## SUMMARY OF PERMUTATIONS & COMBINATIONS

**Formulas:**

No. of ways of selecting $r$ objects out of $n$ objects = ${}^{n}C_{r}$

No. of ways of arranging $n$ objects = $n!$

No. of ways of arranging $n$ objects with $a$ identical & $b$ identical = $\dfrac{n!}{a!\,b!}$

No. of ways of arranging $r$ objects out of $n$ objects = $n(n–1)(n–2)\ ...\ (n – (r – 1)) = {}^{n}P_{r} = {}^{n}C_{r}\, r!$

No. of ways of arranging $n$ objects in a circle = $(n – 1)!$

**Choosing People**

A team of 4 is to be chosen from a group consisting of Anne and Bob and 4 other people. In how many ways can this be done if

(i) there are no restrictions?
(ii) Anne must be in the team?
(iii) Anne and Bob must both be in the team?
(iv) at most one of Anne and Bob in the team?
(v) Anne or Bob or both are in the team?

(i) No. of ways of choosing 4 people = ${}^{6}C_{4} = 15$
(ii) No. of ways of choosing the other 3 people = ${}^{5}C_{3} = 10$
(iii) No. of ways of choosing the other 2 people = ${}^{4}C_{2} = 6$
(iv) Total no. of ways – no. of ways with Anne & Bob both in the team = 15 – 6 = 9
(v) No. of ways with Anne in the team + no. of ways with Bob in the team – no. of ways with both in the team = 10 + 10 – 6 = 14

**Choosing from Different Types of People (e.g. Boys & Girls)**

A team of 3 is to be chosen from a group of 3 boys and 4 girls. How many ways can this be done if

(i) there are no restrictions?
(ii) there must be exactly 1 boy?
(iii) there must be at least 1 boy?
(iv) there must be at least 1 boy and at least 1 girl?

(i) No. of ways of choosing 4 people = ${}^{7}C_{3} = 35$
(ii) No. of ways of choosing 1 boy and 2 girls = ${}^{3}C_{1}\,{}^{4}C_{2} = 18$
(iii) No. of ways = Total no. of ways – no. of ways with no boys = $35 – {}^{4}C_{3} = 35 – 4 = 31$
**Note:** It is wrong to say no. of ways = ${}^{3}C_{1}\,{}^{6}C_{2} = 45$
(iv) No. of ways = Total no. of ways – no. of ways with no boys – no. of ways with no girls
$= 35 – {}^{4}C_{3} – {}^{3}C_{3} = 35 – 4 – 1 = 30$
**Note:** It is wrong to say no. of ways = ${}^{3}C_{1}\,{}^{4}C_{1}\,{}^{5}C_{1} = 60$

**Choosing People to form Groups**

Find the no. of ways in which 6 people can be divided into

(i) two groups consisting of 4 & 2 people,
(ii) two groups consisting of 3 people each.
(iii) group 1 and group 2, with 3 people in each group.

(i) No. of ways = ${}^{6}C_{4}\,{}^{2}C_{2} = 15$
(ii) No. of ways = $\dfrac{{}^{6}C_{3}\,{}^{3}C_{3}}{2!} = 10$ **Note:** Divide by 2! since the 2 groups are of equal size.
(iii) No. of ways = ${}^{6}C_{3}\,{}^{3}C_{3} = 20$ **Note:** Don't divide by 2! since the 2 groups are labeled.

**Choosing Letters, Balls or other Identical Objects**
Seven cards each bear a single letter, which together spells the word "MINIMUM". Three cards are to be selected. The order of selection is disregarded. Find the number of different selections.

Case 1: No. of ways of choosing 3 identical letters = no. of ways of choosing "MMM" = 1
Case 2: No. of ways of choosing "MM" + 1 other letter = no. of ways of choosing I, N, U = $^3C_1 = 3$
Case 3: No. of ways of choosing "I I" + 1 other letter = no. of ways of choosing M, N, U = $^3C_1 = 3$
Case 4: No. of ways of choosing 3 different letters = no. of ways of choosing M, I, N, U = $^4C_3 = 4$
$\therefore$ total no. of selections = 1 + 3 + 3 + 4 = 11

**Arranging People**
Anne and Bob and 2 other people are to sit in a row. How many ways can this be done if
(i) there are no restrictions?
(ii) Anne must sit on the left and Bob on the right?
(iii) Anne and Bob must sit together?
(iv) Anne and Bob must be separate?

