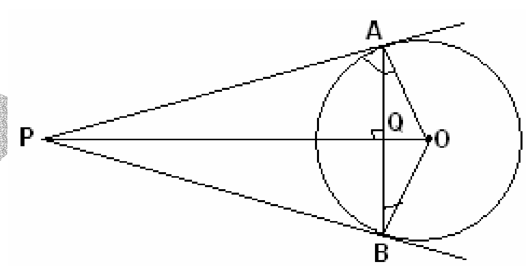
$\Rightarrow \quad OG^2 = \left(\frac{FG}{2}\right)^2 + FG^2 \Rightarrow 25 = 5\frac{FG^2}{4}$

So FG= $2\sqrt{5}$

So area of EFMN = EF × FN = $2\sqrt{5} \times \frac{5\sqrt{2}}{2}$

= $5\sqrt{10}$ sq.units.

113. $\frac{1}{AO^2} + \frac{1}{PA^2} = \frac{1}{25}$



$\frac{PA^2 + AO^2}{AO^2 PA^2} = \frac{1}{25}$

But in ΔOAP, $PA^2 + OA^2 = PO^2$

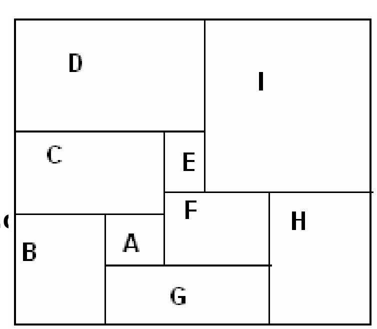
$\Rightarrow \quad \frac{OP^2}{AO^2 PA^2} = \frac{1}{25} \Rightarrow \frac{OP}{AO.PA} = \frac{1}{5}$

$\Rightarrow \quad \frac{AO.PA}{OP} = 5$

Now is ΔOAP, as AQ $\perp$ OP

$\Rightarrow \quad \frac{AO..PA}{OP} = AQ$. So AQ=5 $\Rightarrow$ AB= 10.

114. Side of A = 1cm.



B = 9 cm $\left(i.e. \sqrt{81}\right)$

Side of C = (9+1) = 10cms.
Also side of G = side of B – side of A
= 9-1 = 8cm.
Side of F = side of g – side of A
= 8-1 = 7cm.
Side of H = side of G + side of F = (8+7) = 15cms.
Also side of (B+C) = side of (E + F + G)
Side of E = side of (B + C) – (F + G) = (9 + 10) – (8 + 7) = 4cm.
Side of I = side of (H+F) – side of E
= (15+7) – 4 =18cms.
So Area of I = $18^2 = 324cm^2$

115. The distance traveled by Ant would be minimum if it travels as in the figure.

So total distance = $\frac{1}{4}(2.\pi.1)+1+\frac{1}{4}(2\pi.1)$

= $(\pi+1)$ meteres.

116. CD = 17cm (given)
EC = 8cm (given)

$\Rightarrow$ $DE = \sqrt{17^2 - 8^2}$ = 15 cms.

Also DEC & DHA are congruent hence DH = 8 $\Rightarrow$ Side of square = (15-8) = 7cms.

So area of required square = $7^2 = 49cms^2$

117. The area grazed by the cow is I + II + III + IV

(I + II) is a sector of circle with radius 50m and angle 270^o.

So Area = $\pi.50^2.\frac{270}{360} = 1875\pi m^2$

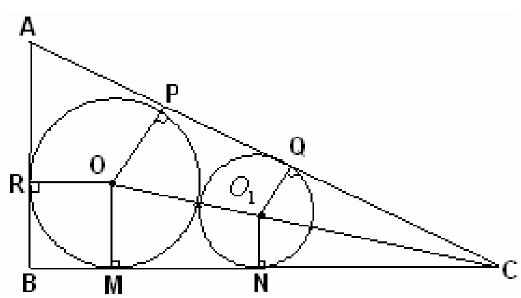
Area III is a sector of circle with radius 210 m and angle 90^o.

So Area = $\pi.20^2.\frac{90}{360} = 100\pi m^2$

Similarly Area IV = $\pi.30^2.\frac{90}{360} = 225\pi m^2$,

So Required Area = 1875 + 100 + 225 = 2200 m^2.

118. Let O & O_1 are centers of the two circles. OM $\perp$ BC,
O_1 N $\perp$ BC.



OR $\perp$ AB, OP $\perp$ AC, O_1Q $\perp$ AC.

In similar triangle's OMC & O_1NC

We have OM = 4cm, O_1N = 1cm.

Also MN = PQ = $\sqrt{(1+4)^2 - (4-1)^2}$ =4cms.

$\Rightarrow$ NC = $\frac{4}{3}$ cms. $\Rightarrow$ CQ = $\frac{4}{3}$ cms.

In square BROM, BR = BM = 4cms.

Let AR = AP = x cms.

So in ΔABC, $AB^2 + BC^2 = AC^2$

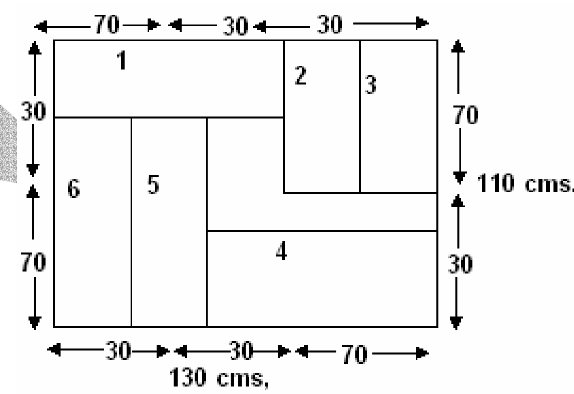
$$(4+x)^2 + \left(4+4+\frac{4}{3}\right)^2 = \left(x+4+\frac{4}{3}\right)^2$$

x = 28cms. ⇒ AB = 28 + 4 = 32.

119. Area of rectangular floor = 110 × 130 = 14300 cm^2

Area of each tile = 70 × 30 2100 cm^2

Now, as $\frac{14300}{2100} < 7$, so maximum no. of tiles that can be laid are 6. Six tiles can be accommodated as given in the figure.

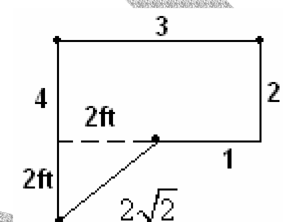


120. If we look at the horizontal orientation only the last step of the 4th floor is $2\sqrt{2}$ ft. away from the ground floor.

Similarly 8th floor is $2\sqrt{2}$ ft. away from the 4th floor. So the top of 100th floor is $25.2\sqrt{2} = 50\sqrt{2}\,ft$ away from the ground floor horizontally.

Now the total height achieved till 100th floor is (1 + 2 + 3+.... 100) = 5050ft.

So the distance between the final and initial point is



$$\sqrt{\left(50\sqrt{2}\right)^2 + (5050)^2} = \sqrt{5000 + 255502500}$$

$$= \sqrt{25505500}$$

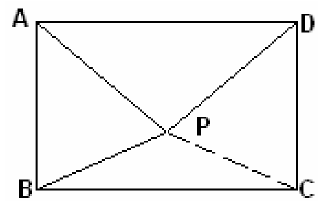
$$= 5050.3\,ft$$

121. In a rectangle

$$PA^2 + PC^2 = PB^2 + PD^2$$

$$4^2 + PC^2 = 3^2 + 5^2$$

$$PC^2 = 18 \Rightarrow PC = \sqrt{18} = 3\sqrt{2}units$$



122. For an acute angled Δ with sides a, b, c

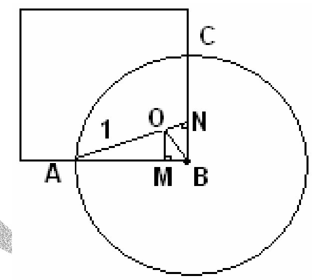
$$a^2 < b^2 + c^2$$

(i.e. sum of the squares of any two sides is always greater than the square of the third side).
So, the sides satisfying the above condition are 11, 60, 59 & 11, 60, 60 only. Hence Answer 2.

123. The ratio of the base areas for the two cylinders is $1^2 : 3^2 = 1:9$.

Now as the empty space will have the same volume in the both the cases, so the height of empty space in the smaller cylinder shaved be 9 times that of the bigger cylinder.
Let 'h' is the height of empty space in bigger cylinder, so 9h mill be that of smaller one.
20 + 9h = 28 + h
h = 1cm.
So height of the bottle = (28 + 1) = 29cms.

124. OM & ON are perpendiculars on AB & BC respectvely.



As AB = BC
OM = ON =x (say)
In ΔOMB $(OM)^2+(MB)^2=(OB)^2$

$$x^2+x^2=\left(\frac{1}{2}\right)^2 \Rightarrow 2x^2=\frac{1}{4} \Rightarrow x=\frac{1}{2\sqrt{2}}cms.$$

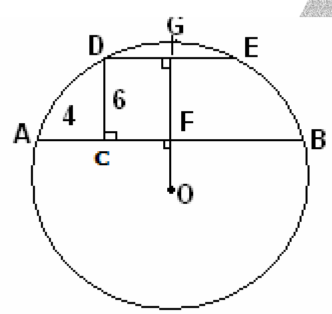
Now, in ΔOAM

$$(AM)^2+(OM)^2=1^2$$

$$AM^2=1-\frac{1}{\left(2\sqrt{2}\right)^2}=AM\,\frac{\sqrt{7}}{2\sqrt{2}}.$$

AB = $\frac{\sqrt{7}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{7}+1}{2\sqrt{2}}$

125. DE || AB
Let 'O' is centre of circle.
OF $\perp$ AB, OG $\perp$ DE
F & G are the midpoints of AB & DE respect.
Also As DP = 6, FG= 6.
Also AB = 17+4=21 $\Rightarrow$ AF=10.5
Now in ΔAFO



$$(OA)^2=AF^2+(OF)^2$$

$$\Rightarrow \quad r^2=(10.5)^2+(OF)^2$$

$$\Rightarrow \quad OF^2=\frac{441}{4}-r^2$$

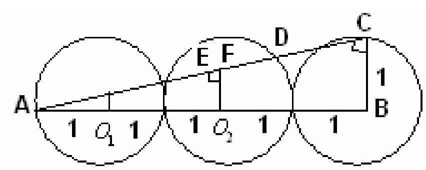
In ΔODG, DG = PF = (10.5 – 4)=6.5
So $OD^2=OG^2+GD^2$

$$r^2=\left(\sqrt{\frac{441}{4}-r^2}+6\right)^2+(6.5)^2$$

Solving above eq^4. We get $r=\frac{65}{6}$.

