ANSWER KEY

EXERCISE 1		EXERCISE 2		EXERCISE 3		EXERCISE 4	
1	a	1	a	1	b	1	c
2	c	2	b	2	b	2	a
3	b	3	c	3	b	3	b
4	b	4	d	4	d	4	b
5	b	5	a	5	d	5	b
6	d	6	b	6	c	6	b
7	c	7	c	7	a	7	d
8	a	8	b	8	b	8	b
9	c	9	b	9	c	9	d
10	c	10	c	10	d	10	c
11	a	11	d	11	a	11	a
12	d	12	c	12	d	12	d
13	a	13	d	13	c	13	b
14	c	14	b	14	b	14	a
15	a	15	c	15	d	15	c
16	a	16	b	16	a	16	c
17	b	17	c	17	d	17	a
18	b	18	d	18	a	18	b
19	b	19	a	19	b	19	b
20	b	20	c	20	c	20	a
21	c	21	c	21	c	21	d
22	b	22	d	22	d	22	c
23	d	23	d	23	c	23	a
24	c	24	d	24	b	24	b
25	b	25	a	25	d	25	d
26	d	26	d	26	c	26	d
27	c	27	a	27	b	27	c
28	b	28	d	28	c	28	b
29	c	29	c	29	c	29	d
30	c	30	c	30	d	30	b
31	b	31	b	31	d	31	b
32	c	32	a	32	a	32	d
33	b	33	a	33	c	33	b
34	b	34	c	34	d	34	b
35	b	35	a	35	b	35	c
36	b	36	d	36	a	36	b
37	a	37	d	37	d	37	c
38	c	38	b	38	d	38	b
39	d	39	c	39	a	39	d
40	c	40	c	40	a	40	b

EXERCISE 5		EXERCISE 6		EXERCISE 7		EXERCISE 8	
1	a	1	a	1	a	1	c
2	a	2	b	2	b	2	b
3	b	3	a	3	a	3	a
4	c	4	b	4	a	4	c
5	d	5	d	5	d	5	c
6	d	6	d	6	c	6	c
7	c	7	c	7	c	7	a
8	b	8	c	8	d	8	d
9	c	9	c	9	d	9	c
10	a	10	b	10	d	10	b
11	c	11	c	11	d	11	d
12	c	12	a	12	a	12	a
13	c	13	c	13	d	13	a
14	b	14	c	14	a	14	d
15	c	15	b	15	b	15	b
16	d	16	c	16	b	16	c
17	b	17	a	17	a	17	d
18	d	18	c	18	c	18	a
19	c	19	c	19	a	19	a
20	c	20	b	20	c	20	a
21	c	21	b	21	a		
22	b	22	a	22	d		
23	c	23	b	23	d		
24	b	24	d	24	c		
25	a	25	a	25	a		
26	b	26	d	26	c		
27	c	27	b	27	d		
28	d	28	c	28	c		
29	b	29	a	29	b		
30	a	30	c	30	a		
31	c	31	a	31	d		
32	c	32	c	32	c		
33	d	33	b	33	a		
34	a	34	c	34	b		
35	b	35	b	35	b		
36	c	36	d	36	c		
37	a	37	a	37	c		
38	d	38	d	38	b		
39	a	39	c	39	a		
40	c	40	b	40	b		

EXPLANATIONS

EXERCISE - 1

1. Unit's digit of x is 3. The number x can be written as 4n + 1, because upon division by 4, the remainder is 1. Therefore,

$$x^{x^{x^{x^{\cdots\infty}}}} = (\ldots..3)^{(4n+1)^{(4n+1)^{\cdots\infty}}} = (..3)^{4n+1}$$

, because (4n + 1) to the power any number will always be of the form 4n + 1 (using binomial expansion).
Therefore, unit's digit is 3.

2. If the can contains x litres of paint, $[(x - 5)/x]^2 = 49: (49 + 15)$.
$(x - 5)/x = 7: 8$. or, X = 40 litres.

3. Sum of squares of first n natural nos. is $\dfrac{n(n+1)(2n+1)}{6}$
Square of sum of first n natural number is $\dfrac{n^2(n+1)^2}{4}$
Ratio = 17 : 325
Solving we get n = 25

4. A = 1.15 B
B = 1.075 C, C = 92 (given), therefore, Put the values & get B = 98.9, $\therefore$ A = 113.735

5. Work done by 1 man in 1 day $= \dfrac{1}{44}$
Work done by 1 woman in 1 day $= \dfrac{1}{88}$
Work done by 1 boy in 1 day $= \dfrac{1}{132}$

$\Rightarrow$ Work done by 1 man, 1 woman and 1 boy in 1 day $= \dfrac{1}{44} + \dfrac{1}{88} + \dfrac{1}{132} = \dfrac{1}{24}$

$\Rightarrow$ Time required to complete the work = 24 days

6. The problem can easily be solved by alligation. In container 1, the ratio of liquid A to the total liquid is 5/(5 + 1) = 5/6. In container 2, this ratio is 1/(1 + 3) = ¼In the final mixture, this ratio will be 1/(1 + 1) = ½Alligating as shown, we get the required ratio as 3 : 4.

$$\begin{array}{ccc} 5/6 & & 1/4 \\ & 1/2 & \\ 3 & & 4 \end{array}$$

7. 12 has to be completely divisible by the ratio, which is not the case with 3:2

8. Suppose 90 units of work have to be done.

$\Rightarrow$ No. of unit done by A in one day = 10 units
B in one day = 9
C in one day = 6

Work done by B and C in 2 days = (9 + 6) x 2
= 30 units

Remaining work = 90 – 30 = 60 units

$\therefore$ Time taken by A = $\frac{60}{10}$ = 6 days

9. Distance =

A		B		C
100	:	75		
		100	:	96

A	:	C		A	:	C
$100x\frac{100}{75}$	:	96	=	100	:	72

A beats C by 100 – 72 = 28m

10. 20 meters- In the time that A covers 360 m ,B would cover 480 m.

11. (a) Milk in final/Milk in original = $[(a-b)/a]^n$,
where a = Quantity of mixture;
b = amount taken out during each operation
n = No. of times the operation is repeated
Milk in final/Milk in original
= $[(40 - 10)/40]^3 = (3/4)^3$
Milk in final solution = 40 × 27 /64 = 16.875
Water = 23.125
Required ratio = 16.875 : 23.125 = 27:37

12. No such pair exists. L.C.M has to be a multiple of H.C.F.

13. $6a^2 = 2(lb + bh + lh)$
$l : b : h = 1 : 2 : 4$ Therefore, $b = 2l$; $h = 4l$
Hence, $6a^2 = 2[2l^2 + 8l^2 + 4l^2]$
$6a^2 = 28l^2$

$$a = \left(\frac{14}{3}\right)^{\frac{1}{2}} l$$

$$\therefore a^3 = \left(\frac{14}{3}\right)^{\frac{3}{2}} l^3$$

Volume ratio of cube and cuboid $= a^3 : lbh$

$$= \left(\frac{14}{3}\right)^{\frac{3}{2}} : 8$$

14 & 15. Assume the volume in each container as 16 lts
Amount of alcohol at the end of the first process = 12 lts.
Amount of alcohol at the end of the second process = 11 lts.
Volume of the 1st vessel at the end of process 1 = 20 lts.
Volume at the end of process 2 = 21 lts.
14—(c) ,15---(a)

16. Nos. ending with 2,3, 7 and 8 cannot be perfect squares

17. No. divisible by 2 ---50
No. divisible by 5 ---20 [That includes 10 div of 2]
No. divisible by 3 ---33 [That includes 13 div of 2 & 6 div of 5]
Then the total Nos. = 50+10+14 = 74
No. not div by 2, 3 or 5 is 100 – 74 = 26

18. $\left(\frac{3}{5}\right)^{th}$ of time = 6 hrs.
Remaining time & Distance = 4 hrs, 40 km
$\Rightarrow$ Speed = 10 km/hr.

19. A player can play a maximum of 3 games. Hence, he can get a maximum of 3 points.

20. Let D_1 and T_1 and D_2 and T_2 denote the diameters and the thickness of the two coins, $V_1/V_2 = (D_1^2\ T)\ /(D_2^2\ T_2) \Rightarrow (V1/V2) = (D_1/D_2^2)^2\ (T_1/T_2)$. Therefore, $4/1 = (4/3)^2\ (T_1/T_2)$ or $T_1/T_2 = 9/4$.

21. Let the amount invested @ 10% = X

$$\Rightarrow \quad X \times \frac{10}{100} \times 5 = \frac{9}{100} \times (2600 - X)$$

$$\Rightarrow \quad X = 1350$$

22. After rationalization, $x = 6 - \sqrt{35}$ and $y = 6 + \sqrt{35}$. Therefore,

$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$= (12)\{[36 + 35 - 2(6)(\sqrt{35})] + [36 + 35 + 2(6)(\sqrt{35})] - (36 - 35)\}$

$= (12)(71 + 71 - 1) = 1692.$

Alternatively: x is less than 1 and y is only marginally less than 12. Therefore, $x^3 + y^3$ means value slightly less than 12^3 or 1728. The only option slightly less than 1728 is 1692. Further, $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$, and $x + y = 12$, therefore number must be a multiple of 12. Therefore 1269 is not possible.

23. Fifth power of any number ends with the unit's digit of that number (by rule of cyclicity). The rule applies when the power is any power of 5. Therefore, required unit's digit is same as the unit's digit of 1 + 2 + 3 + ... + 99, which is equal to $\frac{99 \times 100}{2}$, which ends with 0.

24. Ratio of white mice to total mice.

$= \frac{1}{1/2} : \frac{1}{1/8} = 1.4 = \frac{1}{4} : 1$

Ratio of gray mice to total mice

$\frac{1}{1/3} : \frac{1}{1/9} = 3:9 = 1:3 = \frac{1}{3} : 1$

$\therefore$ Ratio of white mice to grey mice = $\frac{1}{4} : \frac{1}{3} = 3:4$

25. Rebate = 0. 25 × 32 =8

40/8=5

26. Use options.

27. Use options. Moreover, x must be less than 1.

28. $x^2 - y^2 = 145 \quad \Rightarrow \quad (x - y)\ (x + y) = 145$

$$(x - y) = \frac{145}{29} = 5$$

29. Let the no. of persons insured by 100

$\therefore$ annual premium earned = 250×100

= 25000

Given that 1 out of every 100 will incur the hospitalization bill of Rs.15000

It is also given that insurance cover will be 80% of hospitalization bill subject to an upper limit of Rs.18900

Thus the person incurring a bill of Rs.15000 will get Rs.12000 ($\because \frac{80}{100} \times 15000$) from insurance company.