(i) No. of ways of arranging 4 people = 4! = 24
(ii) No. of ways of arranging the other 2 people = 2! = 2
(iii) Treat Anne and Bob as 1 item.
No. of ways of arranging 3 items × no. of ways of arranging Anne & Bob = 3! 2! = 12
(iv) No. of ways = 24 – 12 = 12

**Arranging Different Types of People (e.g. Boys & Girls)**
In how many ways can 3 boys & 3 girls be arranged in a row if
(i) there are no restrictions?
(ii) the 1st person on the left is a boy?
(iii) the person on each end is a boy?
(iv) the boys are together?
(v) the boys are separate?

(i) No. of ways of arranging 6 people = 6! = 720
(ii) No. of arrangements = $^3C_1 \times$ no. of ways of arranging the other 5 people = $^3C_1\ 5! = 360$
(iii) No. of arrangements = $^3C_2 \times 2! \times$ no. of ways of arranging the other 4 people
= $^3C_2\ 2!\ 4! = 144$
(iv) Treat the 3 boys as 1 item.
No. of ways of arranging 4 items = 4!
The boys can be arranged among themselves in 3! ways
$\therefore$ no. of arrangements = 4! 3! = 144
(v) $G_1$ $G_2$ $G_3$
↑ ↑ ↑ ↑
The 3 girls can be arranged in 3! ways.
From the 4 spaces, choose 3 places for the boys in $^4C_3$ ways, and arrange them in 3! ways.
$\therefore$ no. of arrangements = $3!\ ^4C_3\ 3! = 144$
**Note:** It is wrong to subtract no. of ways where boys are together from the total no. of ways.

**Arranging Letters, Balls or other Identical Objects**
(a) Find the number of arrangements of all 7 letters of the word "MINIMUM" in which
(i) there are no restrictions.
(ii) the 3 letters M are next to each other
(iii) the 3 letters M are separate

(iv) the first letter is M
(v) the first & last letters are M
(vi) the first letter is M or the last letter is M or both
(b) Find the number of 4–letter code–words that can be made from the letters of the word "MINIMUM".

(ai) No. of arrangements = $\dfrac{7!}{3!\,2!} = 420$

(ii) Treat "MMM" as 1 item.

No. of arrangements of "MMM", I, N, I, U = $\dfrac{5!}{2!} = 60$

(iii) I N I U

↑ ↑ ↑ ↑ ↑

The letters I, N, I, U can be arranged in $\dfrac{4!}{2!}$ ways

From the 5 spaces, choose 3 places for the 3 M's in ${}^5C_3$ ways.

$\therefore$ no. of arrangements = $\dfrac{4!}{2!}\,{}^5C_3 = 120$

(iv) M _ _ _ _ _ _

No. of ways of arranging I, N, I, M, U, M = $\dfrac{6!}{2!\,2!} = 180$

(v) M _ _ _ _ _ M

No. of ways of arranging I, N, I, M, U = $\dfrac{5!}{2!} = 60$

(vi) No. of arrangements = 180 + 180 – 60 = 300

(b) Case 1: M, M, M & 1 other letter: No. of words = ${}^3C_1 \dfrac{4!}{3!} = 12$

Case 2: M, M, I, I: No. of words = $\dfrac{4!}{2!\,2!} = 6$

Case 3: M, M & 2 different letters: No. of words = ${}^3C_2 \dfrac{4!}{2!} = 36$

Case 4: I, I & 2 different letters: No. of words = ${}^3C_2 \dfrac{4!}{2!} = 36$

Case 5: 4 different letters: No. of words = 4! = 24

Total no. of code–words = 12 + 6 + 36 + 36 + 24 = 114

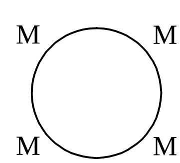
**Arranging People Around a Table**

4 men and 3 women are to sit at a round table. Find the number of ways of arranging them if
(i) there are no restrictions.
(ii) the 3 women must sit together.
(iii) the 3 women must be separate.
(iv) the 3 women must be separate and the seats are numbered 1 to 7.

(i) No. of ways of arranging 7 people around a table = (7 – 1)! = 720

(ii) Treat the 3 women as 1 item.
No. of ways of arranging 5 items around a table = (5 – 1)!
The 3 women can be arranged among themselves in 3! ways
$\therefore$ no. of arrangements = (5 – 1)! 3! = 144

(iii) Let the 4 men sit down first in (4 – 1)! ways.
From the 4 spaces, choose 3 places for the women in ${}^4C_3$ ways.
Arrange the 3 women in 3! ways.
$\therefore$ total no. of arrangements = $(4-1)!\ {}^4C_3\ 3! = 144$




(iv) No. of arrangements = 144 × 7 = 1008