126. Let O_1, O_2 be the centers of circle 1 & 2.

In similar triangle's AO_2F & ABC, $\frac{AO_2}{AB}=\frac{O_2F}{BC}$

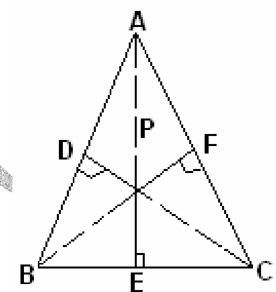


$$\Rightarrow \qquad O_2F = \frac{3.1}{5} = \frac{3}{5}$$

In $\Delta O_2EF, EF = \sqrt{1^2 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

DE= $2.\frac{4}{5} = \frac{8}{5} cms.$

127. AB = 5cm, BC=6cm, CA =7cm, PD= 2cm, PE =3cm.



Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

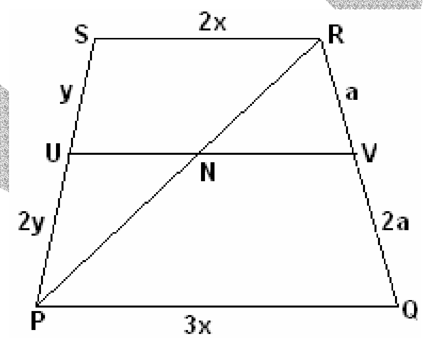
$s = \frac{5+6+7}{2} = 9$ $\qquad \Rightarrow$ Area= $\sqrt{9.4.3.2} = 6\sqrt{6} cm^2$

Also Area of ΔABC = Area of {ΔAPB + ΔBPC + ΔAPC}

$$6\sqrt{6} = \frac{1}{2}.2.5 + \frac{1}{2}.3.6 + \frac{1}{2}.PF.7$$

solving we get $PF = \frac{12\sqrt{6} - 28}{7}$.

128. Let RS=2x, PQ=3x, SU=y so UP=2y



||ly RV = a, so VQ=2a.

In similar triangle's PUN & PSR

$$\frac{PU}{PS} = \frac{UN}{SR} \Rightarrow \frac{2y}{3y} = \frac{UN}{2x} \Rightarrow UN = \frac{4}{3}x$$

In similar triangle's RNV & RPQ

$$\frac{RV}{RQ} = \frac{NV}{PQ} \Rightarrow \frac{a}{3a} = \frac{NV}{3x} \Rightarrow NV = x$$

So length of UN= $x + \frac{4}{3}x = \frac{7}{3}x$

So UV : SR = $\frac{7}{3}x : 2x = 7:6$

129. In similar triangle's AEF & ABC

as EF= $\frac{1}{2}$ BC $\qquad \Rightarrow$ E & F are the midpoints of AB & AC respectively.

Now in ΔFGD & BGC

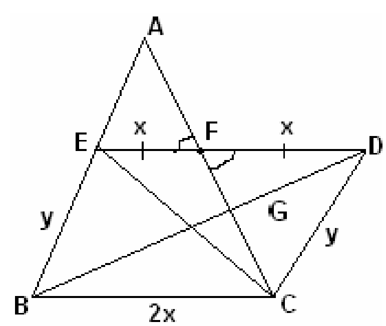
$\angle FGD = \angle BGC$

$\angle FDG = \angle GBC$ ($\therefore$ FD || BC)

FGD ≅ CGB

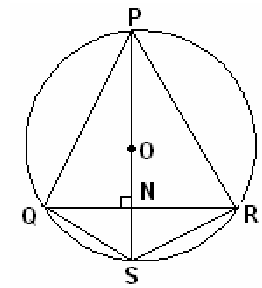
$\frac{BC}{FD} = \frac{CG}{GF}$ $\qquad \Rightarrow$ GF = 105cm.

Hence AC = 2(3+1.5) = 9 ⇒ AG = (9-6) = 3cm.



130. O is the centre of the circle circumscribing the ΔPQR.

side of PQ= $r\sqrt{3}$



PN = $\frac{\sqrt{3}}{2}.r\sqrt{3} = \frac{3}{2}r$

So NS = $2r - \frac{3}{2}r = \frac{r}{2}$.

QS= $\sqrt{\left(\frac{r\sqrt{3}}{2}\right)^2 + \left(\frac{r}{2}\right)^2}$ $\left(In\Delta QSN, QS^2 = QS^2 + NS^2\right)$

So perimeter or PQRS = $2r\sqrt{3} + 2.r = 2r\left(1+\sqrt{3}\right)$

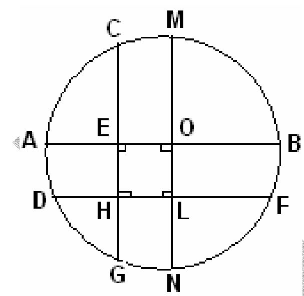
131. AE : EB = 1 : 2

AE + 1cm, EB = 2cm

∏ly NL = 1cm, LM = 2cm.

But AO = ON = 1.5 cms.

EO = OL = 0.5 cms.



In ΔODC, $OD^2 - OC^2 = DC^2$

$$DC = \sqrt{\frac{9}{4} - (0.5)^2} = \sqrt{2}cms.$$

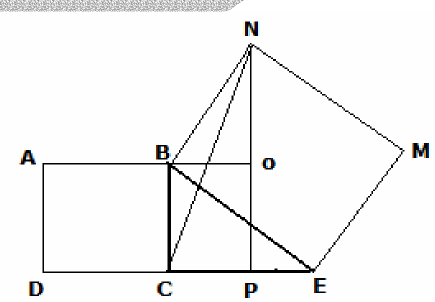
Also in Δ EHC, EH = HC = 0.5cms.

So DH = DC – HC = $\sqrt{2} - \frac{1}{2} = \frac{2\sqrt{2}-1}{2}cms.$

132. From the figure ΔBCE ~ ΔBON

∴ No = CE = 4 cm (angle opposite to $\angle\theta$)

and BO = CP = 3cm (angle opposite to $\angle\left(90^o - \theta\right)$)

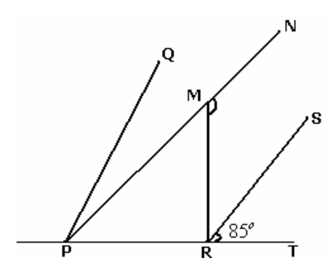


∴ from ΔNCP

$$CN = \sqrt{CP^2 + NP^2} = \sqrt{3^2 + 7^2}$$

$$= \sqrt{58}$$

133. From figure



$\angle QPR = \angle SRT = 85^o$ (corresponding angle)

$\therefore \angle MPR = 25^o - 30^o$

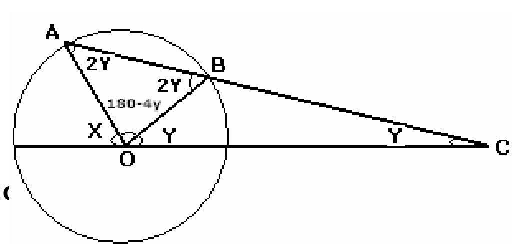
= 55^o

= $\angle PMR$ (Since RM = PR)

$\therefore \angle RMN = 180^o - 55^o$

= 125^o

134. From ΔBOC



$\angle BOC = \angle BCO = Y$ ($\because$ BO = AO = BC)

$\therefore \angle OBC = 180 - 2Y$

$\therefore \angle OBA = 2Y = \angle OAB \quad \angle \because B$

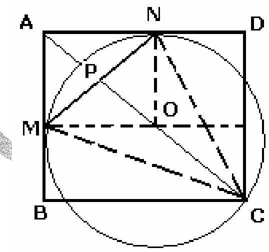
Now From ΔAOC

$\angle AOC = 180^o - 3Y$

and $180^o - 3Y + X = 180^o$

or X = 3Y

135. From the figure



$\angle ANO = \angle AMO = 90^o$ (angle between tangent and corresponding radius)

$\therefore$ Quadrilateral ANOM is a square.

Now, From ΔMON, NM = $\sqrt{2}r$

and MP = $\frac{r}{\sqrt{2}}$

Now From ΔOPM:

OP = $\sqrt{OM^2 - MP^2} = \frac{r}{\sqrt{2}}$

and OC + PO = $r + \frac{r}{\sqrt{2}} = 5$

$\Rightarrow r = 5\left(2 - \sqrt{3}\right)$

Now AP = $\sqrt{AM^2 - MP^2} = \frac{r}{\sqrt{2}}$

$\therefore$ diagonal AC = CP + AP = 5 + 5 ($\sqrt{2}$ - 1)

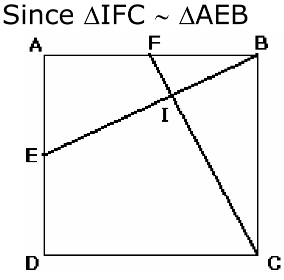
= $5\sqrt{2}$

$\therefore Area = \frac{1}{2} diagonal$

= $\frac{1}{2} 25 \times 2$

= 25

136. Since ΔIFC ~ ΔAEB



$\therefore \frac{AB}{BI} = \frac{AE}{FI} = \frac{BE}{BF}$

$\frac{1}{BI} = \frac{0.5}{FI} = \frac{\sqrt{1.25}}{0.5} = \sqrt{5}$

$\therefore FI = \frac{1}{2\sqrt{5}}$

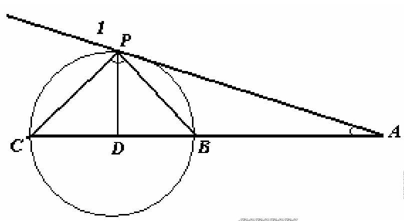
$\therefore BI = \frac{1}{\sqrt{5}}$

$\therefore$ Area of quadrilateral IEDC = Area of square ABCD – Area of ΔABE - Area of ΔBCF + Area of ΔFIB

$$= 1 - \left(\frac{1}{2}\times 1\times 0.5\right) - \left(\frac{1}{2}\times 1\times 0.5\right) + \left(\frac{1}{2}\times\frac{1}{2\sqrt{5}}\times\frac{1}{\sqrt{5}}\right)$$

$$= 1 - \frac{1}{4} - \frac{1}{4} + \frac{1}{20} = \frac{11}{20} unit^2$$

137. $\angle APB = \angle PCB$



Now $\frac{\angle ABP}{\angle APB} = \frac{\angle ABP}{\angle PCB} = \frac{5}{3}$