Also, the insurance to incur an administrative cost of 10% of the revenue

$\frac{10}{100}(25000) = 2500$

$\therefore$ Total cost of the firm = 12000

$$\begin{array}{r} 2500 \\ \hline 14500 \end{array}$$

Hence net revenue of the insurance firm = 25000 – 14500

= 10500

$\therefore$ Profit per person = $\frac{10500}{100} = 105$

30. Given: Instead of 1, 1.6 people incur the hospitalization Bill.

$\therefore$ cut of insurance firm = 12000×1.6

= 19200

Now let the total amount of annual premium be x

$\therefore$ Total cost of the firm = 19200 + 0.1x

Since the firm wants to maintain same profit per person

$\therefore$ x – (19200 + 0.1x) = 10500

0.9x = 29700

x = 33000

$\therefore$ Premium to be charged per person if the firm wants to maintain same level of profit

$= \frac{33000}{100} = 330$

31. Increase = 30 + 20 + (30 x 20)/100

(by A+B+AB concept)

32. $48 \times 48 = 36 \times 64 = 2304$

33. 10%

Rs. 150 x 80% = Rs.120

$\Rightarrow$ Rs. 120 – 108 = 12

$\frac{12}{120}\times 100 = 10\%$

34. Compare $(2^3)^{100}$ and $(3^2)^{100}$

35. (11 + 21) - 4 = 28. Or, $xk_1 + 11$ & $xk_2 + 21$ then $[x(k_1 + k_2) + 32]/x$ leaves remainder 4. $\therefore$ the divisor must be 28.

36. Ratio of work done by A : B : C = $\frac{1}{20}:\frac{1}{25}:\frac{1}{30}$ = 15 : 12 : 10

A – B = $\frac{3}{37}\times 2220 = \text{Rs.}180$

37. Rs.2250

38. 1.1! + 2.2! + 3.3! + ...+ n.n1
=(2-1)1! + (3-1).2! + (4-1).3! +........[(n+1)-1] . n!
= 2! – 1 + 3!-2! + 4! - 3!...............+ (n+1)! - n!
= (n+1)! - 1

39. If a+b+c=0, then $a^3+b^3c^3 = 3abc$

$$\Rightarrow \frac{(x+y+z)^3}{xyz} = \frac{(3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}})^3}{xyz} = 27$$

40. Sum of digits of the numbers at even places = 7 + 6 + B + A = 13 + B + A
Sum of the digit at odd places = B + A + 1 + 1 = B + A + 2
Their difference = 13 + B + A – B – A – 2 = 11
Thus, No. is a multiple of 11.
Ans. (C)

EXERCISE - 2

1. N =

$$\frac{1}{4}\left(\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+......\infty\right)=\frac{1}{4}\left(\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+......\left(\frac{1}{n}-\frac{1}{n+1}\right)\right),$$

when number of terms is assumed to be n.

N = $\frac{1}{4}\left(\frac{1}{1}-\frac{1}{n+1}\right)=\frac{1}{4}\left(\frac{n}{n+1}\right)=\frac{1}{4}\left(\frac{1}{1+(1/n)}\right)$. When n approaches infinity, 1/n approaches 0. Therefore, we get N = $\frac{1}{4}$.

2. Work is done in n + (n – 1) + (n – 2) + (n - 3) + …3 + 2 + 1 = n(n + 1)/2 days. We get n(n + 1)/2 = n(2n/3) days, hence n = 3.

3. checking the options we see that 21 is the factor of both the nos. only in option (c)

4. 12%

5. 1 : 3 : 6

6. Using options.

7. S=1/9 P,
T = R
Thus, S = PxTxR/100
=> 1/9 = RxR/100
=> R = 10/3 = 3. 33

8. In the original solution we have 0.3 litres of salt and 5.7 litres of water then 1 litres of water is evaporated and we are left out with 0.3 litres of salt and 4.7 litres of water. So the percentage of salt in the solution is 6%.

9. 13. Use options.
Let the parts be x and 24 - x.
7x + 5(24-x) = 146
⇨ 2x + 120 = 146
⇨ 2x = 26
⇨ x = 13

10. The given inequality involves a quadratic equation which is greater than zero. Hence the roots are either both positive, or both negative. The inequality can be reduced to (x - 1) (x - 2) > 0. This given (x > 2) as one range

and ($x < 1$) as the other. Thus in-between these two extreme values, i.e. in the range ($1 \le x \le 2$), there is no value of x which satisfies the given inequality.

11. Each lamp burns 12 cubic feet of gas per hour, i.e. 1/5 cubic feet of gas per minute. He takes one minute to go from one lamppost to the other. When he lights the last lamppost, first lamppost will burn for 99 minutes, second for 98 minutes. gas used=1/5(99+98+97+…..+2+1) = 990

12. If IBM initially quoted Rs. 7x lakhs, SGI quoted
4x lakhs. IBM's final quote = (4x –1) lakhs.
$(4x - 1) / 4x = 3/4$
$x = 1$
IBM's bid winning price = Rs. 3 lakhs.

13. The prime factors are 2, 3, 7 and 11.

14. In fact, she takes out 5 % of the existing volume of wine in every operation.
After 1 st operation concentration of wine is 95%
After 2^{nd} operation concentration of wine is $95 \times 0.95 = 90.25\%$
After 3^{rd} operation concentration of wine is $90.25 \times (0.95) = 85.7375\%$
Obviously after the 4^{th} operation the concentration of wine will be less than 85%.

15.

	Ajay	Bunti	Chintu
Step 4	16	16	16
→ Final amounts			
Step 3	16 – 8	16	16 + 8
→ Chintu gives Ajay			
Or	8	16	24
Step 2	8	16 + 12	24 – 12
→ Bunti gives Chintu			
Or	8	28	12
Step 1	8 + 14	28 – 14	12
→ Ajay gives Bunti			
Or	22	14	12
→ Initial amounts			

16 Since 51 = 49 + 2, we have to find the remainder when 2^{138} is divided by 7. Now, $2^{138} = 8^{46} = (7 + 1)^{46}$. Therefore remainder is 1.

17. $10! + 11! + 12! + 13! + ... + 1000! = 10!(1 + 11 + 11\times12 + ...)$
$= 10!(12 + 11\times12 + ...) = 10!\,(12)(1 + 11 + ...)$.
This contain 8 + 2 = 10 number of 2's.

18. $(80 - 2)^2 + (80 - 1)^2 + (80)^2 + (80 + 1)^2 + (80 + 2)^2 = 5 \times 80^2 + 10 = 32010$.

19. Let the Max marks = 100 $\Rightarrow$ Arjun got 40 – 0.1(40) = 36 marks; Bheem got 36 - (1/9) 36 = 32 marks

⇨ Arjun + Bheem = 68 marks

⇨ Marks obtained by Karan = 68 – (7/17) 68 = 40 marks $\Rightarrow$ Karan has just passed.

20. $x^3 - x = x(x^2 - 1) = x\,(x + 1)\,(x - 1)$. x, (x + 1) and (x – 1) being three consecutive numbers, one of them must be divisible by 3. So $x^3 - x$ is divisible by 3. Now as 'x' is odd, let x = 2p + 1.

$\therefore x^3 - x = (2p + 1)\,(2p + 2)\,(2p) = 4p\,(2p + 1)\,(p + 1)$

Now, p and (p + 1) being two consecutive numbers, one of them must be even. So, $x^3 - x$ is divisible by $4 \times 2 = 8$.

Thus, $x^3 - x$ is divisible by $8 \times 3 = 24$

Remainder when $x^3 - x + 1$ is divisible by 24 is 1

21. After first transfer, the amount of wine in 1st = 9

amount of water in 1st = 3

amount of wine in IInd = 3

amount of water in II nd = 1

3 litres drawn from 1st contains $\frac{3}{4} \times 3 = \frac{9}{4}$ litres of wine & $\frac{1}{4} \times 3 = \frac{3}{4}$ litres of water.

3 litres drawn from II nd contains $\frac{9}{4}$ litres of wine & $\frac{3}{4}$ litres of water

$\therefore$ Amount of wine in 1st vessel after 2nd transfer = $9 - \frac{9}{4} + \frac{9}{4} = 9$

Amount of water in 1st vessel after 2nd transfer = $3 - \frac{3}{4} + \frac{3}{4} = 3$

Similarly for II nd vessel, wine $= 3 - \frac{9}{4} + \frac{9}{4} = 3$ water = $3 - \frac{3}{4} + \frac{3}{4} = 3$

$\therefore$ Ratio is same in both the vessels.

22. Plug in the answer choices and verify.

23. $(2.89)^{0.5} = \left(\frac{289}{100}\right)^{1/2} = \left(\frac{17^2}{10^2}\right)^{1/2} = \frac{17}{10} = 1.7$. greatest no. is (d)

24. If they are 50% then the choices should be equal.

25. Let the total volume of each utensil be 28 lt. Thus total milk after mixing = 24+20+21 = 65 lt. Total water = 4+ 8 + 7 = 19 lt. Therefore final ratio = 65 : 19.

26. Multiply the last digits of the nos.

27. If the number of cats is less than 9, and 9 from the total escaped, at least one dog has to escape. Hence , the no. of cats cannot be greater than 9.

28. (d)
2/3 of work was done.
⇨ 1/3 of work is left.
⇨ A does 1/3 work in 20 days.
⇨ A does 1 work in 60 days.

29. (c) Best done from the choices. Choice (a) is wrong because small hand will be at 11 if the hands are interchanged; choice (b) is also wrong because small hand will be at 2 and similarly choice (d) is also wrong because small hand will be at 4 (it should be between 6 and 7 according to the question). Hence the correct answer is (c).