Or $\angle ABP = 5X$ $\therefore \angle PBC = 180^o - 5X$

And $\angle PCB = 3X$

From ΔPBC

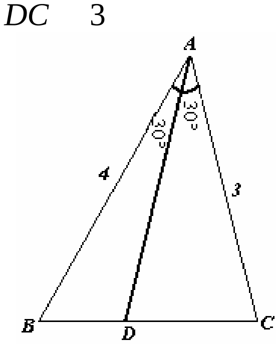
$\angle CPB + \angle PCB + PBC = 180^o$

or $50^o + 3X + 180^o - 5X = 180^o$

or $X = 25^o$

$\therefore \angle APB = \angle PCB = 3X = 75^o$

138. $\frac{BD}{DC} = \frac{4}{3}$



$BC^2 = AB^2 + AC^2 - 2AB\times AC$ Cos 60^o

$= 4^2 + 3^2 - 2\times 4\times 3\times\frac{1}{2}$

= 25 – 12

BC = $\sqrt{13}$

BD = $\frac{4}{7}\sqrt{13}$ & DC = $\frac{3}{7}\sqrt{13}$

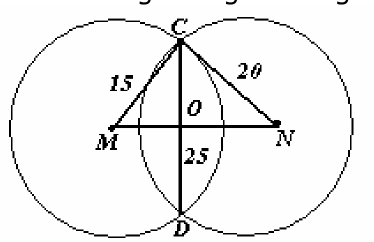
$\frac{SinB}{3} = \frac{SinA}{\sqrt{13}} \Rightarrow SinB = \frac{3\sqrt{3}}{2\sqrt{13}}$

now From ΔADB

$\frac{Sin30^o}{BD} = \frac{SinB}{AD} \Rightarrow \frac{1\times 7}{2\times 4\sqrt{13}} = \frac{3\sqrt{3}}{2\sqrt{13}} \times \frac{1}{AD}$

$\Rightarrow AD = \frac{12\sqrt{3}}{7}$

139. ΔMCN is a right angle triangle.



∴ area of $\Delta MCN = \frac{1}{2}\times 15\times 20 = \frac{1}{2}\times 25\times CD$

$\Rightarrow CD = \frac{15\times 20}{25} = 12$

∴ length of common chord CD = 2 × CO = 24 a.

140. Area of □ABEC = 2 × ΔABC = $14cm^2$

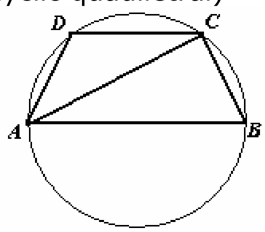
Area of □FECD = 3 × Area of □ABEC ∵ (EC = 3 BE)

= $42cm^2$

∴ Area of □ABCD = 14 + 42 = $56cm^2$

141. $\angle ADC = 180^o - \angle CBA$

(cyclic quadiletral)



$\angle ADC = 180^o - 70^o = 110^o$

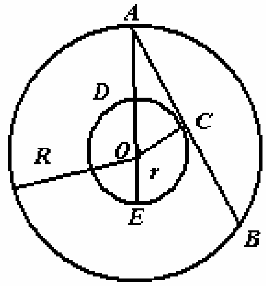
From ΔACD

$\angle ACD = 180^o - (\angle CDA + \angle CAD)$

= $180^o - 110^o - 30^o$

= 40^o

142. $\angle OCA = 90^o$ (angle between tangent and radius) and AB is a chord of outer circle



$\therefore AC = \frac{1}{2}AB = 3a.$

Now $AC^2 = AD \times DE$

= (R – r) (R + R)

= $R^2 - r^2$

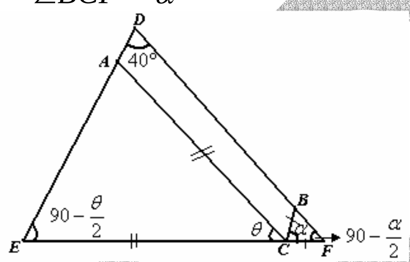
$9^2 = R^2 - r^2$

Since R & r are integer.

$\therefore$ R = 5cm & r = 4cm.

143. Let $\angle ACE = \theta$ and

$\angle BCF = \alpha$



$\therefore \angle CEA = 90 - \frac{\theta}{2}$ and

$\angle BFC = 90 - \frac{\alpha}{2}$

From ΔDEF

$40 + 90 - \frac{\theta}{2} + 90 - \frac{\alpha}{2} = 180^o$

or $\theta + \alpha = 80$

$\therefore \angle ACB = 180^o - \theta + \alpha$

= 100^o

144. $a^2 + b^2 + c^2 = bc + ca + ab$

$\Rightarrow 2a^2 + 2b^2 + 2c^2 = 2bc + 2ca + 2ab$

$\Rightarrow a + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ac = 0$

$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$

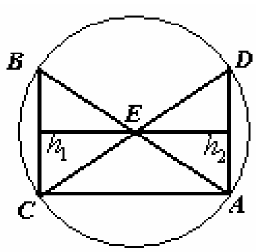
Which means (a - b) = (b - c) = (c – a) = 0

$\Rightarrow$ a = b = c

$\therefore$ ΔABC in an equilateral triangle.

145. PR = PB + RA – AB = 20 + 20 – 5 = 35
Similarly RQ = 40 – 10 = 30
PQ = 40 – 12 = 28.
$\therefore$ Perimeter = 93

146. $\angle DCB = \angle BAD$ (angle drawn by the same segment BD)
$\therefore$ ΔBCE ~ ΔDEA



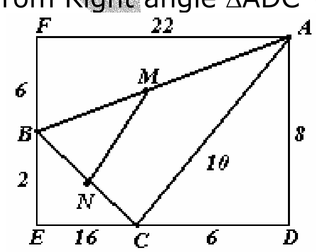
$$\therefore \frac{BC}{DA} = \frac{h_1}{h_2} = \frac{1}{2}$$

$$\therefore \frac{\Delta EBC}{\Delta DEA} = \frac{\frac{1}{2}BC \times h_1}{\frac{1}{2}AD \times h_2}$$

$$= \frac{BC}{AD} \times \frac{h_1}{h_2}$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

147. From Right angle ΔADC



$$AC = \sqrt{8^2 + 6^2} = 10cm$$

Now ΔBMN ~ ΔABC and the proportionality constant = $\frac{1}{2}$

$$\therefore MN = \frac{1}{2}AC = 5cm.$$

148. Since $\angle ACB$ is formed in semicircle
$\therefore \angle ACB = 90°$
and $BC = \sqrt{13^2 - 5^2}$
= 12 cm.

$$\therefore Area of\ \Delta ACB = \frac{1}{2} \times 12 \times 5$$

$= 30cm^2$

149. From $\Delta ABC, \angle A = 30°, \angle B = 90°$

$\therefore \angle C = 60°$

Now From ΔCED

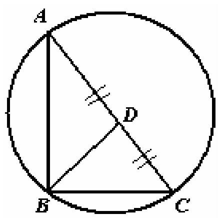
$\angle DEC = \frac{1}{2}\angle C$ (∵ CE is bisector)

$= \frac{1}{2} \times 60° = 30°$

$\angle EDC = 90°$ (given)

$\therefore \angle CED = 60°$

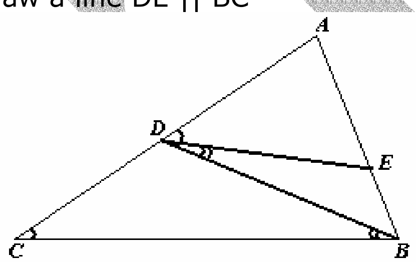
150.



If we form a circle, taking AC as diameter. The circle will pass through vertice B and AD, DC & BD will become radius of the circle as shown in the figure.

$\therefore BD = AD = DC = \frac{1}{2} \times 6 = 3cm$

151. Draw a line DE || BC



From figure $\angle ADE = \angle ACB$

$\angle EDB = \angle DBC$

Since $\angle ABC - \angle ACB = 40°$

Or

$(\angle ABD + \angle DBC) - \angle ADE = 40°$

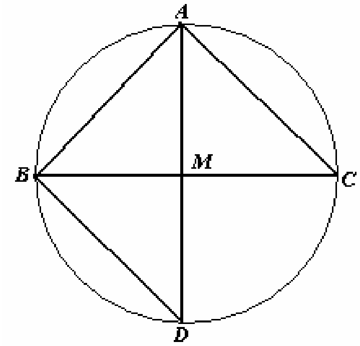
$\angle DBC + \angle ADB - \angle ADE = 40°$ $(\because \angle ABD = \angle ADB)$

$\angle DBC + \angle EDB = 40°$

or $\angle DBC + \angle DBC = 40°$

or $\angle DBC = 20°$

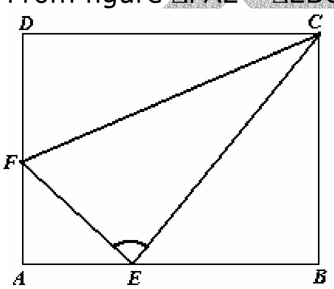
152. If we draw a circle passing through A, B and C, taking BC as diameter and M as center. It will pass through point B & C since $\angle B = 90°$ which means it lies in semicircle and AD in the Diameter.