30. 10. 25%
Use a+b+ab/100
a=b=5%

31. let x be total number of student then 72% of x + 44 % of x – 40 = x and we get x = 250

32. Favourable case is C wins. The probability of this is ½x ½x ½= 1/8.
Sample space comprises wins of all the five people. The probability of this ½ + $(1/2)^2 + (1/2)^3 + (1/2)^4 + (1/2)^5$ = 31/32
Hence, the probability that C wins is 1/8/(31/32) = 4/31.

33. The total no. of terms in the expansion of the given binomial is 7 + 1 = 8 (even). Hence, there'll be two mid-terms : $\frac{(7+1)\text{th}}{2}$ term and $\frac{(7+3)\text{th}}{2}$ term
$T_4 = {}^7C_3(x/2)^4(-4/x)^3 = -{}^7C_3 4x$
$T_5 = {}^7C_4(x/2)^3(-4/x)^4 = -{}^7C_3 32x$

34. Distance between two given points is 3 units. Since the area is 6 units it's second side must be 4 units. Hence third vertex can be (1, 2) or (1, - 6) or (-2, 2) or (-2, 6). Ans c.

35. Given points are the vertices of a right angled triangle with right angle at (a, a). It's sides forming the right angle are a and a. $\therefore$ $a^2/2 = 2 \Rightarrow a^2 = 4$ $\therefore$ a = $\pm$ 2.

36. Since sum of any two sides of a triangle is greater than the third side $\Rightarrow$ (r – 1) + r > r+1 $\Rightarrow$ r > 2.

37. Number of days = Field available / 10 = 22x14x14/ (7x100) = 6.16 days = 6 days

38. Out of 10 position select two, no. of ways = ${}^{10}C_2$. In these two positions A1 and A2 can be arranged in one way only. And on remaining, 8 candidates can be arrange in 8! Ways. So total ways = ${}^{10}C_2$. 8!

39. Number of diagonals of a polygon of n sides = ${}^{n}C_2$ – n = n (n-3)/2 = 10 7/2 = 35.

40. Percentage of the candidates who failed in at least one subject = $n(M \cup E)$ = $n(M) + n(E) - n(M \cap E) = 25 + 20 - 10 = 35$ $\therefore$ Percentage of the candidates who passed in both the subjects = 65.
$\therefore$ 65 x / 100 = 2600 $\Rightarrow$ x = 4000

EXERCISE - 3

1. 626! - 625! = 625!(626 - 1) = (625!)(625). The number of zeros in 625! is 125 + 25 + 5 + 1 = 156. But, the number of 5's from 625 (the other factor) also needs

to be counted, which is 4. Therefore total number of zeros is 160.

2. The cost price of the mixture = (100/137. 5)(75) = 600/11. $\therefore$ Fraction of pure spirit in the mixture = (600/11) ÷ 60 = 10/11. $\therefore$ the fraction of water is 1/11. $\therefore$ Ratio 10 : 1.

3. Since the traveler takes $\frac{4}{5}$th of the time he makes his speed 5/4 of original speed. $\Rightarrow$ He increases his speed by 1/4.
$\Rightarrow$ 1/4 = 1/2 km/hr.
$\Rightarrow$ speed = 2km/hr.
In second case he goes 1/2 km/hr slower.
$\Rightarrow$ 3/4 of original speed
$\Rightarrow$ he takes 4/3 of time.
$\Rightarrow$ 1/3 of time = 5/2 hrs.
$\Rightarrow$ time = 15/2hrs
distance = 2 x 15/2= 15 km

4. L.C.M of 6,9,15 and 18 and 7 is 360.
and 360 + 4 = 364

5. A, B and C are to share profit in the ratio 4 : 3 : 7 But, since A is to receive 10% of profit, only 90% is to be shared)
A's total share = 10% of profit + 4/14 × 90% of profit = 6000
or, (10 + 180/7)% of Profit = 6000
or, 250/7% P = 6000 $\Rightarrow$ P = Rs. 16,800
B and C's share combined
= 10/14 of the 90% profit= $\frac{10}{14} \times \frac{90}{100} \times 16800$

6. The clock would take 66 seconds to strike 12. Between the 1st and the 6th striking of the clock there are 5 intervals of time, each interval being 6 seconds long. Hence for 11 intervals between 11 and 12 it takes 66 seconds.

7. The integral part is 1011 whose octal is obtained as $(1011)_2 = (13)_8$ after forming groups of three digits from right side. Similarly, the fractional part is 1111 which are grouped into groups of 3 starting from left side (contrary to what is done for integral part). Therefore fractional number in base 8 is 74. Therefore, answer is 13.74.

8. Let CP = 100. $\Rightarrow$ MP = 120 Profit = 8
$\Rightarrow$ SP = 108
Hence Discount = (120-108)/120 × 100=10%

9. If Harish delivers 100 glasses safely, he will get 300 paise
Let the no. of glasses he broke = x
Acc. to the Question,
$300 - 9x - 3x = 240$
$\Rightarrow x = 5$

10. Acc. to the Question.

A	**B**	**C**
6000	8000	10000
15%	10%	

remaining 75% divided in proportion.
Let net profits = X
A's share = $1080 = 0.15X + 0.75X \times 6 / 24$
Or X = Rs 3200
$\therefore$ B's share = $0.1X + 0.75X \times 8 / 24$ = Rs 1120
$\therefore$ C's share = $3200 - 1080 - 1120$ = Rs 1000

11. The given number N can be expressed as $N = 899n + 63$ for some integer n. $899 = 29 \times 31$. $63 = 29 \times 2 + 5$. Then $N = 29(31n) + 29 \times 2 + 5$, i.e. the remainder left when divided by 29 is 5.

12. First clock: 33/11 = 3 sec. Second clock: 22/11 = 2 sec. Sixth of the first: 15 sec. Eighth of the second: 14 sec. Difference = 1.

13. Three wheels can complete respectively 60, 36, 24 revolutions per minute. In other words these three wheels will take 1 second, 60/36 i.e. 5/3 seconds and 60/24 i.e. 5/2 seconds respectively to complete one revolution. LCM of 1, 5/3, 5/2 is {LCM of 1, 5, 5}/{HCF 1, 3, 2 } = 5. Thus after every 5 seconds the red spots on all the three wheels touch the ground. Hence they will touch the ground again after 5 seconds.

14. Number of questions in the test taken by team A = 270
Let team A answer x number of questions for y hours
Then $xy = 270$. Team B answers $x + 7$ questions for $y - 3$ hours
Hence, $(x + 7)(y - 3) = 300$
$\therefore x = 18$ and $y = 15$ or, team A answers 18 questions per hour.

15. Check by the options.

16. Let us assume that out of 100, he obtained 72 marks, if he had attempted 4 more questions, he would have made one more mistake. Hence 3 correct answers secures him 12 marks more. This means
each question carries 12/3 = 4 marks and hence number of question is 25.

17. After discount of 40%, amt = 300.
After discount of 36%, amt = 320
After discount of 4%, amt = 307.2.
Hence diff = 7.20

18 Average daily wages = $\dfrac{5x + 3y + y + 2y + y}{5} = x + \left(\dfrac{7}{5}\right)y$

19. Since B is at the units place B can be one of 0, 1, 4, 5, 6, 9. If B = 0 then BE will be only one digit number, then $(BE)^2$ cannot be a three digit number. Also if B is other than 1, then $(BE)^2$ will have more than three digits which is not in accordance with the given problem. Hence B must be equal to 1. Since $(BE)^2$ = MPB and B = 1, then E can be 1 or 9 only. But since 1 is already assigned to B, E must be equal to 9. Then $19^2 = 361$ gives M is equal to 3.

20. The maximum and the minimum five digit number that can be formed using only 0,1,2,3,4 exactly once are 43210 and 10234 respectively. The difference between them is 43210 - 10234 = 32976.

21. X! + (X + 1)! is evidently an even integer if $X \geq 2$.
But $X^3 + X + 5$ = even + even + odd = odd.

22. (12 + 14 + 10) = 36. Hence 100 - 36 = 64

23. 54 - 24 = 30. [100 - (54 + 10 + 12)]

24. A can collect $2 \times 10 + 2 = 22$.

25. D collected 10 (from soln no. 24)

26. Hiring cost per day is Rs.1000. In order to minimize the cost, the truck should be only on the day when the storing cost exceeds Rs. 1000. At the end of the first day, the hired storing cost will be $150 \times 5 = 750$. At the end of the second day the storing cost for the day will be $120 \times 5 = 600$. At the end of the fourth day, the storing cost for that day will be $(120 + 250) \times 5 = 1850$. Since on the fourth day the storing cost is exceeding Rs.1000, the truck must be hired. Further at the end of the fifth day, the storing cost for that day will be $160 \times 5 = 800$. At the end of the sixth day, the storing cost for that day will be $(160 + 120) \times 5 = 1400$. Since on the sixth day the storing cost is exceeding Rs. 1000, the truck must be hired. Then in the end the truck must be hired on the seventh day to complete the job.
But the cost can be reduced further if the truck is hired on the fifth day instead of the sixth day. Storing cost on the fifth day is $160 \times 5 = 800$, but if the truck is hired on the fifth day, the storing cost for the sixth day will be $120 \times 5 = 600$.

27. Hiring cost per day is Rs.1000 while the storage cost is Rs. 0. 8 per cubic feet. In order to minimize the cost, the truck should be hired only on the day when the storing cost exceeds Rs. 1000.
At the end of the first day the storage cost will be $150 \times 0.8 = 120$. At the end of the second day the storage cost for that day will be $(150 + 120) \times 0.8 = 216$. At the end of the third day the storage cost for that day will be $(150 + 120 + 180) \times 0.8 = 360$. At the end of the fourth day the storage cost for that day will be $(150 + 120 + 180 + 250) \times 0.8 = 560$. At the end of the fifth day the storage cost for that day will be $(150 + 120 + 180 + 250 + 160) \times 0.8 = 688$. At the end of the sixth day the storage cost for that day will be $(150 + 120 + 180 + 250 + 160 + 120) \times 0.8 = 784$. Thus there is no point in hiring the truck on any other day than on the seventh to send all the production units to the market.

28. Note that odd number when multiplied with the odd number gives the odd number. Hence if n is odd then n^2 and n^3 both will be odd. Since n^3 is odd, n must be odd and n^2 is odd.