So the circle will pass through all the four points A, B, C & D
Now AM = BM = CM = DM = radius
and $\angle A = 90°$ (angle in semicircle)
Now $\angle BAM = 30°$ $\{\because$ median divides $\angle A$ in 1 : 2$\}$
$\angle MAC = 60°$
From ΔAMC
$\angle MAC = 60°$
AM = MC (radius)
$\angle MCA = \angle MAC = 60°$
$\angle CMA = 60°$
ΔAMC in and equilateral triangle.
$\therefore$ AC = radius.

$$\therefore \frac{AC}{AD} = \frac{radius}{2 \times radius} = \frac{1}{2}$$

153. From figure ΔFAE ~ ΔEBC



$$\frac{FA}{BE} = \frac{AB}{BC} = \frac{1}{2}$$

or $FA = \frac{1}{2}, BE = \frac{1}{4} AB$

Area of $\Delta EBC = \frac{1}{4}$ area of square.

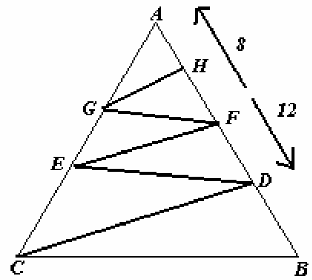
Area of $\Delta FAE = \frac{1}{16}$ area of square

Area of $\Delta FDC = \frac{3}{8}$ area of square

Area of $\Delta FEC =$□$areaofsquare\left[1 - \frac{1}{4} - \frac{1}{16} - \frac{3}{8}\right]$

$= \frac{5}{16}$ of area of square.

154. Draw a line GF || BC



Now, ΔAGF ~ ΔAED

$\therefore \frac{GF}{ED} = \frac{AF}{AD} = \frac{8}{20} = \frac{2}{5}$

Now ΔECD ~ ΔEFG

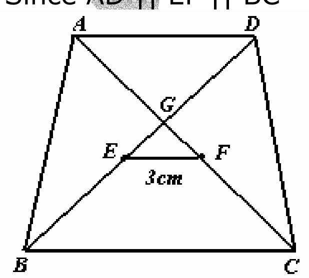
$\therefore \frac{EF}{CD} = \frac{FD}{DB} = \frac{2}{5}$

$\Rightarrow DB = \frac{5}{2}FD$

$= \frac{5}{2} \times 12$

$= 30cm.$

155. Since AD || EF || BC



ΔEFG ~ ΔGBC

$\frac{GB}{GE} = \frac{97}{3}$

$\Rightarrow \frac{BE+GE}{GE} = \frac{97}{3}$

$\Rightarrow \frac{BE}{GE} = \frac{94}{3}$ ----(I)

Now ΔADG ~ ΔGBC

$\frac{BG}{GD} = \frac{97}{AD} \Rightarrow \frac{BE+GE}{ED-GE} = \frac{97}{AD}$ ($\because$ BE = DE)

or $\frac{BE+GE}{BE-GE} = \frac{97}{AD}$ (using componendo-dividendo) or $\frac{2BE}{2GE} = \frac{97 + AD}{97 - AD}$

or $\frac{BE}{GE} = \frac{97 + AD}{97 - AD}$ ---(II)

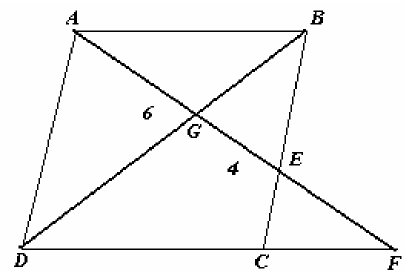
From (I) and (II)

$\frac{94}{3} = \frac{97 + AD}{97 - AD}$

Solving the above expression we get

AD = 91 cm.

156. From ΔAGD and ΔGBE



$\angle AGD = \angle BGE$

$\angle ADG = GBE$

∴ ΔAGD ~ ΔGNE

$\frac{AG}{GE} = \frac{AD}{BE} = \frac{6}{4} = \frac{3}{2}$

$\therefore \frac{AD}{BE} = \frac{3}{2}$

or $\frac{AD}{AD - BE} = \frac{3}{3-2}$

or $\frac{AD}{EC} = \frac{3}{1}$ -----(1)

Since ΔECF ~ ΔADF

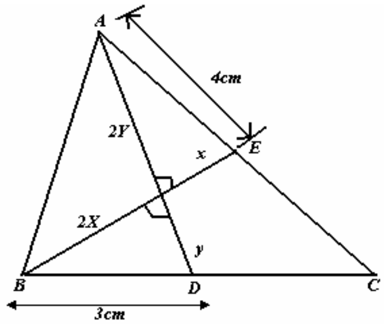
$\therefore \frac{AD}{EC} = \frac{AF}{EF} = \frac{3}{1}$ (From 1)

$\Rightarrow \frac{EF + 10}{EF} = \frac{3}{1}$

$\Rightarrow \frac{10}{EF} = \frac{3}{1}$

$\Rightarrow EF = \frac{10}{3} cm.$

157.



OE = X (Say)
OD = Y (Say)
So OA = 2Y
OB = 2X
From ΔAOE

$4Y^2 + X^2 = AE^2$
$= 16$ -----(1)

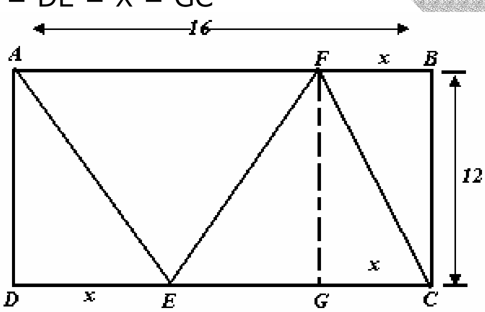
Similarly from ΔBOD

$4X^2 + Y^2 = BD^2$
$= 9$ -----(2)

(1) + (2)
$5\left(x^2 + y^2 = 25\right)$ -----(3)
$X^2 + Y^2 = 5$
From ΔAOB

$AB^2 = AO^2 + BO^2$
$= 4X^2 + 4Y^2$
$= 4\left(x^2 + y^2\right)$
$= 20$
$AB = 2\sqrt{5}cm.$

158. BF = DE = X = GC



$\therefore$ FC = AE = AF = EC
$\therefore \sqrt{12^2 + x^2} = 16 - X$
or $144 + x^2 = 256 + x^2 - 32x$
32 x = 144
X = 4.5 cm.
= GC
EG = EC – GC
= (16 – 4.5) – 4.5
= 7 cm

$EF = \sqrt{12^2 + 7^2} = 144 + 49 = \sqrt{193} cm.$

159. Lets assume the radius & height of the cylinder is r & h respectively.

So its volume = $\pi r^2 h$

The new radius = $\frac{7r}{6}$

It is given that $\pi r^2 h = \pi \left(\frac{7r}{6}\right)^2 h'$ (h′ is the new height)

$\Rightarrow h = \frac{36}{49} h'$

$\Rightarrow h' = \frac{36}{49} h' = \frac{72}{98} h = h - \frac{18}{98} h$

= 18% approx

160. $\pi rh = \frac{1}{2}$ (given)

$\Rightarrow r = 2$ unit

$\frac{r}{h} = \frac{2}{3}$ (given) $\Rightarrow n = 3$ unit

$\therefore$ total surface area = $\pi rh + 2\pi r^2 = \pi[2\times3 + 2\times2^2)$

= 14π

161. The path of the ant is given in the figure below:

$\therefore$ The distance covered by the ant is

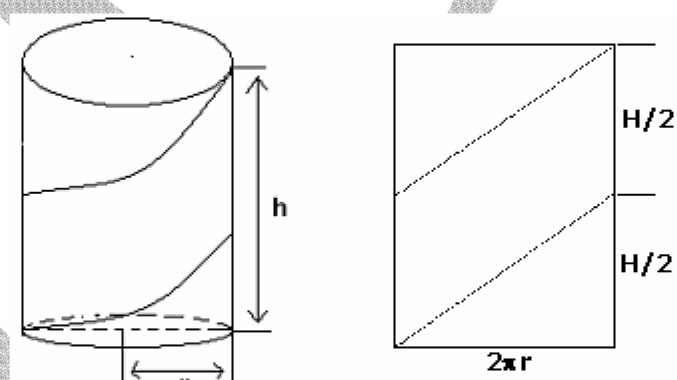
$= 2\times\sqrt{4\pi^2 r^2 + \frac{h^2}{4}}$

$= 4\times\sqrt{4\times\pi^2\times\frac{144}{\pi^2} + \frac{20^2}{4}}$

$= 4\times\sqrt{576+100}$

$= 4\times26$

= 104



162. The radius of such sphere would be same as the radius of in circle of the cross section of come. (as shown in figure)

From ΔABD,

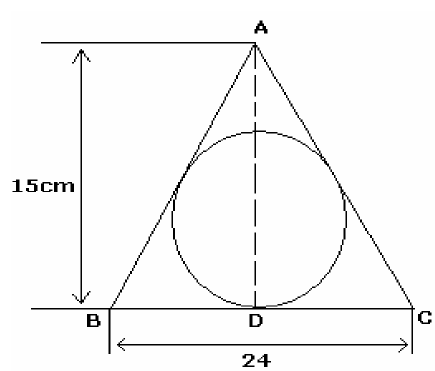
AB = $\sqrt{AD^2 + BD^2}$ = $\sqrt{15^2 + 12^2}$

= $\sqrt{369}$

= 19.20 cm

$\therefore$ = AC

$\therefore$ Semi peremeter (s) of ΔABC = $\frac{19.20 + 24 + 19.20}{2}$ **=** 31.20cm



Area of ΔAB = $\frac{1}{2} \times 24 \times 15$ = 180cm^2

$\therefore$in radius (r) = $\frac{Area}{5} = \frac{180}{31.20}$ = 5.77cm.

$\therefore$Volume of sphere = $\frac{4}{3}\pi r^3$ = $\frac{4}{3} \times \frac{22}{7} \times (5.77)^3$ = 805cm^3

163. Let assume the radius of come = cylinder = sphere = r and the did of sphere (2r) = height of come = height / cylinder

or r= $\frac{h}{2}$ =h

Now the volume of come = $\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$ = P

volume of cylinder = $\pi r^2 h = 2\pi r^3$ = Q

volume of sphere = $\frac{4}{3}\pi r^3$ = R

$\therefore$P+R= $\left[\frac{2}{3}+\frac{4}{3}\right]\pi r^3$ = 2πr^3 = Q

$\Rightarrow$ P-Q+R= 0

164. The isosceles right angle triangle = $2 \times \frac{1}{3}\pi r^2 h = \frac{2}{3}$

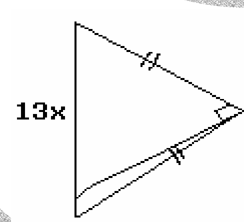
in the figure :- r= h = $\frac{3x}{2}$

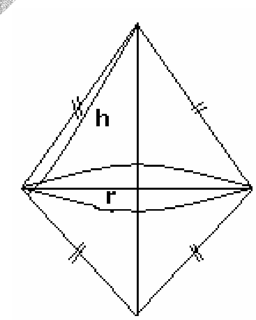
So the volume of the solid generated due to the rotation of a isosceler right angle triangle = $2 \times \left[\frac{1}{3}\pi r^2 h\right]$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi \frac{27}{8} x3$

$= \frac{9}{4}\pi x^3$





165. The volume of the sphere = $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi$ (10.5)^3

The cuboid of maximum is the one which is having all side equal or a cube.
So the volume of the cube= x^3 (where x is the length of one side)

$\therefore$x^3 = $\frac{4}{3} \times 10.5 \times 10.5 \times 10.5$

= 4851

x = 16.92

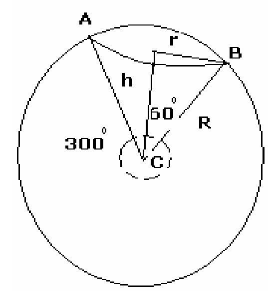
$\therefore$Total surface area of such cube = 6x^2

= 6x(16.92)^2

= 1719.3

≈ 1720

166. The radius of the sphere = R = 6cm



The volume taken out would be $\frac{1}{6}^{th}$ the volume of the sphere. Therefore,

the remaining volume would be $\frac{5}{6}^{th}$ the volume of the sphere, i.e.

$\frac{5}{6}\times\frac{4}{3}\pi\times 6^3 = 240\pi$

167. In this process the volume of n cones. Would be equal to the volume of the cylinder.

$\therefore n\times\frac{1}{3}\times\pi\times 1^2\times 1 = \pi\times 3^2\times 5$

$\therefore$n = $3^3\times 5$ = 135

168. In this process the volume of the new cube would be equal to the sum of volume of the three smaller cube.

$\therefore$ a^3 = 1 + 216 + 512 = 729

$\therefore$ a = 9cm.

$\therefore$ the length of the diagonal = $\sqrt{3}a = 9\sqrt{3}cm.$

169. From the figure it is clear that only half of the volume of cylinder is filled with soft drink and the remaining half is empty.

$\therefore$the capacity of the cylinder = 4.2 ltr.

170. If we consider only one villi. So now the perfected surface area of the villi = $2\pi rh + \pi r^2$

and the surface area of small intestine, projected to food, without villi = πr^2

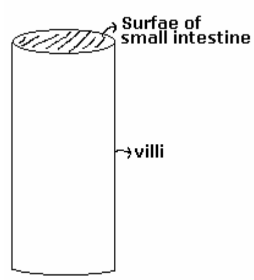
$\therefore$percentage increase in the surface area exposed to food = $\frac{2\pi rh + \pi r^2}{\pi r^2}\times 100$

$= \frac{2\pi rh}{\pi r^2}\times 100$

$= \frac{2h}{r}\times 100$

$= \frac{2\times 1.5\times 10^{-3}}{1.3\times 10^{-4}}\times 100$

$= \frac{30}{1.3}\times 100$



171. The total surface area of the human head = $4\pi r^2$

Area of the head covered with hair = $\frac{5}{8} \times 4\pi r^2$

$= \frac{5}{2}\pi r^2$

$= \frac{5}{2} \times \frac{2}{7} \times 7 \times 7$

$= 385\ cm^2$

$\therefore$Total number of hair on a human head = 385×300 hair.
Since the daily hair loss is one hair/1500 hair.

$\therefore$number of hair, on an average, a person lose = $\frac{385 \times 300}{1500}$

= 77 hair.

172. When we remove cube A we fined five new unit area surfaces named a,b,c,d and e or the surface area increases by $4cm^2$ when we remove cube B we fine four new unit area surfaces named f,g,h and I of the surface area increases by $3cm^2$ Similarly the surface area increases by $2cm^2$ when we remove cube C

$\therefore$The total surface area of the new figure = $[6 \times 4 \times 4] + [4 + 3 + 2]$

$= 96 + 9 = 105cm^2$

173. Since all the solids are made up of same material. So that their weight are directly proportional to their volume. Lets assume that their height = x= radius (given)

Volume of a sphere = $\frac{4}{3}\pi x^3$

Volume of a cylinder = πx^3

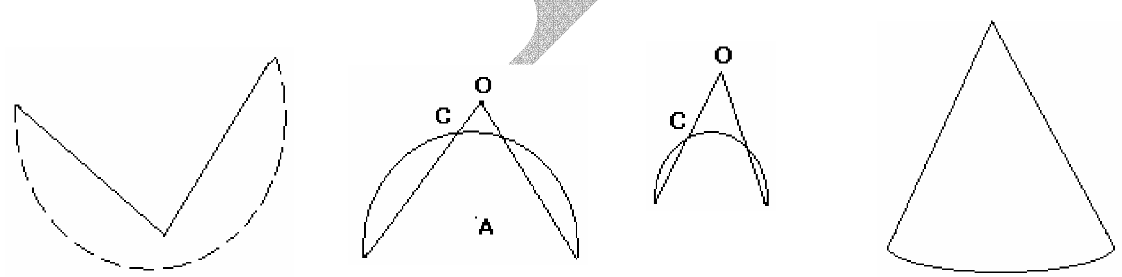
Volume of a cone = $\frac{1}{3}\pi x^3$

$\therefore$Some of volume of two sphere, one cylinder and one cone = $\left[\frac{8}{3} + 1 + \frac{1}{3}\right]\pi x^3$

$= 4\pi x^3$

The same volume can be balanced by:-
(1) – 4 cylinder.
(2) 3 cylinder + 3 cones.
(3) 2 cylinder +2 cones + 1 sphere
So there are 3 ways to balance the bean with the given solids.

174.



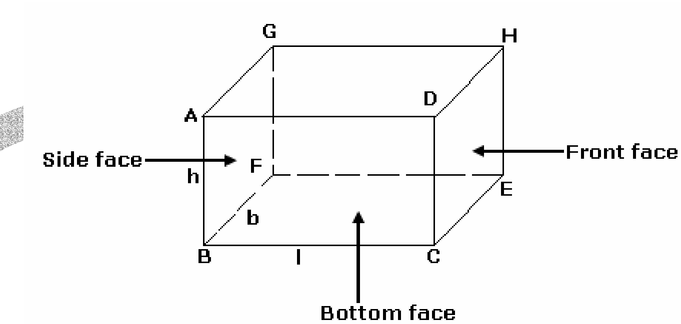
175. Slant height of the cone = 10cm And the perimeter of the base circle = length of the are of semi circular paper ship.

2πr = πR (R= 10cm)
R= 5

$\therefore$height of cone = $\sqrt{100^2 - 25} = \sqrt[5]{3}$

$\therefore$volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 5^2 \times \sqrt[5]{3} = \frac{125\pi}{\sqrt{3}}$

176. When we observe the length of the rod from the side, front and bottom, we get the length of side face diagonal, front face diagonal and bottom face diagonal.



$\therefore 5 = \sqrt{b^2 + h^2}$ (side face diagonal) –(1)

$\sqrt[4]{10} = \sqrt{l^2 + h^2}$ (Front face diagonal) – (2)

$\sqrt{153} = \sqrt{b^2 + l^2}$ (Bottom face diagonal) – (3)

From (1), (2) & (3)

$5^2 + \left(\sqrt[4]{10}\right)^2 + \left(\sqrt{153}\right)^2 = 2\left[b^2 + h^2 + l^2\right]$

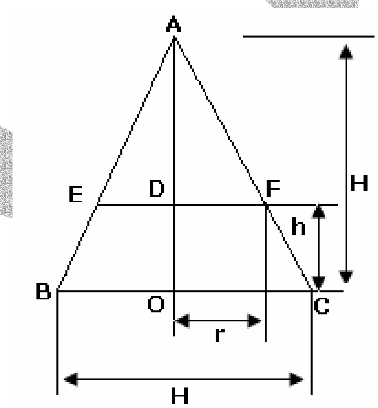
Or

$b^2 + h^2 + l^2 = \frac{1}{2}[25 + 160 + 153] = \frac{1}{2} \times 338 = 169$

Or $\sqrt{b^2 + h^2 + l^2} = \sqrt{169}$ = 13cm. (body diagonal of cuboid)

$\therefore$length of rod= 13cm.

177. From ΔABC and ΔAEF

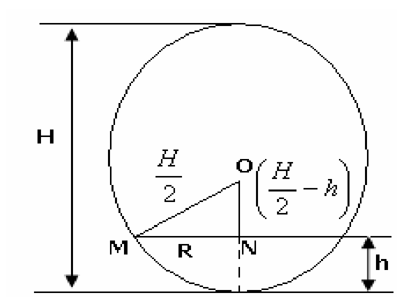


$\frac{AD}{ED} = \frac{AD}{BO}$ (Since both the Δs are similar)

$\therefore$ED= $\frac{AD \times BO}{AO}$

= $\frac{(H-h)\times\frac{7}{2}}{H} = \frac{(H-h)H}{2H} = \frac{H-h}{2} = r$

From ΔMNO



R^2 = $\left(\frac{H}{2}\right)^2 - \left(\frac{H-2h}{2}\right) = \frac{H^2 - H^2 - 4h^2 + 4Hh}{4}$

R^2 = $\sqrt{\frac{4Hh - 4H^2}{4}}$

Since both the cross sections are having equal area

$\frac{H-h}{2} = \sqrt{\frac{4Hh - 4h^2}{4}} \Rightarrow \frac{H^2 + h^2 - 2Hh}{4} = \frac{4Hh - 4h^2}{4}$

$\Rightarrow$ H^2 +$5h^2$-6Hh=0 $\Rightarrow$ H^2-Hh-5Hh+$5h^2$

$\Rightarrow$ H(H-h)-5h(H-h)

$\Rightarrow$ (H-5h) (H-h)=0

Or h=H (which in not possible)

Or h= $\frac{H}{5}$

178. If the initial volume (V)= $\pi r^2 h$

Then the new volume (V^1) = $\pi(1.1r^2) \times 0.9h$

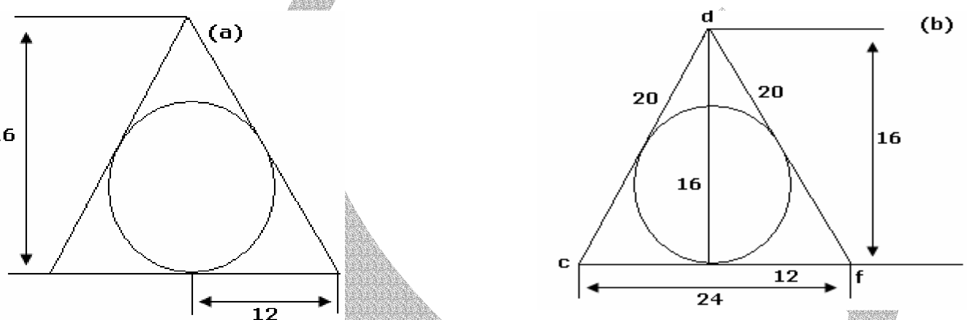
= $1.089\pi r^2 h$

$\therefore V^1 = V + 0.089V$

$= V + \frac{8.9}{100}V$

It means the new volume is 8.9% more then the previous volume.