29. From the figure it is clear that $\Delta BCF \sim \Delta EDF \Rightarrow BC/.ED = CF/DF$. Since side of the square is 20 and CF = 5 $\therefore$ DF = 15 $\therefore$ $6/DE = 5/15 \Rightarrow DE = 18$.

30. By the properties of the numbers the difference between any number and a number obtained by interchanging it's digits is always divisible by 9.

31. The last digit in the product can be one of the required digits if and only if the last digit of each of the numbers is one of the numbers 1, 3, 7, or 9. $\therefore$ Probability of choosing each of the four numbers is 4/10. As last digit of required numbers can be chosen in four different ways and total number of ways of choosing a digit at units place is 10. $\therefore$ Required probability = 2/5. 2/5. 2/5. 2/5 = $(2/5)^4$ = 16 / 625.

32. Only one day per 20 days produces the rainbow. $\therefore$ out of 20 days 19 days can not produce the rainbow. Hence their percentage = 100x(19/20) = 95%.

33. Since the face cards are removed there will be 10 cards of each suit. Total number of ways of drawing four cards = ${}^{40}C_4$. Favourable number of cases will be 10 x 9 x 8 x 7. Hence the required probability is 10 x 9 x 8x7/ ${}^{40}C_4$

34. Possible arrangements are
1 1 3 $\Rightarrow {}^5C_1\,{}^4C_1\,{}^3C_3\,3 = 60$
2 2 1 $\Rightarrow {}^5C_2\,{}^3C_2\,{}^1C_1\,3 = 90$
So total is 150 Ways

35. Draw ZX paralle to AY

$\therefore \Delta CZX - \Delta CAY \Rightarrow CZ/CA = ZX/AY$
$\therefore 2/5 = 4/AY \Rightarrow AY = 10 \therefore B = (0, 10)$.

36. The total volume of the cones will be given by : $1/3(\pi r^2 h_1 + \pi r^2 h_2) = 1/3\pi r (h_1 + h_2)$. Where h_1 and h_2 will be the respective heights which is nothing but the distance between the two vertices of the cones joined base to base which is given to be 12. Thus the volume will be $1/3(\pi x 7^2 x 12) = 616 cm^3$. Thus

37. The perpendicular distance of point (3, 5) from the given line is the magnitude of
$| (4x3 - 3x5) + 1 | / \sqrt{(4^2 + 3^2)} = 2/5 = 0.4$

38. The volume of the larger cylinder will be $1/3 \times \pi\ 3^2 \times h = 3 \times 1/3 \times \pi.3^2.7$
Thus h = 21

39. Let the radii be 2x and 3x. Then the ratio volumes is $4/3 \times \pi \times (2x)^3 : 4/3 \times \pi \times (3x)^3 = 8:27$

40. In the letters D, R, A,U, G, H, T vowels are A and U which can be put together in 2! ways and these two vowels together with the other five letters can be arranged in 6! Ways. $\therefore$ Required number of arrangements = 2! 6! = 1440 different ways.

EXERCISE - 4

1 If, the SP is Rs.80, and the loss is 20%, then the CP has to be Rs.100.

2 Let s,a and d be the speeds at which Shyam,Arvind and Deepak can push the cart respectively. So we have s+a = 5 and a+d = 6, a = 3. Thus s+a+d = 8. Thus time taken = 1 hr 15 minutes.

3 In the first 6 months, the ratio of distribution of profit will be 1:4:3. For the rest of the year, it will be 3:4:3. Thus X gets Rs.4 for every 18 Rs of profit. Since he gets Rs.800, the profit must be Rs.3600.

4 The expression can be written as $12^3 - 3 \times 5 \times 7 \times (5 + 7) = 5^3 + 7^3 = 468$.

5 The amount obtained by Kapil after selling the shares is Rs.27,400 + Rs.2400 = Rs.29,800. Let number of shares be x, then brokerage paid by Kapil = x/2 everytime he buys or sells the shares. Thus he pays a total brokerage of Rs.x. Thus 13x-x = 2400. Therefore x = 200. Thus the market price of each share is 27,400/200 = 137

6 Since C and D have different ages which are again different from that of A and B, and the difference between adjoining brothers is constant, we must have that the twins A and B should stand on either side of C or D, then the remaining one person will stand at an extreme end and one of the twins will stand at the other extreme end. Since A does not stand at the extreme, B must stand at one of the extremes.

7 Using alligation we get that the ratio in which the two types of sugars should be mixed is 3:1 i.e. 72:24. Thus 72 kgs should be mixed.

8 Let the speeds be 2x and 3x. Thus relative speed is 5x. Since time taken is 10 seconds, the distance travelled i.e. the sum of lengths of both train is 50x. Thus length of each train is 25x. Now while passing the stationary train, the total distance travelled would be 500+25x in time 25sec i.e. (500+25x)/3x = 25. Simplifying we get x = 10. Thus length of either train is 25x i.e. 250 meters.

9 Using the formula we get that there is a loss in the overall transaction of $(15)^2/100$ % = 2.25%.

10. The profit is zero implies CP is same as SP. Thus if initial SP was Rs.5, now it is 4 = CP. Thus initial profit is Re 1 for Rs.4 of CP i.e. 25%.

11. Let the work of 5x12x3 units. Half of that is 5x6x30.
Thus number of days taken = 5 x 6 x 30/(15 x ½x 12) = 5 Ans.a.

12. By formula $\theta = 6 (40 - (11 \times 24) / 12) = 108^0$

13.

John

Tough	Simple	Total
19	6	25
Mutiply by 2		
38	12	50

Raymond

Tough	Simple	Total
47	3	50

So, % of tough questions in final paper is (47+38) = 85

14. Suppose the income was 30 then 10 came from this salary, $1/5^{th}$ of the remaining 20 i.e. 4 came from working at week-ends. ½of the remaining i.e. ½ of 16 i.e. 8 came from the royalty of his book. The remaining 8 from the investments which is thus twice of the pay of week-end work. He earns 1200 from weekend-work thus he earns 2400 from investments.

15. The relative speed when they are moving in same direction is 1 on the ratio scale and when they are moving in opposite directions, it is 5. For constant distance, the speed is inversely proportional to time, therefore the required ratio is 5:1. Thus answer is c.

16. Let t min past 3, the minute hand be 4 min behind the hour hand. Using the formula t = (12/11) (m + x) or (12/11) (m – x), substituting m = 15 m.s. and x = 4 m.s., we get t = (12/11) (15 – 4) = 12 min. Ans. is c.

17. Ratio of the passengers is 1/21 : 1/7 : 1/3 = 1: 3 : 7
11a = 385 $\Rightarrow$ a = 35, $\therefore$ the number of I, II and III class passengers is 35 : 105 : 245. Ans a.

18. Let x and y be the number of kgs of tea of rates Rs.24 and Rs.18 per kg respectively.
$\therefore$24x+18y = 0.99 (x + y) 20, $\therefore$ 24 x/y + 18 = 19.8 (x/y + 1) $\Rightarrow$ x/y = 3/7.

19. Let the speed of the man in still water be m km/hr. and the speed of the current be s km/hr.
¾= (m-s) 45/4 and ¾= (m+s) 15/2
15m – 15 s = 1 and 10m + 10s = 1 $\Rightarrow$ m = 5 km/hr.

20. Let he invests Rs x in 5% stock and Rs y in 4% stocks.
On Rs.132 he earns Rs.5. $\therefore$ On Rs.x he can earn Rs.5x/132
Similarly, on Rs.y he can earn Rs.4y/99
5x/132 = 4y/99 $\Rightarrow$ x /16 = y / 15 = (x + y) / 31 = 6200 /31
x = Rs.3200 and y = Rs.3000

21. Given that mn = 100. $\therefore$ m and n can be any of the following pairs 25, 4 or 20, 5 or 50, 2 or 100, 1 whose sum is 29 or 25 or 52 or 101 but there are no factors of 100 whose sum can be 50.

22. Without leakage it can fill 1/3 of the tank per hour and with leakage it can fill 2/7 of the tank per hour ∴ leakage per hour = (1/3 – 2/7) = 1/21. ∴ Time taken to drain all the water = 21 hours

23. Ram lost a bicycle worth Rs.300 + Rs.50 given in cash to the tourist ∴ in all, he lost Rs.350.

24. In four weeks his earning by over time = 432 – 160 x 2.4 = 48.
Number of hours which he worked over time = 48/3.2 = 15 ∴ Total hours he worked = 175.

25. $\log_{10} e + \log_e 10$ = log e/ log 10 + log 10 / log e. If log e/log 10 = x, then the given expression can be written as $|x + 1/x| = (\sqrt{x} - 1/\sqrt{x})^2 + 2 \geq 0 + 2 = 2$. Hence the value of the given expression is always greater than 2.

26. $2^{3015} = 2^3.2^{3012} = 8(2^4)^{753} = 8(17 - 1)^{753} = 8\,[17\,k - 1]$
9 will be the remainder.

27. Profit drops by 5% which is equivalent to Rs.5. Thus 100% corresponds to Rs.100 or the CP is Rs.100.

28. Let the work be 9x24x8 = 1728. Let the number of days required by 16 men be x . Then we have 16xx12 = 1728. Thus x = 9.

29. Suppose Rahul sells the berries at Rs.30 per kg then Sunil sells it as Rs.20 per kg. Since Rahul makes a profit of 20%, the CP of the berries will be Rs.25 per kg. Thus Sunil has a loss of Rs.5 for every Rs.25, thus the loss percentage is 20%.

30. 177776 can be written as 2 * 88888. Hence $5^{177776} = 25^{88888}$. Since the power is same in both the cases. Expression with the greater base will be greater. Ans.b.

31. The relative speed is 4 kmph. Thus the dog will overtake the man by 4 km in one hour. Thus he can overtake 300 m in 4.5 minutes. Ans.b.

32. The two prices are 5 & 11. Mean price is 13. You cannot mix the two quantities such as to make mean price 13, as the main price does not lie between the prices. Ans.d.

33. $a\,(1 + 20/100)^T > 2a$ ∴ $(6/5)^T > 2$, which holds if T = 4 years.

34. Sequence of numbers which are divisible by 8 and lies between 900 and 1700 is {904, 912, .., 1696) which is an A.P. with first term 904, c.d. = 8 and the last

term 1696. If there are n numbers between 900 and 1700 which are divisible by 8,then 1696 = 904 + (n – 1) 8 $\Rightarrow$ n – 1 = 99 $\therefore$ n = 100.