179. If we see the cross section of the come, it would look like the same as shown in the figure (b)



The radius (r) of the sphere would be same as the radius (r) of inscribed circle of Δdef

r= $\frac{Area}{semipe} = \frac{\frac{1}{2} \times 24 \times 16}{32} = \frac{12 \times 16}{32} = 6$

$\therefore$ Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 6 \times 6 \times 6$

= 288π

180. The volume of the liquid in side the container= $\frac{5}{8} \times 16^3$

= 2560cm^3

If we drop a perpendicular from K to line CD, it will divide the water into following two parts.

(1) The cuboida part, which is shown by the cross section BKML.
(2) The prismatic part, which is shown by the cross section KML
It is given that BK=2x & LC = 3x

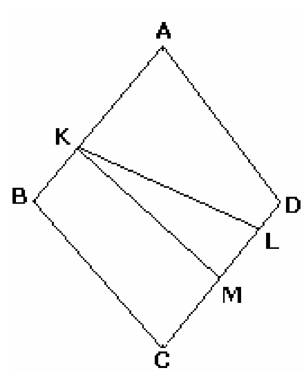
$\therefore$ MC= 2x & ML =x

$\therefore$ Vol of first part = $16 \times 16 \times 2x$

= 512x

And vol of second part = $\frac{1}{2}x \times 16 \times 16$

= 128x



$\therefore$ Since the volume of water is unchanged
$\therefore$ 512x + 128x= 2560
$\therefore$ 640x = 2560

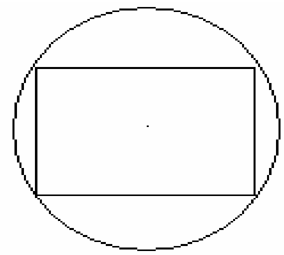
$X = \frac{2560}{640} = 4$

$\therefore$ length of line segment LC= 3x

= 12cm.

181. Let assume the side length of cue C_1= a unit
and the side length of cube C_2 = b unit

radius of circumscribed sphere of cube C_1= $\frac{a}{\sqrt{2}}$

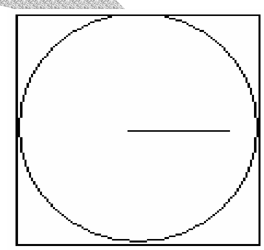


C_1

$\therefore$ Vol V_1 of circumscribed sphere of cube C_1= $\frac{4}{3}\pi\left(\frac{a}{\sqrt{2}}\right)^3$

Similarly, Radius of inscribed sphere of cube C_2 = $\frac{b}{2}$

$\therefore$ Volume V_2 of inscribed sphere of cube C_2 = $\frac{4}{3}\pi\left(\frac{b}{2}\right)_3$



C_2

It is given V_1= $2v_2$ or $\frac{4}{3}\pi\left(\frac{a}{\sqrt{2}}\right)^3 = 2\frac{4}{3}\pi\left(\frac{b}{2}\right)^3$

$$\Rightarrow \frac{a^3}{2\sqrt{2}} = 2\frac{b^3}{8}$$

$$\Rightarrow \frac{a^3}{b^3} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \left(\frac{a}{b}\right) = \left(\frac{1}{2}\right)^{\frac{1}{6}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

$\therefore$ surface area 5, if inscribed sphere of the cube C_1= $4\pi\left(\frac{9}{2}\right)^2$

Similarly surface area S_2 of circumscribed sphere of the cue C_2= $4\pi\left(\frac{b}{\sqrt{2}}\right)^2$

$$\therefore \frac{S_1}{S_2} = \frac{\left(\frac{a}{2}\right)^2}{\left(\frac{b}{\sqrt{2}}\right)^2} = \left(\frac{a}{b}\right)^2 \times \frac{1}{2}$$

From (1) $$\frac{S_1}{S_2} = \left(\frac{1}{2^{\frac{1}{6}}}\right)^2 \times \frac{1}{2} = \frac{1}{2\times\sqrt[3]{2}} = \frac{1}{2^{\frac{4}{3}}}$$

182. Sum of the volumes of cube S_1 and S_2 = $a_1^3 + a_2^3$
Sum of the length of edges of cubed S_1 and S_2 = $12a_1+12a_2$
It is given that :-

$$a_1^3 + a_2^3 = 12a_1 + 12a_2$$

Or $(a_1 + a_2)(a_1^2 + a_2^2 - a_1a_2) - 12(a_1 + a_2) = 0$

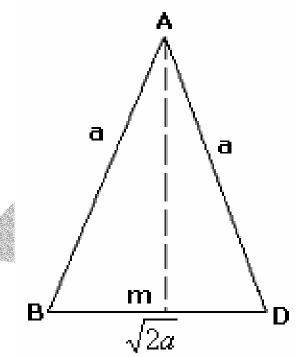
Or $(a_1 + a_2)(a_1^2 + a_2^2 - a_1a_2 - 12) = 0$

Since $a_1+a_2 = 0$ (not possible)

$\therefore a_1^2 + a_2^2 - a_1a_2 - 12 = 0$

Or $(a_1 - a_2)^2 + a_1a_2 = 12$

183. If we take the cross section of the pyramid it would look like the same as shown in the figure:



Length $AM = \sqrt{a^2 - \left(\frac{a}{\sqrt{2}}\right)^2}$ (from pythagorous)

$= \frac{a}{\sqrt{2}}$

$BM = \frac{a}{\sqrt{2}}$

$\therefore \angle ABD = 45^o$

Similarly $\angle ADB = 45^o$

$\therefore \angle BAD = 90^o$

(Second method) if we look at the length of three sides of triangle ABD, they form Pythagorean triplet with BD as hypotenuse.

$\therefore \angle BAD = 90^o$

184. In the above figure CB, represents the water surface. If we till the bowl the water will start spilling as the point B coincide with point e.

Or, as line OB coincide with line oe

To male this happen we have to till the line OB by angle α

Now from triangle OAB

OB = r (radius of bowl)

OA = $\frac{r}{2}$ (given)

$\therefore \angle AOB = 60^o$

$or \angle \alpha = 30^o$

185. The cut plane is ODB

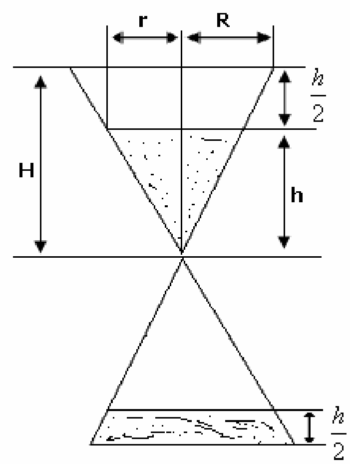
OD and BD are the medium of

ΔOAC and ΔABC respectively.

$\therefore$ OD=BD = $\frac{\sqrt{3}}{2}a$ and OB= a (given) the height of the triangle ODB would be same as height of tetrahedron

$\therefore$ area = $\frac{1}{2} BD \times height = \frac{1}{2} \times \frac{\sqrt{3}}{2} a \times \sqrt{\frac{2}{3}} a$

186. .From the figure $h + \frac{h}{2} = H$



$$\frac{3}{2}h = H$$

$$h = \frac{2}{3}H$$

radius of the conical column of send= $\frac{2}{3}R$

$\therefore$ Volume of sand in upper cone= $\frac{8}{27}V$

Where V in the total volume of sand (or each of cone)

$\therefore$ Volume of sand poured down in the lower cone $= V - \frac{8}{27}V$

$= \frac{19}{27}V$

$\therefore$ since the entire volume of sand takes the to pour down

$\therefore \frac{19}{27}V$ of sand will take $\frac{19}{27} \times 27$ mins

$= \frac{19 \times 20}{9} = \frac{380}{9}$ = 42.23 mins

187. $\frac{1}{2}AC \times BP = \frac{1}{2}AB \times BC$ (area of triangle)

$\therefore BP = \frac{8 \times 16}{10} = 4.8cm.$

From the similar $\Delta BDE and \Delta ABC$

DB= $8 \times \frac{4.8}{10} = 3.84cm$

BE= $6 \times \frac{4.8}{10} = 2.88cm$

$\therefore$ area of shaded portion= area of semi circle – area of ΔBDE

$= \frac{1}{2} \times \frac{\pi(4.8)^2}{4} - \frac{1}{2}3.84 \times 2.88$

188. if we join center of inscribed circles.
We will get an equilateral triangle of side 2a
now the radius R of the outer circle = AD + AD

Where $AO = \frac{2}{3}$ of medium of ΔABC

$\therefore R = a + \frac{2a}{\sqrt{3}}$

$\therefore$ area of outer circle= πR^2

$= \pi\left[a + \frac{2a}{\sqrt{3}}\right]^2$

$$= \frac{\pi\left(2+\sqrt{3}\right)^2 a^2}{3}$$

189. Area of shaded portion = area of circumscribing circle – area of ΔABC

- $3\times\frac{5}{6}$ area of smaller circle

$$= \frac{\pi\left[2+\sqrt{3}\right]^2 a^2}{3} - \frac{\sqrt{3}}{4}\times 2a\times 2a - 3\times\frac{5}{6}\times\pi a^2$$

$$= \frac{\pi\left(7+4\sqrt{3}\right)}{3}a^2 - \sqrt{3}a^2 - \frac{5}{2}\pi a^2$$

$$= \frac{\left[14+8\sqrt{3}-15\right]\pi a^2}{6} - \sqrt{3}a^2$$

$$= \frac{\left(8\sqrt{3}-1\right)}{6}\pi a^2 - \sqrt{3}a^2$$

190. Let the base radius and slant height is r, and l, for the first cone and similarly r_2 and l_2 for the second cone

$\frac{\pi r_1 l_1}{\pi r_2 l_2} = \frac{2}{1}$ (given)

$\frac{r_1}{r_2} = \frac{2}{1}\frac{l_2}{l_1} = 2\times 2$ $\left(\frac{l_2}{l_1} = 2; given\right)$

=4

$\therefore \frac{A_1}{A_2} = 16$ (A_1 and A_2 are the area of bases of cone 1 and cone 2 respectively.)