35. Let the person rides the bicycle for x km. $\therefore$8.5 = x/12 + (72 – x)/ 4.5 $\Rightarrow$ x = 54 kms.

36. Let a be the volume of water in one litre of the mixture. $\therefore$ a1000 + (1- a) 1350 =1250 $\Rightarrow$ 10 = 35 a $\Rightarrow$ a = 2/7. $\therefore$ b = 5/7.

37. To strike 7, clock pendulum has to perform six oscillations for which it takes 7 seconds. To strike 10, pendulum has to perform 9 oscillations. For that it will take 9.(7/6) seconds. = 10.5 sec.

38. P9-P8 = 999..9 (9 times)
= 9 + 90 + 900 + ….+ 900000000
= 9 (1 + 10 + ……+ 10^8) = 10^9 – 1

39. Average of p+1 numbers = (pa + x) / (p+1) = b $\Rightarrow$ x = b (p+1) – pa = p(b-a) + b.

40. Total no. of letters = 4 + 4^2 + 4^3 + ….+ 4^8 = 4 (4^8 – 1)/(4-1) = 87380
Total money spend = (25/100)x 87380 = Rs.21845

EXERCISE - 5

1. Let he invests Rs.x and Rs.y respectively at 3% and 4% stocks. $\therefore$ x + y = 2400 and
3x/75 + 4y/96 = 97.5 $\Rightarrow$ 24 x + 25y = 97.5 x 24 x 25. Solving these two equations we get x = Rs.1500 and y = Rs.900.

2. In one hour it will travel 41 km without stoppage and with stoppages it covers 27 kms. $\therefore$ It stops for the time in which it will travel a distance of 14 km with speed 41 km/hr.
$\therefore$ time of stoppage = (14/41)x60 min = 20.5 min.

3 . Let the pipe A be turned off in t minutes.
$\therefore$ (1/24) t + 16 (1/32) = 1 $\Rightarrow$ t/24 = ½$\Rightarrow$ t = 12. $\therefore$ The first pipe A is to be turned off after 12 minutes.

4. Let P = k-1, Q = k and R = k + 1. $\therefore$ P+Q+R = 3k, which is even or odd depending on whether k is even or odd. Now P+2Q+R = k-1 + 2k + k + 1 = 4k, which is always even.

5. Since the distance is same and with different speeds he can save 12 min = 1/5 hrs.
$\therefore$ 2.5 t = 3.5 (t – 1/5) $\Rightarrow$ t = 7/10 $\therefore$ d = 2.5x(7/10) = 7/4 km = 1 ¾km.

6. Let the initial amount of sugar be x kgs and rate per kg be 5r. Thus 5rx=50 and 6r(x-2) = 50. Thus solving we get x = 12.

7. CP is 50 and loss is 10% i.e. 5. Thus the SP is 50-5 = 45. Ans.(c)

8. In one day the minute hand and the hour hand are at right angles 44 times. Therefore in a week they will be 44 x 7 = 308 times at right angle position. Ans.b.

9. Let l and w be the amounts of liquid and water in the mixture.
$\therefore$(1.25) (15) l = 12.5 (l + w) $\Rightarrow$ 3 (l) = 2 (l + w) $\therefore$ w/ l = 3/2 – 1 = ½

10. A, B and C's work per day is 1/90, 1/40 and 1/12 respectively. Work done per cycle is
1/90 + 1/40 + 1/12 = 43/360 and to complete the work A, B and C each have to work for 9 days.(app) In 9 days A does 1/10th and B does 9/40th of the work. $\therefore$ A's share = 1/10.240 = Rs.24.
B's share = 9/40.240 = Rs.54. $\therefore$ C's share = Rs.162

11. Let the cost price of each article be Rs. x
$\therefore$ (150 x + 50) x 1.38 = 12.5 x 90 + 10 x 60 = 1725.
$\therefore$ 150 x + 50 = 1250 $\Rightarrow$ x = Rs.8.

12. Length of the largest hurdle = HCF of 15547, 17647, 3521 which is 7.

13. The digits which shows some number when turned upside down are 0, 1, 6, 9 and 8. Of the choices 169 and 196 are the perfect squares containing these

digits but when turned upside down these shows 691 and 961 of which 961 = 31^2 is the only perfect square.

14. Amount of water left in container P = p – q/3 – r/2 = (6p – 2q – 3r) / 6

15. From the given data we get a+b = 29 …(I) b + c = 29..(II) c+d = 42..(III) and d + e = 37..(IV) From (I) and (II) a = c and From III & IV c – e = 5, adding I to IV we get a + e + 2 (b+c+d) = 137
$\therefore$a+e = 35 and a-e = 5 $\Rightarrow$ a = 20, b = 9, c = 20, d = 22 and e = 15.
$\Rightarrow$ Total Time = 20 + 9/3 + 20/5 + 22 +15/5 = 52 min

16. Let the number of students be x, therefore total age = 8x. Average of class including the teacher = (8x + 28)/(x+1) = 8.5. Thus solving we get, x = 39. Ans.d.

17. Bozo invests 20,000 in real estate, which becomes 21,000 after an increase of 5%, and he invests 5,000 in bullion, which becomes 6,000 after an increase of 20%. He has 40% of this amount after the stock crash, i.e., 2/5 of 27,000 = 10800. Ans.b.

18. Since the work done is same, $M_1T_1R_1 = M_2T_2R_2$. Thus 24x5x2 = 5x6xd where d is the number of days required. Solving we get d = 8. Ans.d.

19. When Runman runs 1000 m, Bhagatram runs 800 m. When Bhagatram runs 800 m, Padtabhau runs 600 m. So, when Runman runs 1000 m Padtabhau runs 600 m, thus he beats him by 400 m. Ans.c.

20. Alligating we get that the ratio in which acid and water should be mixed is 10:8 i.e. the amount of acid is 270 = 10x27. Thus amount of water should be 8 x 27 = 216 ml. Ans.c.

21. The interests in the first year are the same for both CI and SI. The difference in the second year comes due to the excess interest on the first year's interest that is being charged in the CI deposit. Which means that 5% of 5% of X = 200; or, X = 200x20x20 = 80000. Answer is c.
ALTERNATIVE :
Diff = 200
$P(1+5/100)^2 - (P + (Px5x2/100)) = 200$
P = 80,000

22. Let the distance between two towns is x km and it takes t hours with speed 40 km/hr. $\therefore$ with speed 30 km/hr it will take (t + 6/60) hrs. $\therefore$ x = 401 = 30 (t + 1/10) $\Rightarrow$ t = 3/10 and x = 12 kms.

23. If x + y + z = 0 $\Rightarrow$ x + y = -z $\therefore$ (x + y) / - z =-1. $\therefore$ If (x+y)/z = 1 then x + y + z is not equal to zero.

24. Resultant velocity = 12 km/hr = 10/3 m/sec., and total distance = (60+84) = 144 m. $\therefore$ total time required to fully cross it = 144/(10/3) = 43.2 sec.

25. LCM of 7, 9 and 11 is 693. $\therefore$ In 693 the ratio of spirit to water in each glass is 297 : 396, 308 : 385 and 315 : 378. $\therefore$ Ratio of spirit to water in the mixture is (297 + 308 + 315) : (396 + 385 + 378) i.e. 920 : 1159.

26. Since c is co-prime to a, there will not be any prime factor common in a and c and as c divides ab it must divide b. i.e. c is a factor of b.

27. Let manufacturing cost of the item be Rs.x. $\therefore$ (1.1) (1.2) (1.25) x = 41.25. $\therefore$ x = Rs.25.

28. Required number of plants = 60 k + 1, which is divisible by 7, and smallest of such numbers is 301.

29. Number of boys = $(1089)^{1/2}$ = 33

30. $a/b = c / d = y / x \Rightarrow y / a = x / b$

31. $a < b < 0 \Rightarrow -a > -b > 0 \Rightarrow (-a) / (-b) > 1 \therefore a/b>1.$

32. Let weight of the body as obtained by the fourth experiment be x.
$\therefore$ 53.735 = [(3) (54.005) + x + x – 0.004 + 2 (53.995)] / 7 $\Rightarrow$ x = 53.072.

33. Let b, n and p respectively be the number of books, note books and pencils.
$\therefore b/n = n/p \therefore n^2 = bp = 100. \therefore n = 10.$

34. $m+n = 0 \Rightarrow m = -n. \therefore 1/m + 1/n = 1/m - 1/m$ will be equal to zero if m is well defined non zero number, but $1/m - 1/n \neq 0 \Rightarrow m = 0. \therefore n = 0$, hence m = n.

35. Let the original price be Rs.10x per kg. After reduction by 10%, the price will now be 9x. So that Shyam can buy 1800/9x = 200/x kg rice. Thus 1800/10x + 10 = 200/x. Thus solving we get x = 2. Thus original price is Rs.20 per kg and one can buy 90 kg rice in Rs.1800. Now if the price increases by 12.5% i.e. becomes Rs.22.5 per kg, one can buy 1800/22.5 = 80 kg. Thus Shyam buys 10kg less. Ans.b.

36. Let the sums be x and 2x. Then interest earned will be (xx5x2)/100 + (2xx8x2)/100 = 420. Thus solving we get x = 1000. But total sum is 3x i.e.Rs.3000.Ans.b.

37. In 2 litre mixture, 0.2 litre water is present. Let the amount of water added be x litres. Now the total volume of mixture is 2+x of which water is 20%. Thus we have 20(2+x)/100 = x + 0.200. Solving we get x = 0.25 litres. Thus Ans.a.

38. We have to find the minimum number which when divided by 14, 11 and 12 gives the same remainder 5. This is given by N = LCM (14, 11, 12) x n + 5. For n = 1, we will get the minimum number. Thus N = 929. Then total cows that are given to Brahmins are 929-5 = 924. Thus the no of cows each Brahmin gets is 924/11 = 84. Ans.d.

39. The speed ratios are : X:Y=1:2, and X:Z=3:4. This gives the speed ratio Y:Z as 3:2. Therefore, the time ratio would be the inverse; Y:Z=2:3. Now, if Y takes 10 hrs, Z will take 3/2 times that much i.e., 15 hrs. Answer is a.