191. Let the lines of folds be PQ and QR. The folded piece would be symmetrical about the line of fold.

$\Rightarrow 2\theta + \alpha\alpha = 180^o \Rightarrow \theta + \alpha = 90^o$

or $\theta = 90 - \alpha$

In ΔPQD $\tan\theta = \frac{2x}{60-5x}$

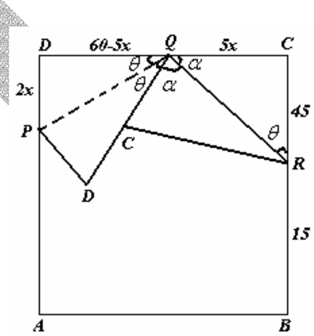
In ΔQRC

$\tan\left(90-\alpha\right) = \tan\theta = \frac{5x}{45}$

$\frac{5x}{45} = \frac{2x}{60-5x}$

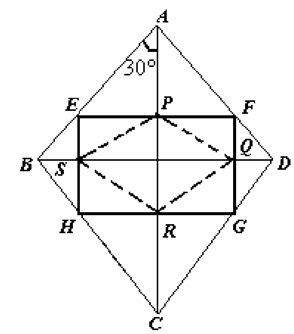
$\Rightarrow 60 - 5x = 18\ x = \frac{42}{5} = 8.4$

area $\Delta PDQ = \frac{1}{2}\times 16.8\times 18 = 151.2$



192. ABD is an equilateral triangle.

Since E and F are the midpoints of the side P would also be a midpoint. Similarly, we can slows that S would also be a midpoint. Therefore the sides of rhombus PQRS would be half the sides of ABCD.



Sum of areas = $2\times\frac{\sqrt{3}}{4}a^2+\frac{\sqrt{3}}{4}a^2+\frac{\sqrt{3}}{16}a^2+....$

$= \frac{\sqrt{3}}{2}a^2\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)$

$= \frac{\sqrt{3}}{2}a^2\times 2=\sqrt{3}a^2=64\sqrt{3}\Rightarrow a=8$

Sum of perimeters =

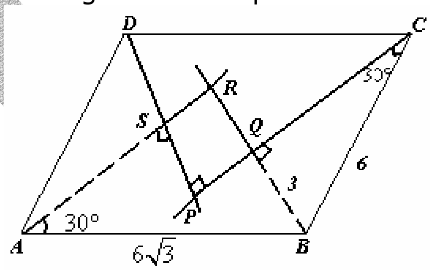
$\left(4\times8+4\times4+4\times2+....\right)+2\left(4+4\sqrt{3}+\frac{4+4\sqrt{3}}{2}+.....\right)$

$= 4\times8\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)+2\left(4+4\sqrt{3}\right)\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)$

$= \left(40+8\sqrt{3}\right)\left(1+\frac{1}{2}+\frac{1}{4}+....\right)=16\left(5+\sqrt{3}\right)$

193. The angle bisectors of two interior supplementary angles intersect at 90^o.

Therefore the quadrilateral formed would be a rectangle. Let the quadrilateral be PQRS as shown in the figure.



QB = 3 (ΔQCB is a 30 – 60 – 90 triangle)

RB = $3\sqrt{3}$ (ΔARB is a 30 – 60 – 90 triangle)

RQ = $3\left(\sqrt{3}-1\right)$

Similarly, AR = 9 and AS = $3\sqrt{3}$

$\Rightarrow RS=3\sqrt{3}\left(\sqrt{3}-1\right)$

$\Rightarrow Area=9\sqrt{3}\left(\sqrt{3}-1\right)^2=9\sqrt{3}\left(4-2\sqrt{3}\right)=18\left(2\sqrt{3}-3\right)$

194. It can be seen that EBCO is a parallelogram. The heights of both ΔBCD and ΔABC are the same.

Therefore the ratio of areas = ratio of bases

$\Rightarrow\frac{AD}{BC}=\frac{8}{3}$

Let AD = 8x, BC = 3x, ⇒ AE = 5x

$\Rightarrow\frac{1}{2}\times8x\times h=8$ $\qquad\Rightarrow hx=2$

$area\Delta AEB=\frac{1}{2}\times5x\times h=5$

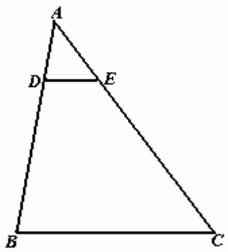
195. Option (c)

196. $\angle ABC=\angle ACB=57.5^o=\angle BEC$ (EB = BC)

$\Rightarrow\angle EBC=180-\left(57.5+57.5\right)=65^o$

Hence data in consistent

197.

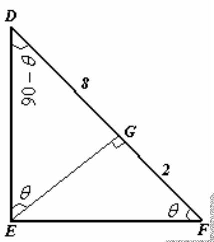


$$\frac{Area\Delta ADE}{Area\Delta ABC} = \frac{AD^2}{AB^2} = \frac{1}{16} \Rightarrow \frac{Area\square DBCE}{Area\Delta ABC} = \frac{15}{16}$$

$$\Rightarrow Area\Delta ABC = 45 \times \frac{16}{15} = 48$$

$$\Rightarrow Area\Delta ADE = \frac{48}{16} = 3$$

198.



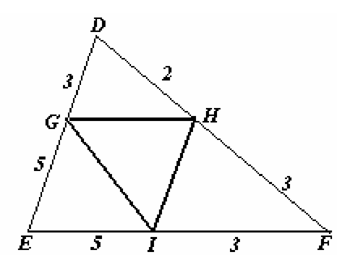
$$\frac{Area\Delta DEG}{Area\Delta EGF} = \frac{DG}{GF} = \frac{8}{2} = 4$$

ΔDEG and ΔEFG are similar

$$\frac{Area\Delta DEG}{Area\Delta EFQ} = \frac{DE^2}{EF^2} = 4 \Rightarrow \frac{DE}{EF} = 2$$

199. Use pythegoreon triplets of (17, 15, 8) and (5, 3, 4).

200.



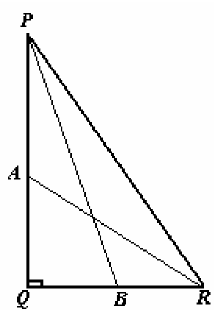
$$\frac{Area\Delta DGH}{Area\Delta DEF} = \frac{5 \times 2}{8 \times 5} = \frac{3}{20}$$

$$\frac{Area\Delta GEI}{Area\Delta DEF} = \frac{5 \times 5}{8 \times 8} = \frac{25}{64}$$

$$\frac{Area\Delta FIH}{Area\Delta DEF} = \frac{3 \times 3}{5 \times 8} = \frac{9}{40} \quad \Rightarrow \frac{Area\Delta GHI}{Area\Delta DEF} = 1 - \left(\frac{3}{20} + \frac{25}{64} + \frac{9}{40}\right) = \frac{15}{64}$$

$$\Rightarrow Area\Delta DEF = \frac{64}{15}\times 45 = 192$$

201.

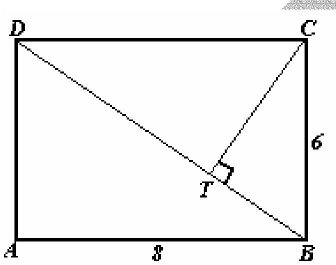


$$PB^2 = PQ^2 + QB^2 = PQ^2 + \frac{QR^2}{4}$$

$$AR^2 = AQ^2 + QR^2 = \frac{PQ^2}{4} + QR^2$$

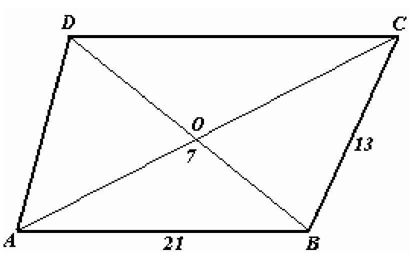
$$PB^2 + AR^2 = \frac{5\left(PQ^2 + QR^2\right)}{4} = \frac{5}{4}PR^2$$

202.



$$\frac{BT}{DT} = \frac{Area\Delta CDT}{Area\Delta CBT} = \frac{CD^2}{CB^2} = \frac{16}{9}$$

203.

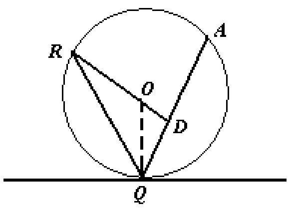


$$AB^2 + BC^2 = 2\left(CO^2 + BO^2\right) = 2\left(\frac{AC^2 + BD^2}{4}\right)$$

$$\Rightarrow AC^2 + BD^2 = 2\times\left(21^2 + 13^2\right)$$

= 1220

204. Join O to Q



$\angle OQD = 45^o \Rightarrow OD = DQ = 10 \Rightarrow OQ = 10\sqrt{2}$

$\Rightarrow RD = 10 + 10\sqrt{2} = 10\left(1+\sqrt{2}\right)$

$Area\Delta QDR = \dfrac{1}{2} \times 10 \times 10\left(1+\sqrt{2}\right) = 50\left(1+\sqrt{2}\right)$

205. At PQR be the given triangle.
In trapezium ABCF, FC = 2a, AB = a

$\Rightarrow PQ = \dfrac{2a+a}{2} = \dfrac{3a}{2} \Rightarrow area\Delta PQR = \dfrac{\sqrt{3}}{4} \times \dfrac{9a^2}{4}$

Area of hexagon = $6 \times \dfrac{\sqrt{3}}{4}a^2$

Ratio = $\dfrac{9}{4}:6$ = 3 : 8

206. Join Q and S

$\angle AQR = \angle ASQ = 60^o$

$\angle AQS = \angle AST = 70^o$

$\Rightarrow QAS = 50^o$

$\angle QPS = 360^o \left(\angle PQA + \angle PSA + \angle QAS\right)$

$= 360^o - \left(120 + 110 + 50^o\right)$

$= 80^o$

207. Length of the side of the square = $a\sqrt{2}$
length of the side of the triangle = $2a\sqrt{3}$
ratio = $1:\sqrt{6}$

208. AM × AD = AP × AQ = AS × AR
⇒ 4 × 10 = 5(5 + SR) ⇒ SR = 3.

209.

$\angle PEH = 180 - \left(95 + 50\right) = 35^o$

$\angle EFQ = 180 - \left(95\right) = 85^o$

$\Rightarrow \angle EQF = 180 - \left(35 + 85\right) = 60^o$

210. $\angle PRQ = \angle \quad RPQ = 45^o \Rightarrow PQ = QR = x$ (say)

RD × PR = RE × RQ ⇒ $3 \times x\sqrt{2} = \left(x - 5\sqrt{2}\right)x$

$\Rightarrow x = 8\sqrt{2} \Rightarrow pr = x\sqrt{2} = 16$

211. $\angle ABC$ is an equilateral triangle

⇒ radius of the circle = $\frac{a}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

Let O be the center.

$\angle BOA = 120^o \Rightarrow areaofshadedregion = \frac{120}{360} \times \pi \times \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16\pi - 12\sqrt{3}}{9} - \frac{1}{3} \times \left(\frac{\sqrt{3}}{4}\right) \times 4^2$

From the figure

$\left(\frac{r}{2} + a\right)^2 = \left(\frac{r}{2}\right)^2 + \left(r - a\right)^2$

$\frac{r^2}{4} + a^2 + ra = \frac{r^2}{4} + r^2 - 2ar + a^2$

$a = \frac{r}{3}$

212. $B = PQ \times a_1$ and $B = QR \times a_2$

$PQ = \frac{B}{a_1}$ and $QR = \frac{B}{a_2}$

Perimeter = 2(PQ + QR) = $\frac{2B\left(a_1 + a_2\right)}{a_1 a_2}$

213. AD = OA = 6 ⇒ DB = 8

$OC^2 = DB^2 + BC^2$

⇒ OC = 10 radius

214. ΔODC is equilateral ⇒ CD = AB = r ⇒ $AD = \sqrt{\left(2r\right)^2 - r^2} = r\sqrt{3}$

$\angle AOD = 180 - \left(30 + 30\right) = 120^o$

Area of the shaded region = $\frac{120}{360} \times \pi r^2 - \frac{\sqrt{3}x^2}{4} = r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

215. Let lb = 4x bh = x hl = 3x

$\Rightarrow l^2 b^2 h^2 = 12x^3 = \left(144\right)^2 \Rightarrow x^3 = 12^3 \Rightarrow x = 12$

⇒ l = 12, b = 4, h = 3

longest diagonal = $\sqrt{l^2 + b^2 + h^2} = \sqrt{144 + 16 + 9} = 13$

216. Surface area = Areas of lateral rectangles + areas of opposite faces

$$= 3\times 6\times 15+12\times 15+\frac{\sqrt{3}}{4}\times 6^2\times 6$$

$$= \; = 270+180+54\sqrt{3}$$

$$\approx 543$$

217. $2\pi r^2+2\pi rh=96\pi \Rightarrow 2\lambda r\left(r+h\right)=96\lambda$

$\Rightarrow$ r = 4 $\qquad\qquad$ $\Rightarrow$ h = 8

218. $\dfrac{h-k}{h}=\dfrac{r_1}{r_2} \qquad r_1=r\left(1-\dfrac{k}{h}\right)$

$$\frac{1}{3}\pi r^2\left(1-\frac{k}{a}\right)^2\times\left(h-k\right)=\frac{1}{3}\pi r^2 h\times\frac{1}{2}$$

$$\left(1-\frac{k}{h}\right)^3=\frac{1}{2} \quad \frac{R}{h}=1-\left(\frac{1}{2}\right)^{\frac{1}{3}} \quad k=h\left(1-\left(\frac{1}{2}\right)^{\frac{1}{3}}\right)$$

219. The portion of the bowl filled = $\dfrac{\frac{1}{3}\pi r^2 h}{\frac{2}{3}\pi r3}$

$$= \frac{h}{2r}=\frac{8}{24}=\frac{1}{3}=33.33\%$$

portion empty = 66.66%

220. $\dfrac{OJ}{DC}=\dfrac{KJ}{BC}\Rightarrow KJ=\dfrac{3}{4}BC\Rightarrow IK=\dfrac{1}{4}BC$

$$\frac{EI}{EB}=\frac{IL}{BC}\Rightarrow IL=\frac{2}{3}BC\Rightarrow LJ=\frac{1}{3}BC$$

$$\Rightarrow KL=\left(1-\left(\frac{1}{4}+\frac{1}{3}\right)\right)BC=\frac{5}{4}=1.25$$

221. ΔSRT is equilateral $\Rightarrow \angle RSR=60^o$

Area of the shaded region

= Area sector TSR + Area sector TRS − Area ΔSTR

$$= \frac{\pi}{6}a^2+\frac{\pi}{6}a^2=\frac{\sqrt{3}}{4}a^2=\frac{\pi}{3}a^2-\frac{\sqrt{3}}{4}a^2$$

$$\therefore Ratio=\frac{\pi}{3}-\frac{\sqrt{3}}{4}$$

222. Join H and I

Area □GHFI = Area ΔEFG

$\Rightarrow$ Area ΔEIJ = Area ΔGHJ (why?)

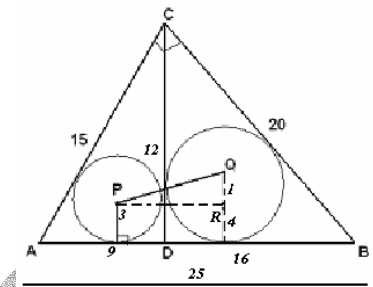
$\Rightarrow$ Area ΔEIJ + Area ΔHIJ = Area ΔGHJ + Area ΔHIJ

$\Rightarrow$ Area ΔEHI = Area ΔGHI
There are two triangle with the same box and same area
$\Rightarrow$ GE || HI $\Rightarrow$ DE || HI $\Rightarrow$ DH : HF = 2 : 3

$\Rightarrow Area\Delta EFH = \frac{3}{5} \times Area\Delta DEF = \frac{3}{5} \times 10 = 6$

223. $\angle AOD - \angle BOC = 2\angle AED = 30^o$

224. $CD = \frac{AC \times BC}{AB} = 12$



In radius of $\Delta CPD = \frac{12+9-15}{2} = 3$

ΔADC and ΔBDC are similarly. Their in radius would be in the ratio of their sides. The ratio = $\frac{AC}{BC} = \frac{3}{4}$

Therefore radius of the in radius of ΔBCD = 4. Drop a perpendicular from P to vertical line passing through Q.

$PQ = \sqrt{1^2 + 7^2} = \sqrt{50}$

225. Join E and F. Let EF intersect OG at H. W is the center of the circle
$\Rightarrow$ H is the midpoint of OG

$\Rightarrow OH = \frac{r}{2}$ OF = r $\Rightarrow \angle OFH = 30^o = \angle BOF$

In $\Delta BOF = 30^o$, OF = OB $\Rightarrow \angle OBF = 75^o$

226. The height and radius of the cone at height 5cm are of height and radius of the original one and volume is one-eighth.

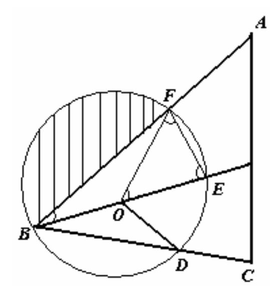
The height and radius of the cone at high 2cm are $\frac{4}{5}th$ of height and radius of the original cone and volume is $\left(\frac{4}{5}\right)^3 = \frac{64}{125}th$

Therefore $V_1 = \frac{1}{8}$ $V_2 = \frac{64}{125} - \frac{1}{8} = \frac{387}{1000}$ $V_3 = 1 - \frac{64}{125} = \frac{61}{125}$

Ratio = 125 : 387 : 488

227 and 288. ΔBFE is a right angled triangle

$\angle FBE = 30^o \Rightarrow \angle BEF = 60^o$



$\Rightarrow$ ΔOEF is an equilateral triangle area of sector BFE = Area of sector FOE + Area ΔBOF

$= \frac{\pi}{6} \times 2^2 + \frac{1}{2} \times 2 \times \sqrt{3} = \frac{2\pi}{3} + \sqrt{3}$

area of shaded region = $\frac{\pi \times 2^2}{2} - \left(\frac{2\pi}{3} + \sqrt{3}\right) = 2\pi - \frac{2\pi}{3} - \sqrt{3}$

$= \frac{4\pi}{3} - \sqrt{3}$

$\therefore$ Total area left = $2\left(\frac{4\pi}{3}-\sqrt{3}\right)=\frac{8\pi}{3}-2\sqrt{3}$

229. We can see that

$$\sqrt{625-x^2} = \sqrt{676-(17-x)^2}$$

$$625-x^2 = 676-289-x^2+34x$$

$$\Rightarrow x=10 \Rightarrow h=\sqrt{625-100}=\sqrt{525}=5\sqrt{21}$$

$$Area=\left(\frac{60+77}{2}\right)\times 5\sqrt{21}$$

230. $AC^2+BD^2 = AB^2+CD^2+2AD.BC$

= 81 + 225 + 2 × 12 × 20 = 786