40. Since he uses equal amounts of two kinds of petrol his cost per gallon = 35 ps. Hence in Rs.350 he will get 10 gallons of petrol. $\therefore$ total distance traveled = 15 x 10 = 150 miles.

EXERCISE - 6

1. (A) $\frac{x}{a} = b - c$

$\frac{y}{b} = c - a$

$\frac{z}{c} = a - b$

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$$

Let $\frac{x}{a}$, $\frac{y}{b}$ and $\frac{z}{c}$ be A, B and C

A + B + C = 0

$A^3+B^3+C^3 = 3ABC$,

2. Only x = 1 and y = 1 therefore, the

3.. Given equation can be written as $-3 < (11 - x) / 2 < 3 \Rightarrow -6 < 11 - x < 6$ i.e. $-17 < -x < -5$ i.e. $5 < x < 17$.

4. Let the length of rectangle be X m and breadth of the rectangle = Y m. Area = XY sq. m.

ICase:

Length = (X + 9) m and breadth = (Y – 5) m

Area = (X + 9) (Y – 5) sq. m.$\Rightarrow$ (X + 9) (Y – 5) = XY

Or -5X + 9Y – 45 = 0 ……….(1)

IICase:

Length = (X – 7)m and breadth = (Y + 5)m $\Rightarrow$ (X – 7) (Y + 5) = XY

Or 5X – 7Y –35 = 0 ……….(2)

Hence length = 63 m and breadth = 40 m. Ans. B)

Note: Assume L = 63 and B = 40 as an option and try checking the conditions

given in the problem. You will see that working backward is exceptionally fast in such cases.

5. Given that $|x|^2 + 5|x| + 4 = 0$

Case I: if $x \geq 0$ then $|x| = x$. $\therefore$ $x^2 + 5x + 4 = 0$

$(x + 1)(x + 4) = 0 \Rightarrow x = -1, -4$ (< 0) Thus there is no solution of the equation when $x \geq 0$

Case II: If x < 0, then $|x| = -x$. $\therefore$ the equation is $x^2 - 5x + 4 = 0 \Rightarrow (x - 1)(x - 4) = 0 \Rightarrow x = 1, 4$ (> 0). Thus there is no solution of the equation when $x \leq 0$.

6. $|x| = y + 5$

When x < 0, -x = y +5 or x + y + 5 = 0

When x > 0, x = y + 5 or x – y – 5 = 0

7. (C) Put the value of a = 1, b = 0.

8. p^2 is odd, (q-r) is even and r is odd

$\therefore$ $p^2 + (q - r) r$ = odd + even = odd number

9. a, b and c are in GP

$\Rightarrow b^2 = ac$, taking log on both sides

$\log_n b^2 = \log_n ac$

$2\log_n b = \log_n a + \log_n c$ ie

$$\frac{2}{\log_b n} = \frac{1}{\log_a n} + \frac{1}{\log_c n}$$

This shows that

$\log_a n$, $\log_b n$ & $\log_c n$ are in HP .

10. $x = \dfrac{\sqrt{3}+1}{2} \Rightarrow (2x - 1)^2 = 3$ or $4x^2 - 4x - 2 = 0$ or $2x^2 - 2x - 1 = 0$.

Now, $4x^3 + 2x^2 - 8x + 7 = (2x^2 - 2x - 1)(2x + 3) + 10$. Since $2x^2 - 2x - 1$ is zero, the given expression $4x^3 + 2x^2 - 8x + 7 = 10$ when $x = \dfrac{\sqrt{3}+1}{2}$.

11. multiply equation (1) with a and subtract from (2)

$x = 0$ and $y = 2$

12. roots of equation are real. Use Discriminant.

13. c $2x + 5y = 12$…(1) and $(x + y) = 3$….(2). Solving the two equations, $2x + 5y = 12$ and $2x + 2y = 6$, we get $3y = 6$; $y = 2$ and substituting in (2) we get $x = 1$. Hence the solution is (1, 2).

14. Multiply the given equation with 4.

15. (d) $1/x = 4 - 1/3 - ½= (24 - 2 - 3)/6 = 19/6$. Hence $x = 6/19$

16. (b) First of all arrange the given equation

$$\frac{x^3 - abc + (ab - bc - ca)x + (a + b - c)x^2}{(a - c)x + x^2 - ac}$$

$$\frac{(x + a)(x + b)(x - c)}{(x + a)(x - c)} = (x + b)$$

Alternate: Put $x = 2$, $a = 3$, $b = 4$, $c = 5$ and check with the options

17. (a) The equation is $R\,[R^2 + 3R+3]$

Add and subtract $1 \Rightarrow R^3 + 1 + 3R^2+ 3R - 1 \Rightarrow (R+1)^3 -1$

Now put $R = 99$ i.e.. $(99+1)^3 -1 = (100)^3 - 1 = 999999$

18. (b) $3^x - 3^{x-1} = 486$ or $3^{x-1+1} - 3^{x-1} = 486$
$3^{x-1}[3-1] = 486$ or $3^{x-1} \times 2 = 486$ or $3^{x-1} = 243 = 3^5$ or $x = 6$

19. Use options

20. $x^2 + 4xy + 4y^2 = (x + 2y)^2$
This takes a minimum value when $x + 2y = 0$ or $x = -2y$

21. 3^{rd} term = a + 2d, 9^{th} term = a +8d
$a + 2d = 10$, $a + 8d = 20 \Rightarrow 2a + 10d = 30$
sum of terms $= \frac{11}{2} (2a + 10d) = \frac{11}{2} (30)$
=165

22. Both equations are the same.

23. $* 5 = 5^2 - 2 = 23$; $* 23 = 23^2 - 2 = 529 - 2 = 527$

24. d Since $\sqrt{8 \times 2} = \sqrt{16} = \pm 4$ which shows that **a** and **b** need not be perfect squares but **c** can be a perfect square. In the given eg., c = - 4 , hence c need not be greater than zero.

25. $\therefore$ $dP / dx = 3x^2 - 12x + 12 = 0$ or $x^2 - 4x + 4 = 0$ or $x = 2$
$\therefore$ The firm should produce 2 units.

26. $x^2 + (k+1)x + 8 = 0$
If the roots are imaginary, then
$(k + 1)^2 - 4 \times 8 < 0$ or $(k+1)^2 - 4(\sqrt{2})^2 < 0$
$\therefore$ The value of k lies between
$-(4\sqrt{2} + 1)$ and $(4\sqrt{2} - 1)$ excluding the extremes.

27. F = age of father. Then $1/3(F + 19) = 1/5(F + 57)$ or $2F = 76$; $F = 38$.

28. c Rohit's age after 10 years = 8 + 3 + 10 = 21, Tabu's father's age after 10 years = 42 + 12 = 54 years
$\Rightarrow$ His present age = 42 years $\Rightarrow$ Tabu's age = 14 years

29. least value = 1 + 1 = 2

30. Using laws of indices, the answer is 32

31. $\text{Log}_5 64 = x$ i.e. $5^x = 64 = 8^{2\cdot}$. Thus $5^{x/2} = 8$. Therefore, $\log_5 8 = x/2$.

32. Let cost of an apple and an orange be a and b respectively and x and y be the number of apples and the oranges respectively. $\therefore$ ax + by = 17 …(I) and bx + ay = 15…(II). Adding (I) & (II) we get (a+b) (x + y) = 32 and x + y = 40 $\Rightarrow$ a + b = Rs.0.80 = 80 paise.

33. Sol. $\text{Log}_2 x^2 + \log_x 2 = 3$
or $2/\log_x 2 + \log_x 2 = 3$
or $(\log_x 2)^2 - 3(\log_x 2) + 2 = 0$
Put $y = \log_x 2$
$\therefore y^2 - 3y + 2 = 0$
or y = 1, 2
For y = 1, $\log_x 2 = 1$ or x = 2
For y =2, $\log_x 2 = 2$ or $x = \pm\sqrt{2}$.
Since the value of base can't be negative, then there are only two valid values of x : 2 and $\sqrt{2}$.

34. The question involves simple theorem that. a!b!, such that a + b = constant, is minimum and a=b if a + b = even.

35. If we put x = 0, we will get y = 2, so answer option a and d, ruled out. y = 3 is not possible from the given expression.

36. Take each answer option and check for values of a in the given range.
Only answer option d satisfies.

37. $\alpha + \beta = 1, \alpha\beta = 1/6$
$\frac{1}{2}(a + b\alpha + c\alpha^2 + d\alpha^3) + \frac{1}{2}(a + b\beta + c\beta^2 + \beta^3)$
$= \frac{1}{2}[2a + b(\alpha + \beta) + c(\alpha^2 + \beta^2) + d(\alpha^3 + \beta^3)]$
$= \frac{1}{2}[2a + b.1 + c\{1 - 2/6\} + d\{1 - 3/6\}]$
$= 1 + b/2 + c/3 + d/4$

38. Let Log x = y, then the given equation becomes $y^2 - 5y + 6 = 0$.
$\therefore$ y = 2 or y = 3. Hence Log x = 2 or Log x = 3. $\therefore x = e^2$ or $x = e^3$

39. $1, \omega, \omega^2$ are three cube roots of unity,
$\therefore \omega^3 = 1$ and $1 + \omega + \omega^2 = 0$
$1 - \omega + \omega^2 = -2\omega$
$1 - \omega^2 + \omega^4 = 1 - \omega^2 + \omega = -2\omega^2$
$1 - \omega^4 + \omega^8 = 1 - \omega + \omega^2 = -2\omega$. Thus each of the two consecutive terms pair up to give $4\omega^3 = 4$. For n/2 terms we have, 4. 4. 4. …..= $4^{n/2} = 2^n$

40. Minimum possible value of any expression inside mode is zero, so we will check for x = 3, -2, and x = 5. At x = 3 we will get minimum value, which is 7.

EXERCISE - 7

1. Put $e^{\sin x} = y$; the equation becomes $y - 1/y - 4 = 0$ or $y^2 - 4y - 1 = 0$
   $y = (4 \pm \sqrt{(16 + 4)})/2 = 2 \pm\sqrt{5}$
   $e^{\sin x}$ cannot be negative ,so only one value of x possible

2. Given expression is $\{[\sqrt{2} + 1)^2 - (\sqrt{2} - 1)^2] \ / \ 2\ (2 - 1)\}^{1/3}\ [4\sqrt{2}/2]^{1/3} = [(\sqrt{2})^3]^{1/3} = \sqrt{2}$.

3. Sol. (**a**)Since $3^2 = 9$ and $0^2 = 0$. The digit in the unit place of the sum must be 9 therefore the digit in the square root is 3 or 7. The sum of the squares of the digits at thousands place ≥ 32 and ≤ 50 $\therefore$ Ans. 6467.

4. Sol. **(a)**Let $S = 1^2 + 2^2/2! + 3^2/(2!)^2 + 4^2/(2!)^3 + 5^2/(2!)^4 + \ldots$.(1)

Put ½= x

$\therefore$ S = 1 + 4x + 9x^2 + 16x^3 + …….. (2)

Multiply (2) with x

$\therefore$ xS = x + 4x^2 + 9x^3 + ….. (3)

Subtracting (3) from (2) we get

(1-x) S = 1 + 3x + 5x^2 + 7x^3 + …….(4)

Multiply (4) with x

x(1-x) S = x + 3x^2 + 5x^3 + 7x^4 + …..(5)

Subtracting (5) from (4) we get

(1-x)^2 S = 1 + 2x + 2x^2 + 2x^3 + ….

= 1 + 2x/(1-x) = (1+x)/(1-x)

or S = (1 + x)/(1-x)

Put x = ½

S = (1 + ½/ (1-1/2)^3

$$= \frac{3/2}{1/8} = 12$$

5. Sol. Let a = x/(x-1) and b = x

Then a + b = x/(x-1)+ x = (x + x^2 – x)/(x-1) = x^2/(x-1)

Thus the given equation becomes $|a| + |b| = |a + b|$

But this hold if and only if ab $\geq$ o i.e. if and only if x^2/(x-1) $\geq$ 0 i.e. if x ε (0) U (1, ∝)

6. As product of terms equidistant from the centre is constant, $g_1\, g_{2n+1} = g_2\, g_{2n} = .. = g_{n+1},\, g_{n+1} = g_{n+1}{}^2 = 2500$.

7. The equation 3x^2 + 2 (a^2 + 1)x + (a^2 – 3a+2) = 0 will have two roots of opposite sign if it has real roots and the product of the roots is negative, that is,

If 4(a^2 + 1)^2 – 12(a^2 – 3a + 2) $\geq$ 0 and (a^2 – 3a + 2)/3 < 0

Both of these conditions are met if

a^2 – 3a + 2 < 0 i.e. if (a – 1) (a – 2) < 0 or 1 < a < 2

8. f(23) is the remainder when f(x) is divided by x - 23. Using Vedic Maths method of division,

23	1	-25	49	-73	87	113	54
		23	-46	69	-92	-115	-46
	1	-2	3	-4	-5	-2	8

Therefore, remainder is 8, which is f(23).

9. We can see from the data, that the man walks in all 5 km to the south and 12 km to the west, thus forming a right angled triangle of sides 5 and 12. So the shortest distance is 13 km.

10. $4^{x-5+b} = 2^{a+b} \times 2^{b} \times 2^{a-4} - 63$
$4^{(a+(b)-5} = 2^{2(a+(b)-4} - 63$
$4^{(a+(b)-5} - 4^{(a+(b)-2} = -63$
$4^{(a+(b)}-2[4^{-3} - 1] = -63$
$4^{(a+(b)-2}[1/64 - 1] = -63$
$4^{(a+(b)-2}[-63/64] = -63 \Rightarrow 4^{(a+(b)-2} = 64 = (4)^{3}$
or, a+(b) - 2 = 3 Therefore, a + (b) = 5

11. For all positive numbers,
AM $\geq$ GM $\geq$ HM
Equality holds good when numbers are equal. Here they are unequal.
Hence $\frac{(a+b)}{2} > \sqrt{ab} > \frac{2ab}{a+b}$

12. L (3.1) – H (6.1) = 3 – 7 = - 4, H (- 4) = - 4
H (7.3) – L (1.9) = 8 – 1 = 7, L (7) = 7 $\Rightarrow$ A[- 4, 7] = 1.5 Ans. A

13. $\because \frac{1}{x^2-1} = \frac{1}{x(x-1)} = \frac{1}{(x-1)} - \frac{1}{x};$

$\therefore \frac{1}{100^2-100} + \frac{1}{101^2-101} + \ldots\ldots\ldots\ldots + \frac{1}{(9999)^2-9999}$

$$= \left(\frac{1}{99} - \frac{1}{100}\right) + \left(\frac{1}{100} - \frac{1}{101}\right) + \left(\frac{1}{101} - \frac{1}{102}\right) + \ldots\ldots\ldots\ldots + \left(\frac{1}{9998} - \frac{1}{9999}\right)$$

$$= \frac{1}{99} - \frac{1}{9999} = \frac{100}{9999}$$

14. We have ((1-a)/a)((1-b)/c)((1-c)/c) = [1- (a+b+c) + bc + ca + ab – abc]/abc
1/a + 1/b + 1/c – 1 [as a+b+c = 1]
But since AM $\geq$ HM
1/3(1/a + 1/b + 1/c) $\geq$ 3/(a+b+c) = 3
we get 1/a + 1/b + 1/c $\geq$ 9
Thus ((1-a)/a)((1-b)/c((1-c)/c) $\geq$ 9 – 1 = 8

15. The nth term = (a – n) + i(b – n)
If the term is real, (b – n) = 0, b = n

16. At n = 1, we get the first term as a+b. At n = 2, we get the sum of first two terms as 2a+4b. Hence the common difference = (2a + 4b) – 2(a+b) = 2b.

17. Since a, c and b are in HP, c = 2ab/(a+b) = 2ab/2 = ab. Geometricmean of a and b is √(ab). Thus geometric mean of a, b is √c. Ans.a.

18. Let the nos. be a-2d, a-d, a, a+d , a+2d which are same as a/r^2, a/r , a, ar, ar^2. Solving these we get that r = 1 and d = 0. Which means that the progression is constant or each term is equal to a. Thus Ans.c.

19. simply plug in options or take $6x^2 – 5x$ as y and solve
(y+11) + (y-25) = 12, y + 11=12- (y-25)
squaring both sides,24(y-25) = 108, (y-25) = 9/2
squaring both sides, y = 181/4

20. Let speed of the escalator be x steps per second. ∴ In 30 second the escalator will come down by 30 x steps and in 18 seconds it will come down by 18 x steps. Since height of the stair way is same 26 + 30(x) = 34 + 18(x) ∴ x = 2/3.
∴ height of the stair way = 26 + 30 (2/3) = 46 steps.

21. 2/9, ¼2/7, 1/3…..
This is an H.P. series. The corresponding AP will be
9/2, 4/1, 7/2, 3/1, …..
or 4.5, 4, 3.5, 3…..
i.e. this is an AP with first term 4.5 and common difference – 0.5.
Hence, T_{11} = 4.5 + 10 (-0.5) = -0.5
The corresponding T_{11} H.P. is 1/(-0.5) = -2

22. a, b, c are in AP, so $2b = a + c \Rightarrow 4b^2 = a^2 + c^2 + 2ac$
$\Rightarrow 4b^2 – 4ac = a^2 + c^2 + 2ac – 4ac \rightarrow 4 (b^2 – ac) = (a – c)^2$
Discriminant of given equation is $4 (b^2 – ac) = (a – c)^2 > 0$
Therefore answer will be d.

23. $y + x^3 + xy = y^3 + y^2 + x \Rightarrow y^3 – x^3 + y^2 – xy + x – y = 0 \Rightarrow (y – x) (y^2 + xy + x^2 + y – 1) = 0 \Rightarrow x = y \therefore x^2 = y^2$.

24. The fraction can be written as F = (4 + c) (2 + c) / (5 + c)
Put 5 + c = t or c = t – 5
$\therefore F = (t – 3) (t – 1)/t = (t^2 – rt + 3)/t$
$= (t^2 – 4t + 4 – 1)/t = (t – 2)^2/ t – 1/t$
Hence, the given expression is minimum, if the square term is zero
∴(t-2)/t = 0 or t = 2

25. Let a boy has x brothers and x sisters. ∴ Number of the children in the family = 2x + 1. ∴ A girl in the family has (x – 1) sisters. ∴ Number of her brothers = 2(x – 1). Hence total number of children in the family = 3 (x – 1) +

$1 = 3x - 2$. $\therefore$ $3x - 2 = 2x + 1 \Rightarrow x = 3$. Total number of boys in the family = 4 and total number of girls in the family = 3.

26. $\alpha + \beta = (1/\alpha^2) + (1/\beta^2) = (\alpha^2 + \beta^2)/\alpha^2\beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]/\ ^2\beta^2$
$-b/a = [(b^2/a^2) - 2c/a]/(c^2/a^2) = (b^2\text{-}2ac)/c^2$
$2a/c = b^2/c^2 + b/a = ab^2 + bc^2/ac^2$
$2a^2c = ab^2 = bc^2$; $2a/b = b/c + c/a$ [dividing by abc]
c/a, a/b, b/c are in A.P.; a/c, b/a, c/b are in H.P.

27. Given that $xy + yz + zx = 5$. Since $(x - y)^2 \geq 0 \Rightarrow x^2 + y^2 \geq 2xy$. Similarly we get $x^2 + z^2 \geq 2xz$ and $z^2 + y^2 \geq 2\ zy$, adding them we get $x^2 + y^2 + z^2 \geq xy + yz + zx = 5$, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2\ (xy + yz + zx)$. As x, y and z are sides of a triangle satisfying $xy + yz + zx = 5$; Maximum value of $x + y + z$ is 5. Using this inequation, we get $x^2 + y^2 + z^2 \leq 15$.
$\therefore\ 5 \leq x^2 + y^2 + z^2 \leq 15$.

28. 99 # 1/99 = (99 x 1/99) + [1/(99 x 1/99)[= 1+1 =2. Similarly ½# 2 = 1+1 = 2.

29. c##d = (c + 1/c) (d + 1/d) = cd + c/d + d/c + 1/cd = (cd + 1/cd) + [c/d + 1/(c/d)] = (c#d) + (c#1/d).

30. Number of pounds of oranges will be maximum if it's cost is minimum i.e. 15 ps/pound. In the case number of pounds of the oranges = 500/15 = 33.33. Hence required answer = 33.

31. The two speeds are 500 and 200, and the mean speed is 1200/3 = 400. On alligation, the time ratio comes to 2:1, that is, 2 hrs at 500 and 1 hr at 200. So, he travels for two hrs at 500 km/hr to reach Rome from Pairs, which makes that distance 1000 km.

32. Suppose that initially Manu takes 6x toffees from the jar. Then he puts back 3x of them in jar and he admits to have taken only 2x. Thus there should have been 4x toffees in the jar but there are only 3x toffees. Thus x = 5. Thus initially Manu took 6x = 30 toffees.

33. $\cos(\log x) = 3^{2x} + 3^{-2x}$
$= (3^x)^2 + (3^{-x})^2 - 2 \cdot 3^x \cdot 3^{-x} + 2$
$= (3^x + 3^{-x})^2 + 2$
≥ 2 . hence, no soln exist since value of cos lies between +1 and -1

34 $T_{2r+1} = {}^{15}C_{2r} \cdot 1^{15-2r}\ (x)^{2r}$
$T_{r-2} = {}^{15}C_{r-3} \cdot 1^{18-4}\ (x)^{r-4}$
Given that the coefficients of these are equal.
$\therefore\ {}^{15}C_{2r} = {}^{15}C_{r-3}$
Either $2r = r - 3$ or $2r + r - 3 = 15$
$r =$ **- 3** or $r = 6$

Accepting the +ve value of r.

$T_6 = {}^{15}C_5\, 1^{10}\, x^5 = {}^{15}C_5 x^5$

35. If $a_1 + a_2 + \ldots + a_3 = k$ (constant), the value of $a_1\, a_2\, a_3 \ldots a_n$ is greatest when $a_1 = a_2 = \ldots = a_n$

So that the greatest value of $a_1 a_2 a_3 \ldots a_n$ is $[k/n]^n$

Given yz + zx + xy = 12 (constant), the value will be greatest when yz = zx = xy

$\therefore$ yz = zx = xy = 4

So greatest value of (yz) (zx) (xy) is 4.4.4.

Hence greatest value of xyz is 8

36. $x \geq 0$; $\sqrt{(x + 3)} \geq \sqrt{3} > 1$, so the value of expression will be more than 1, so no solution is possible.

37. i). $7^7 > 1.3.5.7.9 \ldots.13$

this is in the form of $n^n > 1.3.5\ldots (2n - 1)$; substitute small values of n and see this is always true.

ii). $2^n > 1 + n\sqrt{2^{n-1}}$

check by substituting the values of n

iii). $1/(7+1) + 1/(7+2) + \ldots. + 1/(2.7) > ½$

Now $1/(n + r) > 1/n\,(2n)$ for r = 1, 2, 3, …..(n – 1). Therefore

$1/(n + 1) + 1/(n + 2) + \ldots + 1/(n+n) > 1/(2n) + 1/(2n) + \ldots + 1/(2n)$ (n times) = $n/(2n) = ½$

38. For such problems we can substitute the values of n

Put n = 1, then Sum of one term of series is (2n – 1) = 1, put n = 2

Then sum of two terms of series is (2n – 1) + 2 (2n – 3) = 5.

When we put n = 1 and 2 in option b, we will get 1 & 5 respectively. So answer will be b

.

39. $(a + 1/a)^2 = 3$ or $(a + 1/a) = 3^{1/2}$ and $(a + 1/a)^3 = a^3 + 1/a^3 + 3(a + 1/a) \Rightarrow (a^3 + 1/a^3) = 3\sqrt{3} - 3/3 = 0$

40. $x/y = (a_2 + a_4 + \ldots\ldots + a_{400}) / (a_3 + a_5 + \ldots. + a_{401})$

$= (a_2 + a_4 + \ldots.. + a_{400}) / \{r\,(a_2 + a_4 + \ldots. + a_{400})\}$

$y/x = r$

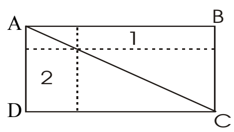
EXERCISE - 8

For Q1- Q4

Let x be the number of students playing all the games.
$\therefore$ Number of students playing only cricket = 20- (15-x) –x –(10-x) = x –5
Number of students playing only football = 25 – (15-x) – x – (12- x) = x – 2
Number of students playing only TT = 18 – (10 – x) - x – (12 – x) = x – 4
$\therefore$ (x – 5) + (15-x) + (x – 2) + (10 –x) + (12 – x) + (x – 4) = 33
$\therefore$ x = 7. Hence,

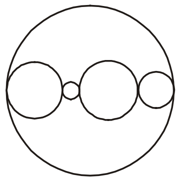
1. c
2. b
3. a
4. c

5.



OQCS and APOR are also rectangles. The diagonal OC divides the area into equal halves. Therefore, area of OQC = area of OSC. Similarly, area of APO = area of ARO. Since ABCD is a rectangle and diagonal AC divides it into equal areas, Area of ADC = area of ABC. Therefore, area 1 = area 2, no matter where is the point O located.

6.



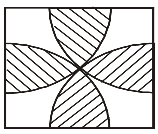
Radius of the outermost circle, let us say, is r and the side of the largest triangle (which will be equilateral) that can be inscribed in it is a. Then,

$r = \left(\frac{2}{3}\right)\frac{\sqrt{3}}{2}a = \frac{a}{\sqrt{3}}$ or $a = r\sqrt{3}$. Therefore, area of the triangle

$= \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{4}r^2 = \frac{3\sqrt{3}}{4}$. Therefore, r = 1 cm.

Let the radius of the inner circles be r_1, r_2, r_3 and r_4. Then,
$2r_1 + 2r_2 + 2r_3 + 2r_4 = 2r$ or $r_1 + r_2 + r_3 + r_4 = r = 1$.
Total circumference of all the 5 circles = $2\pi(r_1 + r_2 + r_3 + r_4 + r) = 4\pi$

7.



Qn 6

Unshaded area, x inside the square

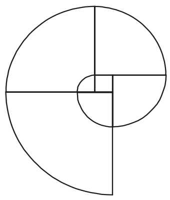
$= \frac{1}{2}$(area of square – areas of 2 vertically opposite quarter circles)

$= \frac{1}{2}\left(100 - 2 \times \frac{\pi(5\sqrt{2})^2}{4}\right)$

Required area = (area of square – 4x) = 50π – 100.

8. It is quite obvious p-q = PQ

9.



Radius of the smallest quarter circle is 2 cm. Therefore, its length = $\frac{2\pi r}{4} = \pi$.The radius of next level quarter circle is 2 + 2 = 4 cm, the next level radius = 2 + 4 = 6, etc. Therefore, radii of the quarter circles starting with the smallest one are 2, 4, 6, 8, 10 and 12.

Total length of the curve = $\frac{2\pi}{4}$ (2 + 4 + 6 + 8 + 10 + 12) = 21π.

10. The total no. of arrangements in which the letters of the word SUNRISE can be arranged are 7!/2!. The no. of arrangements in which the vowels are always together are 5!/2!.
Hence, the no. of ways in which they are never together are 7!/2!. –5!/2!. But these arrangements include the arrangement SUNRISE hence we need to subtract this one.

11. The smallest radius means the smallest circumference. The circumference is $2\pi r$. With six points equidistant with distance between them to be integer, this integer has to be the smallest, i.e., 1. Therefore, circumference is 6 units. Therefore, $2\pi r = 6$ or $r = 3/\pi$.

12. The height of the cliff will be 200m. The side opposite to the 30^0 angle will be $200\sqrt{3}$. Thus the height of the tower will be $200 - 200\sqrt{3}$.

13 $2/3^{rd}$ of the post is initially filled with water therefore the total volume of the pebbles is equal to $1/3^{rd}$ of the volume of the pot. Thus if r is the radius of the pot then $1/3 \times 4\pi r^3/3 = 576 \times 4/3\Pi R^3$. Subtituting R = 1, we get r = 12. Thus answer a.

14. Since area of circle with BC as radius is 154 sqcm., $\Pi(BC)^2 = 154$, so, BC = 7 cm. Triangles ABC and ADE are similar triangles. Therefore, AB/AD = BC/DE. As BC = 7 and DE is 14 cm, BC/DE = ½so AB/AD must also be ½ So, as AD = 15, AB must be 7.5 cm. Thus Ans.d

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15. Let the corresponding sides of the other triangle be a, b, and c.
$\therefore a/2 = b/3 = c/4 = (a+b+c)/9 = 81/9 = 9$ $\therefore$ Required sides are 18 cm, 27 cm and 36 cm.

16, Let the no. be ababa
Given a + b = 10
And $a^3 b^2$ has to be maximum
$\therefore$ a/3 = b/2
$\Rightarrow$ a : b = 3 :2
Hence, the no. is 64646

17. Let radius of the circle be r $\therefore$ PT = 3r/2 and TQ = r/2.
$\therefore$ (3r/2) (r/2) = 6 x 2 = 12. $\therefore$ $3r^2/4 = 12 \Rightarrow r^2 = 16$. $\therefore r = \pm 4$. Hence the diameter of the circle is 8 cm. But it is impossible to draw a chord of length 8 cm (other than the diameter) in a circle of diameter 8 cm. The circle must be imaginary.

18. In these type of problems, the place that is more restricted should be filled first. Unit digit place can be filled in two ways (by 3 or 5 only) while the hundred's place can be filled in three ways (by 2 or 3 or 5).
Therefore starting from the unit's place we can form the nos. in 2 x 2 x 2 ways.

19. There are 13 letters of which Himanshu knows position of three correctly. This means that now he has to arrange only 10 letters. No. of ways in which these 10 letters can be arranged are 10!/(3!.3!). Out of these there is only one

arrangement of correct message. So the chance that he deciphers the message correctly is $\frac{1}{10!/(3!3!)}$

20. The five digit numbers (not containing zero) is ${}^{9}C_{5}.{}^{4}C_{2}$ (the greatest of the five digits gets fixed at the central place) and those containing zero ${}^{9}C_{4}.{}^{3}C_{2}$ (zero will be the last digit).